



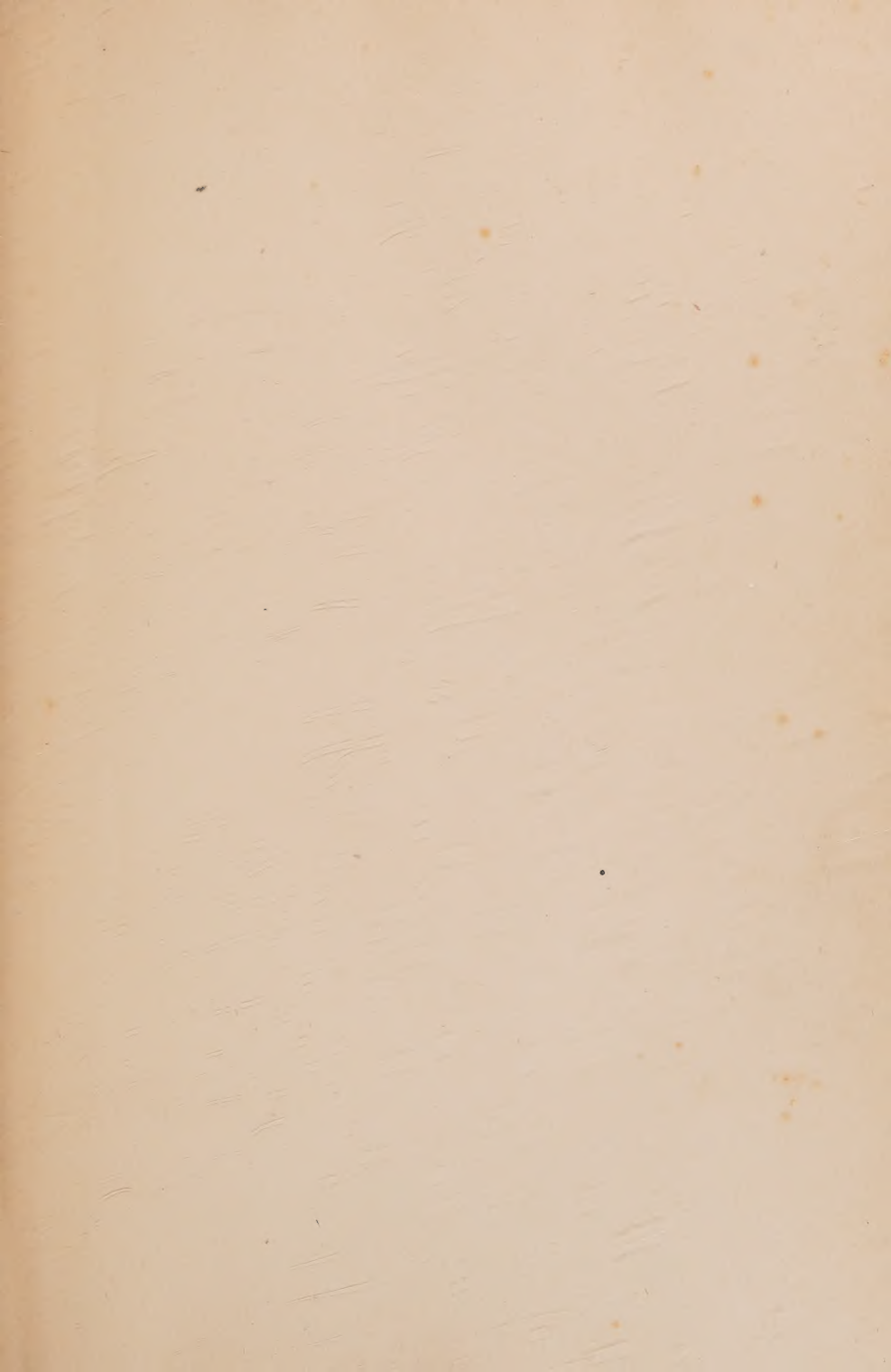
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BUSINESS ARITHMETIC

REVISED
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B U S I N E S S A R I T H M E T I C

By

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FOR USE IN ALL SCHOOLS OFFERING
THE SUBJECT OF BUSINESS ARITHMETIC

SECOND EDITION

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P R E F A C E

The purpose of this text is to serve as a basis for a course in the arithmetic needed by the individual and the business man in the solution of problems arising from the completion of business transactions. The fundamental principles of arithmetic are applied in the same manner as in all arithmetics. Special emphasis is given to accuracy and speed in the application of these principles.

The authors have undertaken to make the text interesting and instructive by:

- (1) The selection and arrangement of practical and interesting material.
- (2) The use of practical problems for illustrating the application of the principles.
- (3) A change in the approach to the various subjects.
- (4) A discussion of the business questions covering the problems of business transactions.

Problems have been selected that correlate with the everyday activities of school, home, society, and business. These problems are presented in exercises, each of which relates to a specific phase of arithmetic. A model solution is provided in connection with each group with a view of emphasizing the systematic solution of the problems and the use of efficient methods in the solution. Sufficient problems are provided to insure accuracy and speed, the two important elements in the solution of business problems.

The discussion in the text is arranged in four principal groups. The first group relates to the fundamental principles, the second to fractions, the third to percentage, and the fourth to interest. Each group contains two or more chapters, one or more of which consists of problems requiring the application of the principles discussed.

The necessity for proof of mathematical calculations and the means of effecting this proof is developed early in the course.

Accuracy in the solution of mathematical problems applicable to industry is essential to successful business operation. The business man must know the facts regarding the operations of his business and he can know that the facts submitted to him are correct only when their correctness has been proved by a reasonable test. Speed in the solution of mathematical problems can be obtained only through continued practice and quick thinking, hence the necessity for numerous oral drills in an arithmetic which emphasizes accuracy and speed.

Fractions are necessary in business calculations. Special emphasis is given to this phase of arithmetic. Seven chapters are devoted to common and decimal fractions, with two chapters of problems requiring their application to business transactions.

Percentage and interest, which are applications of decimal fractions, are thoroughly developed in five chapters, two of which are practical problems. Three chapters are devoted to Banking and Interest, Trade and Cash Discount, and Commission and Brokerage.

The purpose of the revision was to group the problems in exercises, each exercise relating to a specific phase of arithmetic. All of the problems have been brought up to date, and many new problems added. The interest manifested by our friends through helpful suggestions and problems is very much appreciated. Their cooperation has helped to make the text more teachable and more interesting.

THE AUTHORS

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CHAPTER I

INTRODUCTION

As soon as primitive man began to compare one group of objects with another group he began to observe and think in numbers. Very soon, then, he sought to express his number-thoughts, and having neither words nor figures for the purpose he very naturally used his fingers and toes, or made *marks*, *scores*, just as the baseball umpire of today uses his fingers to indicate *strikes* and *balls* and keeps the *score* by *tallies*; thus, $\text{||||} \text{|||}$ ($=8$).

The naming of each unit in a succession probably came next. Counting naturally developed along the *ten* or *decimal* scale. If man had had twelve fingers instead of ten it is quite likely that all counting would go in twelves instead of tens.

The greatest advance step in Arithmetic came with the selection of signs for the different groups, and the perfection of our present system of writing and reading numbers. Many explanations have been offered concerning the origin of the ten digits,* figures. The following appears to be the most probable, and is quite acceptable. Certain arbitrary forms were selected, and their values known by the number of angles.

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

It is easy to understand how these awkward angular forms came to be written as they are today. Some of the old earmarks remain, however, for we often see the figure *one* with its flag at the top, and many of the latin peoples still flag the staff of the *seven*.

The idea of *place values* was a very remarkable discovery and has meant much for civilization. That a 2 in the first position should mean two; in the second position twenty, etc., seems

*Digits originally meant fingers.

simple to us today, but think how long it would take you to develop the plan for yourself.

Having so far mastered the problem of counting and numbering, man turned to the comparison of distance. He found it necessary to have a unit of measure as a basis, and having nothing so unvarying or so handy as his own body, he used *it*. Distance, he measured in foot-lengths, *feet*; or in step-lengths, *paces*; in hand-breadths, *hands*; or hand-reaches, *spans*; in elbow-to-finger-tips, *cubits*; or finger-tip-to-finger-tips, *fathoms*.

From time to time, standards of measurement have been set up as the progress of civilization required. *Area* began to be measured when land became scarce; *capacity*, when drink other than water came into use, and *weight* when precious metals and other commodities were exchanged in trade. The last came slowly, however; only in very recent years have fruits been sold by the pound.

It is interesting to notice the development of measurements. Today we measure the *air* pressure and determine altitudes, measure its density and predict the weather; we measure the temperature, and the speed of the wind—in fact the speed of everything that moves. *Power*, too, is measured; the word *horse-power* stands as a monument to the passing horse; steam has its *pounds-pressure* and electricity, its *volts*, *amperes*, *ohms* and *watts*.

Measuring has become so nearly a fine art that variations of more than a thousandth part of an inch in machine and tool manufacturing are practically out of the question. Mechanical devices for making these measurements have been invented, also numerous machines for the common computations of business. The automatic reel has replaced the old yardstick at the dry goods counter; the automatic measuring pump fills the gasoline tank, and the adding-calculating machine does all the detail figuring of the office.

BUSINESS ARITHMETIC

Fundamentally, arithmetic consists of but three general processes: the *increasing*, *diminishing*, and *comparison* of numbers. These are included in the four fundamental operations: addition, subtraction, multiplication, and division. Through the process of addition, numbers are increased; through the process of subtraction, numbers are decreased; through the processes of multiplication and division, numbers are compared.

Business Arithmetic does not involve any new mathematical principles; it refers to the application of the four fundamental operations to the solution of business problems. These problems arise from the completion of business transactions which occur in the operation of a business. The results affect the individual as well as the business. You need a knowledge of the arithmetic of business, because of the business transactions which you will perform while attending school, and after you have finished school and entered the business world.

The first essential in business arithmetic is accuracy, and the second is speed. The application of the principles is not sufficient for you or for the business man, unless the results are correct. The value of your services to the business world will depend, to a great extent, upon your ability to apply accurately and quickly the fundamental operations of arithmetic to the solution of business problems.

Mention is made in this text of the procedure used by bookkeepers for recording transactions so that the results may be proved, because practically all transactions you perform now, or will perform when you enter the business world, will be affected by the application of this procedure. The correlation of business arithmetic with the problems of bookkeeping will help you to better understand arithmetic, and to appreciate the necessity for accuracy in all mathematical calculations. A knowledge of the fundamental principles of bookkeeping is not necessary in order to understand business arithmetic, but a knowledge of business arithmetic, and accuracy in its application, is essential to the successful completion of a course in bookkeeping.

The ability to reason is necessary in the solution of all business problems. A knowledge of the principles involved, the four fundamental operations of arithmetic, and even the ability to apply these quickly and accurately, does not always indicate the ability to apply them to the solution of business problems. You may know how to add, subtract, multiply, and divide, and be able to perform each process with accuracy and speed, but this ability might not enable you to apply these principles to the statements of a real estate agent regarding the purchase of a home, and the saving that might be effected thereby. As a merchant you might apply these fundamental operations to each sale of merchandise with the assurance that the results will be correct,

but it will require reason to apply the results of these sales to the purchasing of merchandise with which to supply future customers.

Memory should be developed only in connection with the meaning of arithmetical signs and terms, and such tables as may be necessary in order to make an application of your problems to denominate numbers, interest, and the fixed ratio existing between diameter, circumference, etc. The application of the information given in these tables will require a knowledge of the principles. If you undertake to memorize the application of these principles to all the fundamental operations, your knowledge of arithmetic will be limited to the length of your memory. It may be possible to have access to the tables which contain the information to be memorized, but it is not always possible to have access to information regarding the principles. The one who is thoroughly familiar with the fundamental principles of arithmetic and understands the reason for short-cuts can always apply them, but the one who memorizes them from the statements given in the text without knowing the reason will find them difficult to remember.

Machines are available for the solution of practically all mathematical problems resulting from business operations but no machine has brains, hence the brains of the operator alone can make its operation successful. A knowledge of business arithmetic is necessary in order to successfully operate any machine provided for mathematical calculations. The phonograph, player piano, and radio have not eliminated the trained musician; the calculating machine has not eliminated the trained mathematical mind.

With the exception of a few problems necessary to emphasize the principles, all the exercises in this text relate to mathematical problems which have arisen through the completion of business transactions. If you will visualize these problems and place yourself in the position of those connected with the transactions from which the problems arose, you will find arithmetic more interesting and instructive. Even now you are performing transactions which require the application of business arithmetic for their solution. When you enter the business world to earn your own living, you will be continually performing transactions which require a knowledge of business arithmetic, no matter whether you choose your occupation from the professions or business.

CHAPTER II

ADDITION

The purpose of this chapter is to provide practice in addition through the application of some of the more popular short-cut methods, also to provide information regarding proof of the sum, or total. All the information given is based on the assumption that you understand the mathematical principles involved in connection with addition.

Addition is the process of finding the sum of two or more numbers, and expressing this sum in one number. The *sum*, or as it is often referred to in business, the *total*, is the number which expresses the value of the two or more numbers.

The amounts added, and the sum should refer to the same thing. If it is desired to ascertain the weight of three barrels of sugar, and the weight of each barrel is given, the sum of these three amounts would be the total quantity of sugar.

Combination. The process of addition is greatly facilitated by combining figures. Accuracy is essential to addition, but speed is necessary to secure accuracy and efficiency. Combinations will effect both accuracy and speed in addition.

At the top of the next page is given a list of the forty-five two-figure combinations that may be used in addition. There are nine hundred possible combinations in adding any one of the numbers, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to numbers from one to one hundred. These nine hundred combinations include the forty-five two-digit combinations given in the table. If practiced diligently, they will be of great aid in rapid, accurate addition. You are not expected to memorize any of the combinations, but to practice combining figures until they can be read as readily as words.

With practice, you can learn to read the sum of four or five digits without mentally repeating the sum of each two digits. To acquire efficiency in combining more than two digits, it is necessary to begin with two digits and add additional digits as you become proficient.

EXERCISE No. 1, COMBINATIONS

The following problems provide oral practice in ascertaining the sum of two digits. Problem 1 shows the forty-five two digit

1. Two-figure combinations: combinations arranged in regular order; in problems 2-7, the combinations are arranged in a different order. Practice adding these until you can read the sum of any two digits.

1. Two-figure combinations:
 1 2 3 4 5 6 7 8 9
 1 1 1 1 1 1 1 1 1
 - - - - - - - - -

2 3 4 5 6 7 8 9
 2 2 2 2 2 2 2 2
 - - - - - - - -

3 4 5 6 7 8 9
 3 3 3 3 3 3 3
 - - - - - - -

4 5 6 7 8 9
 4 4 4 4 4 4
 - - - - - -

5 6 7 8 9
 5 5 5 5 5
 - - - - -

6 7 8 9
 6 6 6 6
 - - - -

7 8 9 2. 2 5 3 4 1 3 2 4 1 3
 7 7 7 2 4 1 1 3 2 3 1 4 3
 - - - - - - - - - - - -

8 9 3. 1 5 4 3 5 6 1 4 5 7
 8 8 5 1 3 4 2 1 6 4 3 1
 - - - - - - - - - - -

9 4. 4 5 6 7 8 4 2 8 3 5
 9 5 4 3 2 1 5 7 1 6 4
 - - - - - - - - - -

5. 7 8 9 6 7 8 4 9 6 5 5 6
 4 3 2 6 5 4 8 4 7 4 6 5
 - - - - - - - - -

6. 7 8 8 8 9 7 9 8 5 6 7 8
 7 7 8 9 8 6 4 9 9 7 9 7
 - - - - - - - - -

7. 2 3 4 3 8 7 6 5 9 3 3 4
 9 7 5 6 1 8 6 5 9 9 8 7
 - - - - - - - - -

When we first began to read we said b-o-y-s, *boys*, r-u-n, *run*. Now we read much faster for we have stopped spelling the words.

Why say "2 and 3 are 5 and 4 are 9," when it is just as easy to see that 2, 3, 4 are 9 as it is to see that r-u-n is *run*. Read your results.

EXERCISE No. 2, COMBINATIONS

The following problems provide oral practice in combining two figures the sum of which is ten. These combinations occur quite frequently, hence their use will mean speed and accuracy in addition.

1.	7	6	5	3	8	4	9	6	2	1	7	5
	3	4	5	7	2	6	1	4	8	9	3	5
	-	-	-	-	-	-	-	-	-	-	-	-
2.	1	8	6	9	4	8	3	5	6	7	6	2
	9	2	4	1	6	2	7	5	4	3	4	8
	-	-	-	-	-	-	-	-	-	-	-	-
3.	6	7	5	3	8	4	9	6	8	1	3	5
	4	3	5	7	2	6	1	4	2	9	7	5
	-	-	-	-	-	-	-	-	-	-	-	-
4.	5	8	7	6	3	2	7	9	8	4	5	6
	5	2	3	4	7	8	3	1	2	6	5	4
	-	-	-	-	-	-	-	-	-	-	-	-

EXERCISE No. 3, COMBINATIONS

The following problems provide oral practice in combining three or more figures, the sum of which is ten. With practice, the eye can grasp three figures, or even four, as well as two. The ability to see three or four figures which equal ten will enable you to attain greater speed when adding a column of figures.

1.	4	2	5	1	3	6	3	4	3	1	7	1
	5	3	1	6	4	3	2	1	6	8	2	5
	1	5	4	3	3	1	5	5	1	1	1	4
	-	-	-	-	-	-	-	-	-	-	-	-
2.	7	1	3	4	3	2	5	6	7	3	5	1
	2	8	6	1	2	3	4	1	1	6	2	1
	1	1	1	5	5	5	1	3	2	1	3	8
	-	-	-	-	-	-	-	-	-	-	-	-
3.	8	5	7	1	2	6	5	7	4	3	1	6
	1	2	2	8	3	1	3	1	3	1	8	3
	1	3	1	1	5	3	2	2	3	6	1	1
	-	-	-	-	-	-	-	-	-	-	-	-
4.	1	2	1	4	3	5	2	1	3	2	1	2
	2	1	3	1	2	1	3	5	4	1	3	1
	2	4	2	3	2	2	3	2	2	4	1	2
	5	3	4	2	3	2	2	2	1	3	5	5
	-	-	-	-	-	-	-	-	-	-	-	-

NOTE: It is customary to add from the bottom to the top and when re-adding for proof, to add from the top to the bottom. You are to follow this method unless otherwise instructed.

The following problems provide oral drill in combining three or more figures, the sum of which does not equal ten.

EXERCISE No. 7, COMBINATIONS

The following problems provide oral drill in combining two figures the sum of which equal ten with two figures the sum of which may or may not equal ten.

1.	6	3	4	5	6	2	1	3	5	8	7	2
	4	7	3	8	3	8	9	4	5	2	5	8
	5	8	5	3	4	8	3	6	6	5	5	4
	8	2	5	7	6	3	7	4	8	7	5	3
	—	—	—	—	—	—	—	—	—	—	—	—
2.	4	5	5	6	6	3	8	4	3	5	8	5
	3	5	7	8	4	7	3	6	7	5	2	8
	8	5	2	5	4	9	8	3	8	3	7	4
	2	7	8	5	5	3	2	4	2	8	6	6
	—	—	—	—	—	—	—	—	—	—	—	—
3.	5	7	8	4	7	3	6	7	5	2	8	3
	5	5	6	6	3	8	4	3	5	8	5	4
	8	3	6	8	3	9	2	4	8	4	7	3
	2	7	4	1	8	1	8	5	2	7	3	7
	—	—	—	—	—	—	—	—	—	—	—	—
4.	3	9	4	3	7	8	7	3	1	7	6	7
	2	1	6	8	3	9	3	9	9	3	7	9
	8	1	4	6	9	4	8	9	7	3	5	5
	7	6	9	4	1	6	8	1	8	9	5	5
	—	—	—	—	—	—	—	—	—	—	—	—

EXERCISE No. 8, COMBINATIONS

The following problems provide oral or written practice in combining two figures when adding a column of figures.

The brackets in the column of figures at the left indicate the combinations and the numbers to be read mentally as the column is added. Thus, the addition in this column would be expressed mentally as 10, 20, 27, 38.

Place a sheet of tablet paper just below the ruled line, and write your total on this sheet. Do not write in the text.

1.		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>
7	} 11	5	4	2	5	8	4	3	9	6	8
4		4	9	9	7	4	3	9	1	4	2
2	} 7	5	6	4	8	4	9	3	9	9	5
5		4	4	6	5	6	1	7	3	8	6
9	} 10	4	6	2	9	5	8	5	4	7	7
1		6	4	8	1	3	2	4	7	3	3
7	} 10	7	5	1	6	7	7	3	6	7	8
3		3	5	4	4	8	5	6	4	6	1
		—	—	—	—	—	—	—	—	—	—

EXERCISE No. 10, COMBINATIONS

When adding a column consisting of two or more figures, the figures in each column should be combined. When the sum of the figures in the units column is more than ten, the excess tens are carried to the tens column, and included in the first combination of that column.

$$\begin{array}{r} 10 \left\{ \begin{array}{c} 85 \\ 25 \end{array} \right\} 10 \\ 12 \left\{ \begin{array}{c} 96 \\ 34 \end{array} \right\} 10 \\ 15 \left\{ \begin{array}{c} 84 \\ 73 \end{array} \right\} 7 \\ 13 \left\{ \begin{array}{c} 36 \\ 74 \end{array} \right\} 10 \\ \hline 507 \end{array}$$

The method of combining in each column is shown in the illustration at the left. The two figures which are combined are indicated by brackets, and the sum at the right or left of the brackets. The mental process when adding the first column should be "10, 17, 27, 37". Write the 7 and carry the 3. The process in the second column is "13, 28, 40, 50".

The following problems provide written practice in adding two-figure numbers. Write only the sum. Do not write in the text.

1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
	17	29	22	86	36	24	74	63	14	55	33	47
	28	81	88	25	74	56	56	47	26	75	72	23
	36	46	36	47	75	38	38	57	78	23	28	68
	16	83	14	23	67	72	72	68	39	87	16	29
	92	37	93	65	98	81	21	22	23	68	46	37
	85	83	17	54	16	24	84	86	45	97	34	51
	63	14	35	72	22	75	75	43	78	36	48	42
	48	35	26	38	36	35	35	77	46	74	92	14
	23	89	85	36	41	98	41	96	94	26	73	33
	87	21	65	74	69	89	69	48	82	34	39	23

2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
	12	56	98	28	36	88	96	47	78	28	52	81
	98	58	89	49	74	92	14	63	32	82	58	29
	53	26	53	46	57	14	36	38	48	43	99	34
	41	35	75	78	68	36	22	72	63	67	14	76
	38	72	26	54	22	68	61	54	58	74	47	69
	72	79	84	32	88	45	89	76	92	36	35	41
	82	14	21	68	63	25	67	32	14	79	82	85
	46	13	72	55	47	63	75	47	34	31	28	25
	81	87	45	66	37	47	15	57	93	33	45	82
	29	78	65	44	73	18	95	53	83	77	66	18

EXERCISE No. 11, COMBINATIONS

The following problems provide written practice in combining three figures in a group.

1.		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>
	$\left\{ \begin{array}{l} 27 \\ 32 \\ 51 \end{array} \right\} 10$	72	32	83	91	56	26	42	56	67	82
		43	56	75	36	13	65	33	24	22	34
		15	42	12	24	44	89	95	31	31	69
		63	12	34	82	97	31	24	82	45	13
	$\left\{ \begin{array}{l} 21 \\ 24 \\ 67 \end{array} \right\} 12$	14	32	33	16	41	25	12	34	29	75
		33	68	48	12	32	54	74	44	44	22
		21	73	66	74	45	32	21	67	89	96
		66	15	32	12	52	47	61	12	38	23
	$\left\{ \begin{array}{l} 96 \\ 23 \\ 31 \end{array} \right\} 10$	23	22	12	24	13	85	28	31	42	41
		82	86	48	59	98	55	23	96	25	75
		74	14	65	33	56	21	39	43	51	63
		54	19	23	24	62	34	95	28	34	52
	372										

2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
	67	82	59	26	91	18	24	43	76	54	67	19
	22	34	93	65	41	31	33	15	13	35	32	83
	31	69	32	89	68	61	96	52	21	21	45	14
	45	31	82	13	22	98	24	42	79	87	55	82
	29	75	16	52	15	43	12	38	14	63	23	67
	44	22	13	45	73	62	74	89	32	42	32	41
	89	96	74	32	86	21	21	42	13	25	86	25
	38	23	12	41	23	57	61	29	52	64	45	87
	42	41	24	97	21	32	28	45	45	91	23	23
	25	75	59	44	24	78	32	22	67	31	59	21
	51	63	33	31	56	86	93	67	22	12	94	77
	34	52	42	56	32	57	55	31	31	88	38	12

3.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
	64	78	78	56	32	47	56	98	45	85	25	83
	46	14	56	34	78	73	82	12	82	13	58	14
	91	72	45	32	13	83	78	47	37	78	52	82
	78	37	65	78	85	92	56	47	73	16	97	74
	32	57	29	15	67	18	74	35	14	89	75	14
	14	83	81	95	34	41	36	92	83	92	35	22
	96	27	64	34	76	73	14	89	98	67	48	36
	76	16	37	76	39	85	82	36	89	43	72	82
	54	94	85	13	61	25	67	74	37	93	57	18

EXERCISE No. 12, WRITING THE SUM

When adding a column containing many numbers with two or more figures in each number, it is advisable to indicate the sum of the figures in each column as it is added. This plan avoids the necessity of re-adding columns in case the one who is adding is disturbed during the process of addition.

4827	In the illustration at the left the sum of each column is written at the bottom, and arranged so that they may be added, thus avoiding carrying the excess.	2038
9638		5839
4924		6382
3725		1487
5125		3154
6849	In the illustration at the right the excess units are carried when adding the tens column, and the excess tens carried when adding the hundreds column. The total of each column is written at the right, units first, tens second, etc. The sum is the top number, and the right-hand figures in the numbers below it. In either method, the one adding knows the total of the last column added if he is disturbed before he has finished.	5537
7524		3944
3579		1458
2845		4157
2454		3648
60		3148 59
33		4958 68
61		6178 82
45		7893 91
51490		59821

The following problems provide written practice in addition. Write on tablet paper the totals in problem 1 as in the illustration at the left, and in problem 2 as in the illustration at the right.

1.	a	b	c	d	e	f
	28,427	18,572	38,725	37,928	27,846	49,385
	78,492	61,424	49,281	41,672	52,317	42,345
	27,463	37,296	30,427	37,097	74,283	97,485
	95,927	75,735	35,729	25,562	37,208	38,475
	42,379	82,046	86,428	37,981	52,172	94,857
	28,446	27,593	72,823	27,439	78,539	38,476
	48,937	41,275	67,129	41,924	37,296	98,723
	29,452	39,256	43,552	27,096	73,814	78,454
	38,247	52,938	27,949	35,243	28,359	59,287
	40,542	43,725	37,472	17,241	46,294	51,823

2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
	34,857	78,394	56,783	79,385	69,456	43,756
	78,923	23,785	54,238	78,365	31,584	41,385
	34,867	87,325	48,375	67,745	74,837	81,385
	92,867	89,486	98,736	32,286	82,937	29,487
	47,385	47,832	23,474	11,374	14,658	45,179
	65,784	63,258	87,264	18,476	24,158	38,576
	98,375	89,375	23,645	47,647	14,785	14,783
	63,847	78,263	89,465	63,463	53,856	37,859
	34,576	45,732	45,784	42,374	56,793	72,364
	74,573	65,325	74,327	74,583	54,387	37,462
	46,681	45,723	81,375	84,632	73,487	38,344
	<u>23,437</u>	<u>38,473</u>	<u>47,435</u>	<u>26,431</u>	<u>38,983</u>	<u>89,483</u>

EXERCISE No. 13, PROGRESSIVE ADDITION

Progressive addition gives excellent practice in combining numbers. The following drills provide oral practice in progressive addition. You are to add in each case to approximately 100.

1. Begin with 3 and count by 2's; 4's; 6's; 8's
2. Begin with 2 and count by 3's; 5's; 7's; 9's
3. Begin with 9 and count by 5's; 7's; 9's; 3's
4. Begin with 11 and count by 4's; 6's; 8's; 2's
5. Begin with 14 and count by 6's; 2's; 4's; 8's
6. Begin with 19 and count by 7's; 3's; 5's; 9's
7. Begin with 16 and count by 9's; 7's; 5's; 2's
8. Begin with 11 and count by 8's; 2's; 4's; 6's
9. Begin with 13 and count by 3's; 5's; 7's; 9's
10. Begin with 14 and count by 7's; 3's; 5's; 9's
11. Begin with 8 and count by 9's; 5's; 7's; 3's
12. Begin with 5 and count by 7's; 3's; 5's; 9's
13. Begin with 6 and count by 2's; 4's; 6's; 8's
14. Begin with 4 and count by 5's; 3's; 7's; 9's
15. Begin with 9 and count by 8's; 2's; 4's; 6's
16. Begin with 7 and count by 6's; 4's; 6's; 8's
17. Begin with 6 and count by 4's; 6's; 2's; 8's
18. Begin with 8 and count by 7's; 3's; 5's; 7's
19. Begin with 7 and count by 3's; 5's; 3's; 7's
20. Begin with 5 and count by 8's; 4's; 2's; 6's

EXERCISE No. 14, COMBINATIONS

The following columns are to be added and the total written on a sheet of paper. Use either of the methods illustrated in Exercise No. 12.

1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
	3,156	7,894	8,745	6,734	7,145	8,947	5,638	1,385
	4,287	3,857	1,389	1,378	3,785	2,938	3,847	5,748
	3,159	3,756	1,048	4,582	1,482	5,647	2,947	9,283
	8,937	9,283	4,873	1,375	6,628	9,283	5,647	6,743
	4,278	4,756	2,126	8,145	1,385	6,473	3,847	9,827
	1,385	2,847	4,111	6,137	1,748	6,758	3,892	7,465
	6,734	1,478	8,148	4,973	3,362	4,637	5,648	7,485
	7,836	7,831	8,375	1,375	3,785	1,384	4,635	3,847
	9,456	3,279	1,375	9,725	9,837	9,726	2,938	5,647

2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
	8,356	9,834	9,316	5,831	5,748	5,167	2,938	4,158
	9,283	4,278	7,823	9,415	1,378	5,748	8,392	3,482
	1,478	1,582	1,287	4,837	7,745	1,483	5,647	3,128
	5,613	9,528	1,476	9,274	8,364	9,625	3,847	2,321
	9,356	4,735	5,614	1,457	5,354	7,846	4,583	5,661
	7,649	9,256	7,215	8,765	8,238	3,264	1,483	3,869
	5,167	4,175	8,475	2,736	1,574	1,475	9,137	9,384
	7,395	3,441	6,748	5,647	9,536	9,147	5,746	8,956
	4,576	3,857	6,364	9,375	1,574	5,713	3,847	1,478

3.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
	7,345	5,893	7,458	5,784	7,483	5,476	5,473	6,487
	8,246	6,587	5,673	8,473	5,674	5,352	6,432	6,837
	2,564	4,523	9,375	6,432	8,345	8,957	8,574	7,452
	4,985	7,993	8,475	8,947	7,745	2,147	9,847	3,564
	6,347	5,893	2,649	2,433	7,894	4,675	6,453	5,748
	7,763	3,574	4,356	5,372	8,564	7,436	7,574	4,376
	8,392	2,323	5,938	7,465	6,463	8,948	8,235	4,785
	4,675	8,213	6,422	9,376	4,563	4,574	6,574	3,287
	4,756	8,493	7,478	8,584	3,289	6,463	7,463	7,483
	6,354	5,783	6,934	3,857	9,521	6,483	9,487	5,475
	5,837	6,489	5,134	6,542	5,473	8,984	5,674	2,376
	5,374	8,945	5,842	8,475	8,234	5,326	4,837	6,473

EXERCISE No. 16, MONEY COLUMNS

The following problems provide written practice in adding money columns. Show on tablet paper the totals of each column by either method illustrated in Exercise No. 12.

The process of addition where dollars and cents are involved is the same as when adding abstract numbers. Cents must be kept in line with cents and dollars with dollars in the order of their unit value.

1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
	\$10.79	\$18.42	\$ 1.23	\$49.15	\$13.26	\$46.50
	2.18	.75	.94	.37	1.50	.98
	1.62	1.60	6.17	2.45	9.75	1.00
	19.10	.57	9.18	21.16	4.80	9.50
	.32	9.63	10.10	3.40	28.75	15.50
	1.51	8.70	9.16	.84	1.12	7.56
	.08	.15	.13	17.20	8.75	.45
	63.07	96.24	3.21	46.35	.05	1.75
	14.26	9.63	9.46	12.54	87.50	71.25
	12.39	17.27	98.45	22.75	1.39	1.72

2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
	\$ 7.45	\$23.10	\$ 1.98	\$17.06	\$.03	\$48.56
	32.85	.05	10.14	23.50	23.94	.32
	.92	4.50	1.75	3.15	12.39	9.63
	46.67	91.52	8.45	2.35	9.63	10.11
	1.34	13.07	1.76	2.15	18.20	3.40
	.02	.41	1.50	37.26	1.39	28.74
	14.91	8.00	25.50	18.20	61.04	1.16
	14.76	.25	7.50	39.47	.10	71.83
	3.34	.45	4.23	14.50	1.37	5.03
	.17	19.75	1.57	16.53	73.48	30.00

3.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
	\$45.23	\$14.89	\$49.90	\$25.75	\$.95	\$ 1.65
	1.93	4.50	.09	41.63	34.15	41.45
	5.05	49.90	4.50	44.42	4.23	28.42
	45.50	8.16	6.35	9.35	23.48	42.68
	16.53	.93	18.25	.02	9.57	6.75
	93.47	4.75	9.31	1.50	5.75	.75
	91.00	34.92	5.79	23.45	25.35	39.95
	3.50	9.45	15.31	94.35	95.50	3.25
	.50	93.76	23.42	1.50	4.72	13.89

CHAPTER III

PROOF OF ADDITION

The purpose of this chapter is to explain and illustrate some of the methods that may be utilized for proving addition. The definition of addition contemplates correct results because it requires the sum of two or more numbers. When you are adding a column of figures you are naturally interested in knowing that the sum is correct. The methods of proof discussed in this chapter are in the order of their importance.

EXERCISE No. 17, PROOF BY REVERSE ADDITION

In Exercise No. 3 you were advised to add from the bottom to the top but told that the results would be the same if the addition was from top to bottom. This means that when the addition of a column of figures is completed by the process of adding from the top to bottom, the columns would be re-added from bottom to top. The process of doing this and checking the results is explained in the illustration below.

$$\begin{array}{r} 17642 \checkmark 26 \\ 97868 \checkmark 26 \\ 44591 \checkmark 33 \\ 92865 \checkmark 29 \\ 13427 \checkmark 23 \\ \hline \checkmark 266393 \end{array}$$

The sum of the figures in the units column (23) adding from top to bottom is written at the right of the bottom amount; the sum of the figures in the tens column including the 2 carried from the units column (29) is written above the sum of the units column; in the same manner the hundreds, thousands, tens of thousands are added, the amount carried being included in each addition. The sum of all the numbers is 266,393 which is equivalent to reading the top number and the units digits in the totals column. The check mark ($\checkmark$) indicates that the columns have been re-added and the sum of each is correct. If when re-adding, the sum of a column is not the same by the second addition as the first, it is necessary to continue to re-add until the two are the same. The addition is completed by one process before checking by reverse addition.

The problems on page 27 provide written practice in adding and proving the results as illustrated and explained above. You

are to show on tablet paper (a) the total of each column with the excesses carried, and (b) proof of the addition by a check mark at the left of the total of each column.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
65,291	95,243	42,961	37,925	75,689	92,543
34,296	17,845	30,542	59,821	49,100	17,762
59,783	85,391	59,672	20,076	81,047	69,043
45,574	27,956	68,734	47,372	52,165	22,598
67,523	70,123	75,643	19,372	48,623	34,256
33,756	36,824	90,179	65,033	21,809	76,895
95,273	68,462	22,981	96,751	40,675	26,174
76,754	45,980	37,549	71,023	39,172	55,167

EXERCISE No. 18, PROOF BY REVERSE ADDITION

In the preceding exercise the sum of each column was written at the right and the amount carried by mental process. If desired the amount to be carried may be written at the top of the column and included in the addition of the column. This process is illustrated below.

$\sqrt{\sqrt{\sqrt{\sqrt{}}}}$
 2322
 17642
 97868
 44591
 92865
 13427

 266393

The figures in the units column are added from bottom to top, the 3 written under the line and the 2 written at the top of the tens column. The figures in the tens column are added, the 2 carried and written at the top, being included in the total of the column which is 29; the 9 is written as the tens in the sum and the 2 carried written above the hundreds column. The same plan is followed in adding the other columns. The check mark above each column indicates re-addition and proof.

The following problems provide written practice in adding and indicating the amount to be carried as illustrated above. Write the sum, amount carried and check marks as proof.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
28,735	76,543	22,154	76,253	31,754	31,416
39,601	20,174	39,701	90,125	27,590	29,847
77,825	37,592	51,579	85,704	20,016	37,645
43,526	52,164	26,438	44,123	49,385	55,102
90,285	31,756	99,753	78,991	75,309	60,813
21,746	27,539	78,522	21,375	21,804	82,148
37,059	98,076	54,876	45,804	37,512	78,695
98,462	54,163	77,145	39,567	28,543	41,906

EXERCISE No. 19, PROOF BY REVERSE ADDITION

If desired, the sum of each column may be written so that these sums may be added to obtain the sum of the numbers. When this plan is followed the excess tens, hundreds, etc., are not carried and a check mark is written at the left of the sum of each column to indicate proof of addition.

$$\begin{array}{r}
 7642 \\
 7868 \\
 4591 \\
 2865 \\
 \hline
 \sqrt{16} \\
 \sqrt{25} \\
 \sqrt{27} \\
 \sqrt{20} \\
 \hline
 \sqrt{22966}
 \end{array}$$

The sum of the figures in the units column is written at the right; the sum of the figures in the tens column, without carrying, is written below the sum of the figures in the units column and at the left; the same plan is followed in adding the figures in the other columns. The sum of the five numbers is the sum of the units, tens, hundreds, etc. columns. The check marks at the left show re-addition and proof.

The following problems provide written practice in adding and indicating the totals of each column as illustrated above. You are to show on tablet paper (a) the sum of each column arranged as in the illustration, (b) the sum of these sums, and (c) the check marks at the left to indicate re-addition and proof.

1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
	45,239	59,837	31,697	92,873	34,691	22,145
	37,865	21,784	43,501	47,321	12,978	39,864
	90,746	40,976	70,463	19,843	50,013	52,017
	76,904	35,791	99,443	40,125	49,678	49,176
	32,175	24,680	27,756	72,591	85,049	67,589
	95,483	97,531	34,527	87,645	76,142	61,678
2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
	66,349	20,981	88,645	31,802	64,178	22,198
	40,581	34,896	40,826	75,184	45,019	34,567
	75,491	15,135	41,389	44,510	65,233	32,415
	27,116	19,763	65,482	31,709	31,746	98,765
	34,763	43,579	91,534	16,718	41,578	12,345
	56,708	74,350	64,351	34,902	29,713	19,005

EXERCISE No. 20, PROOF BY CHECK FIGURE NINE

A *check figure* is one which, when eliminated from each number to be added and from the sum, provides a key number that will prove the addition. *Nine* and *eleven* are used most frequently as check figures. The use of nine is illustrated below.

$$\begin{array}{rcl}
 742583 & = 29 = 11 = & 2 \\
 52425 & = 18 = 9 = & 0 \\
 3155 & = 14 = & 5 \\
 126 & = 9 = & 0 \\
 38527 & = 25 = & 7 \\
 24004 & = 10 = & 1 \\
 752642 & = 26 = & 8 \\
 750 & = 12 = & 3 \\
 1824 & = 15 = & 6 \\
 250000 & = 7 = & 7 \\
 \hline
 1866036 & = 30 = 3 & 39 = 3
 \end{array}$$

The illustration at the left shows the method of proving addition through the elimination of the 9's. To eliminate the nines from a number (a) divide by 9, or (b) add the digits in the number. The mental process of eliminating the nines in the first number is as follows: "7+4+2+5+8+3=29; 2+9=11; 1+1=2." The same mental process is followed with each number to be added and with the sum. The remainders after eliminating the nines of each number are added and the digits in this sum added. Referring to the illustration you will observe that the key figure "3" is the same in both the

sum of the numbers and the sum of the remainders in each number after eliminating the nines. While the nines may be eliminated from a number by dividing the number by nine, the quicker process is to add the digits as illustrated.

The following problems provide written practice in adding and proving the addition by the use of the check figure nine. Arrange your work in the same order as in the illustration except it will not be necessary for you to copy the figures in the problems.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
23,497	67,623	23,109	76,348	20,146	37,987
5,634	5,987	175	52,901	59,738	4,231
976	437	4,586	6,754	35,764	103
70,891	35,291	366	3,633	4,018	92,053
4,795	2,770	98,421	771	375	2,298
374	143	5,294	398	421	509
87,924	74,126	637	7,520	5,890	14,450
4,762	8,214	85,729	1,098	91,243	7,765
352	952	4,013	36,217	76,021	371

EXERCISE No. 21, PROOF BY CHECK FIGURE ELEVEN

The illustration on page 30 shows the process of proof when the elevens are eliminated from each number to be added and from the sum. You will observe the process of arriving at the key

figure is different from that illustrated in the preceding exercise, but the use of the key figure is the same.

The illustration at the right shows the method of proving addition by eliminating the 11's. The elevens may be eliminated from the number (a) through dividing by 11, or (b) by adding the digits in the odd places beginning at the right and the digits in the even places beginning at the right, and subtracting the sum of the even from the sum of the odd; if the sum of the digits in the odd places is less than that of the even, add 11 or a multiple of 11 so that the subtraction is possible. The mental process of

742583	$12 = 23 - 17 = 6$
52425	$14 = 14 - 4 = 10$
3155	$6 = 17 - 8 = 9$
126	$7 = 7 - 2 = 5$
38527	$15 = 15 - 10 = 5$
24004	$6 = 6 - 4 = 2$
752542	$12 = 23 - 13 = 10$
750	$7 = 7 - 5 = 2$
1824	$12 = 12 - 3 = 9$
250000	$5 = 5 - 2 = 3$
1865936	$= 22 - 16 = 6 \quad 61 = 6$

eliminating the elevens in the first number in the illustration is as follows: " $3 + 5 + 4 = 12$; $8 + 2 + 7 = 17$; $12 + 11 = 23$; $23 - 17 = 6$." Here the sum of the digits in the odd places is less than the sum of the digits in the even places, hence it is necessary to add 11 before subtracting. The same mental process is applied in eliminating the elevens from the second and third numbers. In the fourth number the mental process is " $6 + 1 = 7$; $7 - 2 = 5$."

Eliminating the elevens from the sum and from the sum of the remainders provides the same key figure, "6", hence the addition is correct.

The following problems provide written practice in adding and proving the addition by the elimination of elevens. You will arrange your work on tablet paper in the same form as the illustration, except you need not copy the figures to be added.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
39,582	45,201	31,507	75,591	98,103	66,733
74,730	2,098	62,543	20,148	3,514	21,437
16,029	765	5,170	82,146	952	6,274
4,827	14,380	2,168	5,910	50,147	3,156
367	5,601	402	987	3,217	307
359	225	517	674	758	221
3,875	77,189	3,214	3,819	43,761	3,198
75,103	4,571	5,764	37,165	3,758	2,468
87,549	367	98,473	84,307	324	91,406

NOTE: Either the check figure nine or eleven will prove the addition of an independent column, but when the numbers are copied in separate records, the check figure nine will not detect errors in transposed figures. Since the check figure eleven will detect errors in transposed figures, it is considered the better one to use.

EXERCISE No. 22, VERTICAL AND HORIZONTAL ADDITION

Vertical and horizontal addition refers to the adding of two or more separate columns vertically and horizontally to prove the correctness of all additions. The problem below and its solution illustrates the application of this method.

$$2 + 7 + 3 + 5 + 6 + 9 = 32$$

$$4 + 3 + 7 + 8 + 6 + 3 = 31$$

$$2 + 7 + 9 + 4 + 8 + 5 = 35$$

$$5 + 4 + 3 + 6 + 8 + 2 = 28$$

$$13 + 21 + 22 + 23 + 28 + 19 = 126$$

Each column is added horizontally and vertically. The sum secured by the vertical addition is written under each column, and by the horizontal addition at the right of each column. The process of addition should be from bottom

to top for the vertical and from left to right for the horizontal. The proof is effected by adding the totals at the bottom horizontally, and the totals at the right vertically.

The following problems provide written practice in proof of addition through vertical and horizontal addition. You will show on tablet paper the totals obtained by adding vertically and horizontally and the proof through the addition of these totals.

1

$$2 + 9 + 4 + 7 + 8 + 3 + 9 = ?$$

$$8 + 4 + 3 + 8 + 5 + 7 + 3 = ?$$

$$4 + 2 + 7 + 4 + 6 + 4 + 5 = ?$$

$$1 + 3 + 7 + 4 + 7 + 3 + 7 = ?$$

$$9 + 4 + 6 + 3 + 6 + 9 + 6 = ?$$

$$7 + 2 + 3 + 5 + 8 + 2 + 3 = ?$$

$$7 + 8 + 4 + 9 + 6 + 3 + 8 = ?$$

$$6 + 5 + 3 + 8 + 9 + 5 + 6 = ?$$

$$\begin{array}{r} \text{---} \\ ? + ? + ? + ? + ? + ? + ? = ? \end{array}$$

2

$$7 + 4 + 7 + 4 + 8 + 2 + 4 = ?$$

$$1 + 4 + 6 + 3 + 7 + 2 + 6 = ?$$

$$2 + 4 + 5 + 3 + 3 + 4 + 9 = ?$$

$$4 + 7 + 2 + 7 + 4 + 6 + 3 = ?$$

$$9 + 6 + 7 + 4 + 3 + 7 + 2 = ?$$

$$1 + 4 + 3 + 6 + 7 + 3 + 5 = ?$$

$$4 + 7 + 5 + 6 + 8 + 7 + 9 = ?$$

$$5 + 7 + 4 + 6 + 8 + 2 + 3 = ?$$

$$\begin{array}{r} \text{---} \\ ? + ? + ? + ? + ? + ? + ? = ? \end{array}$$

3

$$5 + 6 + 5 + 7 + 4 + 3 + 5 = ?$$

$$6 + 4 + 7 + 8 + 2 + 9 + 6 = ?$$

$$3 + 6 + 4 + 5 + 7 + 5 + 3 = ?$$

$$5 + 3 + 7 + 4 + 6 + 7 + 4 = ?$$

$$8 + 4 + 5 + 6 + 7 + 3 + 6 = ?$$

$$9 + 6 + 7 + 8 + 8 + 2 + 8 = ?$$

$$4 + 5 + 6 + 7 + 3 + 2 + 2 = ?$$

$$6 + 5 + 4 + 3 + 7 + 8 + 5 = ?$$

$$\begin{array}{r} \text{---} \\ ? + ? + ? + ? + ? + ? + ? = ? \end{array}$$

4

$$4 + 6 + 5 + 7 + 8 + 9 + 7 = ?$$

$$5 + 9 + 8 + 7 + 4 + 2 + 9 = ?$$

$$1 + 4 + 2 + 3 + 6 + 8 + 3 = ?$$

$$4 + 7 + 5 + 6 + 7 + 9 + 5 = ?$$

$$3 + 6 + 4 + 4 + 3 + 6 + 8 = ?$$

$$5 + 3 + 7 + 4 + 5 + 6 + 4 = ?$$

$$2 + 1 + 3 + 3 + 5 + 4 + 2 = ?$$

$$3 + 6 + 9 + 3 + 4 + 7 + 6 = ?$$

$$\begin{array}{r} \text{---} \\ ? + ? + ? + ? + ? + ? + ? = ? \end{array}$$

5. The Model Laundry has 5 drivers who collect and deliver laundry. During the week beginning January 9, they collected cash as listed below.

	<i>M</i>	<i>T</i>	<i>W</i>	<i>T</i>	<i>F</i>	<i>S</i>	
Jones	\$26.50+	\$27.85+	\$35.40+	\$24.16+	\$18.54+	\$52.98=	?
Davis	21.19+	25.32+	31.48+	25.04+	17.36+	51.18=	?
Brown	28.11+	25.15+	33.65+	27.92+	15.81+	55.45=	?
Dooley	20.25+	32.85+	14.62+	31.18+	22.65+	55.87=	?
Ray	24.14+	38.19+	27.62+	22.09+	19.87+	49.23=	?
	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>
	?	+	?	+	?	+	?

Ascertain: (a) the total amount of cash collected each day; and (b) the total amount collected by each driver. Prove by vertical and horizontal addition.

EXERCISE No. 23, COPYING FIGURES

Copying figures correctly is equally as important as adding correctly, because if incorrectly copied, the sum will be incorrect even though the process of addition were correctly applied.

The following problems provide written practice in: (a) copying figures; (b) addition; and (c) proof of addition.

1. Copy each of the following columns on a sheet of paper, add, and prove by the check figure 9. Arrange your work as in the illustration of check figures on page 29.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
326	726	317	174	428	942	715	257
432	398	243	316	747	163	168	533
515	407	815	175	215	206	206	956
679	259	912	815	164	721	721	128
248	117	748	994	928	832	832	314
<u>143</u>	<u>243</u>	<u>463</u>	<u>375</u>	<u>314</u>	<u>415</u>	<u>415</u>	<u>725</u>

2. Copy each of the following columns on a sheet of paper, add, and prove by the check figure 11.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
517	249	752	321	213	613	234	158
861	424	944	234	431	132	311	206
912	375	407	231	524	235	532	891
383	242	367	423	413	443	131	185
559	573	134	435	213	214	321	215
<u>602</u>	<u>703</u>	<u>357</u>	<u>342</u>	<u>321</u>	<u>312</u>	<u>212</u>	<u>112</u>

3. Copy each of the following columns on a sheet of paper, add and prove by reverse addition.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
124	431	231	412	1,244	4,213	3,124
216	513	431	134	3,152	2,531	5,113
341	142	424	244	3,541	4,315	1,234
234	353	321	123	1,124	1,241	6,725
261	425	212	125	3,225	3,632	1,043
514	112	324	412	6,321	4,016	2,120
315	126	124	351	1,232	6,521	3,216
442	451	242	113	1,525	1,234	5,451
312	332	134	142	5,342	2,245	2,320

EXERCISE No. 24, CALLING NUMBERS

When it is necessary to call numbers, two persons are involved, one to call, and one to make the record of the numbers called. A code should be established to facilitate rapid calling and correct interpretation. Thus, if the amount to be called is 1,645, it would require too much time to read this as one thousand six hundred forty-five. Sixteen hundred forty-five would be sufficient information for the writer and he could call back one six four five. The person writing the figures should call them back so that the reader may know he has correctly interpreted them. Each may use the same code, but it is better to use a different one. The illustrations below will suggest a code for calling figures:

1,645 is read sixteen hundred, forty-five,
and called back one, six, four, five.

10,675 is read ten thousand, six hundred seventy-five
and called back one, naught, six, seven, five.

\$12.80 is read twelve dollars eighty cents
and called back twelve point eighty.

\$1,280 is read twelve hundred eighty dollars
and called back one, two, eight, naught, dollars.

\$200.60 is read two hundred dollars sixty cents
and called back two hundred point sixty.

138,460 is read one thirty-eight four sixty
and called back one, three, eight, four, six, naught.

NOTE: If the figures called all relate to dollars and cents, it is not necessary to designate this each time, hence time can be saved when calling.

The figures in the following columns provide practice in calling numbers, writing figures, calling back, and adding. Prove the addition of each problem.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
\$5,807.63	\$142,780.67	\$958,324.55	454,394	354,283,496
1,807.49	1,356.50	34,432.25	34,565	45,921,003
933.61	33,734.35	420.01	231,495	1,354,211
21.00	42.92	3,529.00	1,599	716,752,498
9,910.86	1,329.52	10.00	10,050	45,007
746.84	92,423.75	342,537.19	501,345	4,710,903
1.64	3,000.02	31,563.91	595	4,057
8,327.50	837,703.50	989,768.92	14,678	455,942,405
427.43	745,324.15	78,000.10	983,435	4,032,001
73.00	4,389.59	1,002.05	3,234	45,647,810
1,502.67	345,103.22	101,563.02	375,641	32,445,108
39.14	37,656.15	1.25	42,107	1,675,890

EXERCISE No. 25, PAY ROLLS

Factory employees are usually paid weekly. The information regarding the amount due each employee is obtained from a record of his time maintained in the accounting department. It is necessary for the paymaster to know on pay day (a) the amount due each employee, (b) the total amount necessary to pay all employees, and (c) the change necessary to make up the pay roll so that the amount due each employee may be inclosed in an envelope ready to hand to him.

The following problems provide written practice in ascertaining the information needed by the paymaster. You are not required to record the time because this involves bookkeeping.

1. The Brown Manufacturing Company employs twelve employees in one department of its factory. The pay-roll sheet on page 35, shows the daily earnings of each employee. You are required to ascertain (a) the amount due each employee by horizontal addition, (b) the total amount of the daily pay roll by vertical addition, (c) the proof of this amount, and (d) the change necessary to place the amount due each employee in an envelope.

Name	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total
Brown	6.37	7.45	6.25	6.75	6.00	6.47	
Clark	5.95	5.45	6.25	5.89	5.79	5.73	
Davis	6.75	6.95	6.55	6.34	6.47	6.50	
Ellis	7.00	7.25	7.45	6.95	7.43	7.59	
Ely	5.78	5.85	5.48	5.15	5.98	6.50	
French	7.35	7.25	6.89	6.95	6.12	5.59	
Garrett	6.89	7.84	7.57	7.24	7.18	7.31	
Howard	5.45	5.68	5.79	5.85	5.95	5.79	
Innes	7.35	7.47	7.61	7.83	7.98	7.55	
Jones	6.47	6.63	7.05	6.95	6.47	6.59	
King	5.89	5.74	6.47	6.53	6.63	6.51	
Miller	6.21	6.36	6.63	6.55	6.37	6.49	
TOTAL							

The method of ascertaining the correct amount of change due each employee is illustrated below.

Amount	20's	10's	5's	2's	1's	50c	25c	10c	5c	1c
39.29	1	1	1	2			1			4
35.06	1	1	1						1	1
39.56	1	1	1	2		1			1	1

The illustration shows the amount due the first three employees. You will observe that columns are provided for the amount due each employee and the various denominations of currency and change. On a line with the amount due each employee is indicated the number of pieces necessary to pay the employee, using the least possible number of pieces.

2. The Union Manufacturing Company had fifteen employees. The pay roll for the week ending January 9 is as follows:

Name	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total
White	10 02	9 78	11 07	9 66	9 83	9 17	
Dorley	9 30	10 19	8 73	9 78	6 82	9 46	
Hayes	9 63	11 78	10 32	10 16	9 70	9 65	
Duggan	10 87	9 74	10 09	9 31	9 87	9 14	
Smith	8 68	10 98	10 15	10 02	9 76	9 02	
Foster	9 15	10 31	10 37	10 23	9 78	9 27	
Evens	9 39	10 41	9 87	9 27	9 11	9 36	
Burns	7 99	11 56	9 34	9 73	10 74	9 71	
Kraus	10 78	6 21	9 17	9 88	9 43	9 80	
Fisher	10 13	7 41	9 39	9 18	9 17	9 72	
Grant	11 54	9 19	10 37	10 37	9 97	9 72	
Fleet	9 37	10 43	10 02	9 78	9 46	9 60	
Jones	11 31	9 34	10 15	10 18	10 74	9 74	
Miller	10 32	9 87	10 37	9 75	10 21	9 43	
Wade	11 83	8 18	9 43	6 17	10 27	9 86	
TOTAL							

You are to indicate on tablet paper (a) the amount due each employee, (b) the total daily wages, (c) the total due the fifteen employees at the end of the week, and (d) the number of pieces of each denomination of money required to make up the payroll.

3. The Central Garage employs six men in its repair department. Each man keeps a record of his time on a card, this record being checked with the time-clock and the charge sheets for repair work. The amount due each employee from the card records for the week ending Saturday, May 14, is as follows:

James Moore, Monday \$7.60, Tuesday \$8.80, Wednesday \$9.75, Thursday \$9.87, Friday \$8.97, Saturday \$9.45; John Brown, Monday \$7.75, Tuesday \$8.85, Wednesday \$9.85, Thursday \$9.95, Friday \$8.75, Saturday \$7.75; Ray Whitney, Monday \$8.85, Tuesday \$9.45, Wednesday \$9.65, Thursday \$8.95, Friday \$7.75, Saturday \$8.80; Don Clark, Monday \$8.85, Tuesday \$9.25, Wednesday \$9.75, Thursday \$8.75, Friday \$9.50, Saturday \$8.95; Harry Miller, Monday \$9.75, Tuesday \$8.85, Wednesday \$7.95, Thursday \$9.87, Friday \$8.68, Saturday \$9.56; Frank Smith, Monday \$8.87, Tuesday \$8.95, Wednesday \$9.53, Thursday \$9.78, Friday \$8.98, Saturday \$9.75.

You are required (a) to rule a form similar to that in problem 1 and arrange the names of the employees and the amounts due each in the same manner, (b) to ascertain the amount due each employee on pay day, (c) the total amount necessary to make up the pay roll, and (d) the change required by the paymaster to pay each employee.

EXERCISE No. 26, RECEIVING AND SHIPPING MERCHANDISE

The following problems provide written practice in applying addition to the solution of some of the problems encountered in receiving and shipping merchandise. You are to show on tablet paper the information desired.

1. March 7, Brown Brothers, manufacturers of men's clothing, received an order from A. L. Day, one of their customers. This order was referred to the shipping clerk with instructions to pack in cases and give the office a statement of the weights in each case. After packing the merchandise ordered, the shipping clerk reported that it required twelve cases to contain the merchandise. These cases were numbered in regular order beginning with one and the weights reported as follows: 256 lbs., 319 lbs., 408 lbs., 271 lbs., 400 lbs., 353 lbs., 455 lbs., 485 lbs., 316 lbs., 389 lbs., 412 lbs., 487 lbs. Assuming that the cases are numbered in the order in which the weights are given, show on tablet paper the number and weight of each case and the total weight. Prove by reverse addition.

2. The Jones Mercantile Company received a consignment of merchandise contained in ten cases. The total weight of these cases as shown on the bill of lading was 1,886 pounds. The weight of each case in pounds shown on the invoice was as follows: 199, 156, 142, 237, 219, 86, 155, 181, 148, and 263. You are to list the

weights on tablet paper, add, and show by the elimination of nines that your total is correct.

3. Robert Decker delivered ten truck loads of wheat to an elevator. The net weight in pounds of the wheat in each truck was as follows: 4,165, 4,221, 3,962, 4,144, 4,355, 3,998, 4,455, 4,687, 4,563, and 4,471. List these weights on tablet paper, and ascertain the total weight of wheat delivered.

4. James Dodson delivered to Brown Commission Co., twenty-five bags of potatoes, weights, in pounds as follows: 125, 119, 121, 115, 124, 117, 122, 116, 123, 118, 121, 115, 118, 125, 119, 124, 122, 117, 116, 123, 115, 119, 125, 117, and 122. List these weights, add, and prove by the check figure eleven.

5. J. W. Dillman delivered to a cannery twelve truck loads of green corn, weights, in pounds, as follows: 3,756, 3,942, 4,291, 4,372, 4,150, 3,890, 3,650, 4,097, 4,400, 3,854, 3,905, and 4,321. List these weights, add, and prove the sum by reverse addition.

EXERCISE No. 27, SALES MANAGERS' PROBLEMS

A sales manager has charge of sales and of those who make the sales. The following problems relate to sales and expenses of salesmen.

1. The Davis Manufacturing Company employs five traveling salesmen each of whom reports his weekly expenses at the end of the week. The following are the expenses for each of these salesmen during thirteen weeks beginning January first.

<i>Date</i>	<i>Ray</i>	<i>Day</i>	<i>May</i>	<i>Lay</i>	<i>Kay</i>
January 7.....	\$67.89	\$75.80	\$62.50	\$84.79	\$76.77
January 14.....	7.25	67.89	75.50	78.54	68.90
January 21.....	75.98	81.45	80.45	72.43	83.45
January 28.....	67.50	73.56	83.29	67.89	70.21
February 4.....	83.45	84.56	77.22	75.49	75.76
February 11.....	74.50	6.54	68.75	62.45	79.80
February 18.....	65.43	83.67	72.75	74.56	82.34
February 25.....	78.95	79.80	70.89	77.86	7.50
March 4.....	73.45	80.75	81.76	83.45	80.98
March 11.....	62.34	77.87	65.78	65.89	65.90
March 18.....	69.70	61.89	74.96	9.25	77.75
March 25.....	84.39	65.40	69.70	67.80	68.40
April 1.....	81.70	82.43	7.65	84.35	82.90

Show on tablet paper (a) the horizontal addition with the information it shows, (b) the vertical addition with the information it shows, and (c) the final totals and proof.

2. The Sales Manager for the Kroger Grocery Company wishes to know the total sales made by each of the six traveling salesmen employed by the firm during the first six months of the year, also the total sales of all the six salesmen for the same term. These sales are recorded in the books of the company as follows: J. O. Richards, \$551.65, \$423.38, \$647.27, \$675.42, \$582.26, and \$743.25; Louis Garrison, \$789.45, \$675.25, \$725.31, \$752.47, \$712.46, \$863.22; R. T. Welch, \$864.33, \$784.22, \$792.23, \$825.89, \$789.95, \$835.42; C. T. Gray, \$646.25, \$595.29, \$651.94, \$642.24, \$603.33, \$755.11; John Hume, \$932.49, \$764.48, \$827.72, \$832.46, \$789.72, \$843.68; T. B. Wood, \$865.22, \$798.63, \$825.26, \$851.82, \$846.88, \$875.50. Arrange these figures on tablet paper so that you can show the facts desired and prove by vertical and horizontal addition.

3. The sales report for the Pleasant View Dairy during the twelve months beginning with January shows the following monthly sales of butter, cream, certified milk, sweet milk, and buttermilk.

<i>Month</i>	<i>Butter</i>	<i>Cream</i>	<i>C. Milk</i>	<i>S. Milk</i>	<i>B. Milk</i>
January	\$175.46	\$278.95	\$98.75	\$248.90	\$69.50
February	179.75	295.35	89.25	239.45	57.65
March	164.55	265.47	85.65	245.39	74.80
April	189.76	235.90	64.34	237.56	70.10
May	192.45	256.78	78.90	225.49	44.50
June	155.90	249.55	55.50	215.48	53.24
July	158.75	229.85	69.89	210.56	62.45
August	176.89	239.55	72.34	206.57	66.67
September	179.87	289.76	80.15	229.65	55.19
October	195.67	298.75	95.67	234.90	72.68
November	186.50	281.65	99.21	241.35	59.85
December	199.45	273.66	92.35	249.15	71.62

You are to show on tablet paper (a) the total value of each product sold during the year, (b) the total value of all products sold during each month, and (c) the total of all products sold during the year.

CHAPTER IV

SUBTRACTION

The purpose of this chapter is to provide practice in subtraction, and proof of the difference. The explanation and problems are given under the assumption that you understand the mathematical principles involved in the process of subtraction.

Subtraction is the process of finding the difference between two numbers. The number to be diminished is the *minuend*. The number subtracted is the *subtrahend*. The result obtained in subtraction is known as the *difference*, the *balance*, or the *remainder*.

EXERCISE No. 28, SUBTRACTION

The purpose of the following problems is to provide oral practice in determining the difference between two numbers of one or more figures. With practice you can learn to read numbers and subtract them as rapidly as you can read combinations in addition.

Subtraction is a process of addition. Thus, in the first problem, you are required to subtract 3 from 6, which is equivalent to adding 3 and 3. You should prove all subtraction by adding the difference to the subtrahend.

$$\begin{array}{r} 1. \quad \begin{array}{cccccccccccc} 6 & 5 & 8 & 7 & 5 & 9 & 4 & 3 & 8 & 6 & 5 & 9 \\ 3 & 2 & 4 & 5 & 2 & 3 & 3 & 2 & 5 & 2 & 4 & 7 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} 2. \quad \begin{array}{cccccccccccc} 17 & 14 & 17 & 16 & 18 & 15 & 19 & 12 & 16 & 13 & 19 & 15 \\ 6 & 3 & 4 & 5 & 4 & 3 & 7 & 2 & 5 & 2 & 8 & 4 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} 3. \quad \begin{array}{cccccccccccc} 19 & 16 & 18 & 14 & 12 & 16 & 18 & 14 & 17 & 14 & 18 & 13 \\ 13 & 10 & 14 & 12 & 10 & 14 & 17 & 13 & 12 & 12 & 16 & 11 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} 4. \quad \begin{array}{cccccccccccc} 25 & 28 & 35 & 27 & 36 & 39 & 27 & 24 & 37 & 28 & 24 & 35 \\ 22 & 26 & 34 & 22 & 33 & 25 & 23 & 21 & 31 & 25 & 21 & 32 \\ \hline \end{array} \end{array}$$

(Exercise No. 28—Continued from page 40)

5.	38	49	36	28	37	47	53	58	28	35	39	49
	<u>26</u>	<u>36</u>	<u>22</u>	<u>16</u>	<u>23</u>	<u>33</u>	<u>31</u>	<u>35</u>	<u>15</u>	<u>11</u>	<u>16</u>	<u>23</u>
6.	35	29	47	67	36	28	47	46	27	38	24	57
	<u>23</u>	<u>22</u>	<u>31</u>	<u>33</u>	<u>24</u>	<u>13</u>	<u>24</u>	<u>13</u>	<u>15</u>	<u>24</u>	<u>11</u>	<u>35</u>
7.	38	29	47	68	89	58	49	67	88	49	97	95
	<u>13</u>	<u>11</u>	<u>23</u>	<u>25</u>	<u>47</u>	<u>24</u>	<u>16</u>	<u>24</u>	<u>36</u>	<u>26</u>	<u>43</u>	<u>62</u>
8.	68	35	57	89	49	37	36	57	39	69	45	42
	<u>49</u>	<u>37</u>	<u>28</u>	<u>49</u>	<u>36</u>	<u>29</u>	<u>18</u>	<u>29</u>	<u>35</u>	<u>21</u>	<u>57</u>	<u>28</u>

EXERCISE No. 29, SUBTRACTION AND PROOF

The purpose of this exercise is to provide written practice in subtraction. Each problem should be proved by addition as explained on page 40. Place a sheet of paper under each problem, subtract, write the difference on this sheet and prove by adding the difference to the subtrahend. Place a check mark at the left of each difference to indicate the proof.

1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
	478	\$7,847	59,847	\$587.46	584,736	\$4,564.35	483,766
	<u>127</u>	<u>5,849</u>	<u>48,372</u>	<u>376.28</u>	<u>483,769</u>	<u>2,736.58</u>	<u>138,476</u>
2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
	983	\$5,838	84,736	\$837.47	685,746	\$8,475.67	685,746
	<u>595</u>	<u>2,876</u>	<u>27,365</u>	<u>574.38</u>	<u>328,746</u>	<u>3,837.37</u>	<u>451,678</u>
3.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
	938	\$7,839	58,473	\$587.48	857,463	\$2,837.57	685,743
	<u>473</u>	<u>3,765</u>	<u>47,362</u>	<u>384.37</u>	<u>584,736</u>	<u>1,476.38</u>	<u>415,876</u>
4.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
	634	\$8,473	84,937	\$587.39	584,736	\$5,847.84	796,854
	<u>489</u>	<u>6,857</u>	<u>57,438</u>	<u>513.28</u>	<u>293,874</u>	<u>1,345.68</u>	<u>148,376</u>
5.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
	529	\$9,721	56,723	\$365.27	697,642	\$4,923.15	620,713
	<u>385</u>	<u>3,498</u>	<u>34,962</u>	<u>198.75</u>	<u>576,333</u>	<u>2,168.75</u>	<u>396,752</u>

EXERCISE No. 30, PROOF OF SUBTRACTION

When it is desired to ascertain the sum of the differences between two or more numbers, the proof may be effected by combining addition and subtraction as illustrated below. This illustration shows the solution and proof of the problem $34,256 - 13,475$ and $67,483 - 37,485$.

<i>a</i>	<i>b</i>	<i>c</i>
34,256	67,483	20,781
<u>13,475</u>	<u>37,485</u>	<u>29,998</u>
20,781	29,998	50,779
<i>d</i>	<i>e</i>	<i>f</i>
34,256	13,475	101,739
<u>67,483</u>	<u>37,485</u>	<u>50,960</u>
101,739	50,960	50,779

Solutions (a) and (b) at the left show the subtraction required in the problem; (c) shows the addition of the differences; (d) shows the addition of the minuends; (e) shows the addition of the subtrahends. The subtraction is proved to be correct when (c) the sum of the differences of the numbers equals (f) the difference between the sum of the minuends and the sum of the subtrahends. This method of proof is applied to subtraction and addition in connection with controlling

accounts, a phase of accounting in the standard bookkeeping process.

The following problems provide written practice in applying proof of subtraction by combining the process with addition as illustrated above. Arrange your work on tablet paper the same as in the illustration, except you will have six groups of figures instead of two groups.

- | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>e</i> | <i>f</i> |
|-----------------|---------------|-----------------|---------------|-----------------|---------------|
| \$7,261.44 | \$624.25 | \$2,481.36 | \$642.39 | \$4,283.62 | \$286.24 |
| <u>5,271.30</u> | <u>271.33</u> | <u>1,987.81</u> | <u>489.27</u> | <u>3,762.28</u> | <u>175.56</u> |
- | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>e</i> | <i>f</i> |
|-----------------|---------------|-----------------|---------------|-----------------|---------------|
| \$9,762.35 | \$834.45 | \$5,839.46 | \$385.54 | \$6,598.84 | \$587.45 |
| <u>4,837.59</u> | <u>432.67</u> | <u>4,684.34</u> | <u>123.34</u> | <u>4,774.39</u> | <u>146.78</u> |
- | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>e</i> | <i>f</i> |
|-----------------|---------------|-----------------|---------------|-----------------|---------------|
| \$7,845.56 | \$898.39 | \$3,847.75 | \$348.87 | \$4,158.14 | \$534.82 |
| <u>6,734.84</u> | <u>248.47</u> | <u>3,538.88</u> | <u>145.64</u> | <u>2,938.12</u> | <u>246.88</u> |
- | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>e</i> | <i>f</i> |
|-----------------|---------------|-----------------|---------------|-----------------|---------------|
| \$8,495.56 | \$837.57 | \$5,748.38 | \$758.34 | \$5,748.34 | \$869.46 |
| <u>8,238.47</u> | <u>635.34</u> | <u>2,839.47</u> | <u>746.28</u> | <u>3,738.27</u> | <u>726.38</u> |

Subtraction is usually indicated by writing the smaller number under the larger one from which it is to be subtracted. Sometimes in business problems, the position is such that it is necessary to subtract the top number from the bottom number, because the bottom number is the larger.

The following problems provide written practice in subtraction when the figures are in the reverse position.

1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
	82	17	84	51	57	82	79	73	71	93	87
	<u>99</u>	<u>38</u>	<u>96</u>	<u>63</u>	<u>71</u>	<u>98</u>	<u>87</u>	<u>97</u>	<u>82</u>	<u>99</u>	<u>99</u>
2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
	15	16	33	40	81	16	19	13	16	19	7
	<u>75</u>	<u>24</u>	<u>56</u>	<u>66</u>	<u>98</u>	<u>61</u>	<u>91</u>	<u>32</u>	<u>60</u>	<u>81</u>	<u>63</u>
3.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
	18	15	75	37	18	26	57	33	17	13	15
	<u>66</u>	<u>33</u>	<u>90</u>	<u>84</u>	<u>73</u>	<u>81</u>	<u>62</u>	<u>54</u>	<u>75</u>	<u>68</u>	<u>47</u>
4.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
	73	21	66	45	27	32	37	93	76	17	46
	<u>88</u>	<u>46</u>	<u>84</u>	<u>94</u>	<u>57</u>	<u>44</u>	<u>46</u>	<u>98</u>	<u>85</u>	<u>46</u>	<u>59</u>

The Austrian method of subtraction is made by adding to the smaller number a number which will make it equal to the larger number, the number added being the difference. The following problems provide oral practice in applying this method.

1.	9 7	7 3	4 2	8 6	5 2	8 5	7 4	6 3	9 4	9 2
2.	18 13	14 12	15 13	19 16	13 11	19 13	17 14	18 10	14 11	18 14
3.	38 29	39 23	47 29	57 58	32 27	68 39	48 35	69 24	38 32	57 39
4.	42 26	54 25	65 38	45 27	63 39	47 28	32 16	67 29	41 35	56 27

EXERCISE No. 33, SUBTRACTION BY ADDITION

The following problems provide written practice in applying the Austrian method of subtraction. Arrange your work on tablet paper as in the illustration.

<i>Subtraction</i>	<i>Explanation</i>
1235	6 and 9 are 15
-786	9 and 4 are 13
449	8 and 4 are 12

Unless you are accustomed to subtracting by the Austrian method, it will seem more difficult than the process of subtracting the subtrahend from the minuend. However, with

practice, you will become more proficient, and it is claimed, more accurate through using this method.

1.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
	4,375	21,847	\$38,576.25	\$45,287.37	\$28,374.45
	283	3,146	2,873.48	2,837.79	4,726.28
2.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
	5,847	41,387	\$83,746.88	\$82,736.56	\$73,846.26
	378	3,847	3,746.57	2,837.28	3,746.58
3.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
	8,572	41,587	\$57,362.84	\$38,473.32	\$39,283.45
	398	6,453	4,857.68	9,485.86	3,847.38
4.	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
	9,384	39,487	\$57,382.25	\$47,565.72	\$48,372.21
	478	3,847	5,837.37	3,848.56	1,384.38

EXERCISE No. 34, RETROGRESSIVE SUBTRACTION

Retrogressive subtraction provides excellent practice in obtaining the difference between two numbers.

The following drill provides oral practice in retrogressive subtraction. You will subtract in each case until the remainder is equal to or less than the number to be subtracted.

1. Begin with 47 subtract by 2's; 4's; 6's; 8's
2. Begin with 92 subtract by 3's; 5's; 7's; 9's
3. Begin with 98 subtract by 5's; 7's; 9's; 3's
4. Begin with 78 subtract by 4's; 6's; 8's; 2's
5. Begin with 88 subtract by 3's; 5's; 7's; 9's
6. Begin with 64 subtract by 6's; 8's; 4's; 2's
7. Begin with 73 subtract by 5's; 7's; 3's; 9's
8. Begin with 96 subtract by 8's; 4's; 6's; 2's

EXERCISE No. 35, PROFIT AND LOSS

The profit or loss made through the purchase or sale of property is determined by subtraction, or addition and subtraction. If the purchase price and selling price are the only two amounts involved, the profit or loss is ascertained by subtraction. If a number of amounts are involved in connection with the cost, or the income, it is necessary to add them in order to ascertain the total cost or total income.

The following problems provide written practice in ascertaining the profit or loss resulting from the purchase and sale of specific property.

1. D. W. Cross purchased a farm of 160 acres for \$16,000. January 25, he paid taxes, \$225.50; February 12, he paid for repairing buildings and fences, \$565.75; March 1, he sold the farm for \$18,750. Show on tablet paper all the calculations necessary to ascertain his profit or loss on this farm.

2. May 10, A. L. Young paid \$2,150 for an automobile. He kept it three years and then traded it in on another automobile, the purchase price of which was \$2,575. He gave his old machine and \$1,685.50 cash in payment for the new machine and accessories amounting to \$135.50. Show on tablet paper the loss on the first automobile through its use for three years.

3. C. H. Martin was paying \$60 monthly rent for his home. On April 1, he paid \$8,575 for the property. July 1, he paid \$427.50 for repairs and painting; December 31, he paid taxes \$119.85 and insurance \$75. On March 31, he sold property for \$10,000. How much was his profit taking into consideration the rent which he would have paid had he not purchased the property?

4. A farmer paid \$387.50 for hogs and sold them for \$637.65. How much was his profit, if he spent \$187.78 for feed?

5. C. C. Ming sold to a cannery the tomatoes grown on his farm for \$987.45. How much profit did he make on this crop of tomatoes after deducting the cost of planting, \$75.50; cultivating, \$279.87; and gathering, \$204.62?

6. O. H. Barclay began the chicken business, January 2, with one hundred chickens for which he paid \$75. During the first six months of the year he sold eggs and chickens to the value of \$137.55, and bought additional chickens for \$75. How much was his profit, if the amount paid for feed was \$42.38 and the value of the chickens on hand was \$116.95?

RAILROAD TIME-TABLES

It is customary for the railroads to provide a time-table showing detailed information regarding all trains operated on each division of its roads. The time-table given below shows the schedule of trains on the Cincinnati Division of the Big Four Railroad.

Table
No. 13 CINCINNATI TO INDIANAPOLIS AND CHICAGO

Miles	STATIONS	15	19	43	31	35	5	31	33	27
	Central Standard Time	Chi- cago Lim. Daily	White City Sp'cl Daily	Chi- cago Exp. Sun.	Royal Palm Daily	Chi- cago Night Sp'cl Daily	The Sycamore Daily	Chgo. Mail and Exp. Daily	Ind- pls. Lim. Daily	Exp. Daily
0	CINCINNATI.. Lv	AM 9 00	Noon 12 00	PM 8 30	PM 9 45	PM 11 55		AM 7 30	PM 6 00	PM 3 00
17	Valley Jct.							8 15		3 35
22	Lawrenceburg Jc			G 9 04	10 17			8 30		3 44
27	Guilford.							8 40		3 50
31	Bonnell.							8 46		4 00
36	Weisburg.							8 59		4 10
39	Sunman.							9 08		4 20
41	Spades.							9 12		4 25
45	Morris.			9 42	10 52			9 20		4 35
47	Batesville.	10 11						9 26		4 42
54	New Point.							9 37		4 55
58	McCoy.							9 45	J 7 25	5 00
62	Greensburg.	10 35	1 26	10 05	11 12	1 14		10 00		5 05
67	Adams.							10 11		5 12
72	St. Paul.							10 19		5 16
75	Waldron.							10 24		5 23
82	Shelbyville.	11 04	1 51	10 33	11 41	Q		10 43	7 49	5 40
89	Fairland.		E 2 01					10 54		5 50
96	Acton.							11 05		6 20
103	Beech Grove.							11 20		
109	IND'NAPOLIS AR	11 50	2 35	11 20	12 25	2 25		11 45	8 30	
109	IND'NAPOLIS LV	12 00	2 45	11 35	12 40	2 35	5 00	12 15	PM	PM
120	Augusta.							12 35		
125	Zionsville.							12 51		
130	Whitestown.							1 05		
138	Lebanon.		3 28	12 22	H 1 31		5 42	1 12		
143	Hazletrigg.							1 20		
147	Thorntown.		3 40	12 38	H 1 50			1 30		
153	Colfax.							1 44		
158	Clark's Hill.							1 58		
161	Stockwell.							2 20		
165	Crane.							2 52		
174	LAFAYETTE.	1 30	4 18	1 45	2 55	4 16	6 25	2 59		
192	Templeton.							3 04		
196	Atkinson.			f 2 30				3 16		
198	Swanington.			5 02	H 3 46			3 31		
202	Fowler.		5 13	2 54	H 3 58			3 37		
209	Earl Park.							3 43		
211	Sheff.			f 3 01	H 4 08			3 58		
214	Raub.			3 14	H 4 18			4 10		
219	Sheldon.		5 28	G 3 23	H 4 27			4 24		
223	Iroquois.			f 3 31	H 4 36			4 36		
227	Donovan.			f 3 41	H 4 46			4 51		
232	Beaversville.			f 3 51	H 4 56			5 02		
238	St. Anne.			G 4 00	H 5 06			5 20		
244	Aroma Park.			4 20	5 15	6 10	8 05	5 20		
248	Kankakee. AR	3 20	6 16					5 20		
284	Harvey.				H 6 18			H 6 17		
290	Kensington.				H 6 32			H 6 25		
295	Grand Crossing									
296	63d Street.	4 36	7 41	5 47	6 45	7 30	9 21	6 38		
298	Hyde Park 53d St	4 40	7 44	5 51	6 48	7 33	9 24	6 42		
299	43d Street.	4 43	7 47	5 55	6 51	7 36	9 27	6 46		
303	CHICAGO. AR	4 55	7 59	6 10	7 05	7 50	9 40	7 00		
	(Central Sta.— Mich. Ave. & Roosevelt Rd.)	PM	PM	AM	AM	AM	PM	PM		

The description of each train by number and name is given at the top of the column which shows its running time. Each train leaves Cincinnati at the time mentioned on a line with Cincinnati and arrives at the other stations at the time given on a line with the names of the stations. No time, indicates that the train does not stop. The figures at the left of the names of the towns indicate the number of miles between the town and Cincinnati.

EXERCISE No. 36, SUBTRACTION OF DISTANCE AND TIME

The following problems in subtraction are based on the railroad time-table on the preceding page. These should be oral, unless your instructor requires them to be written.

1. Ascertain the distance between Morris and Kankakee.
2. Ascertain the distance between Spades and Chicago.
3. How long does it take Train No. 15 to go from Batesville to Lafayette?

NOTE: Time printed on the time-table in lightface type is between 12 midnight and 12 noon, and that printed in blackface type, between 12 noon and 12 midnight.

4. How long does it take Train No. 1 to go from Cincinnati to Sheldon?
5. Ascertain the distance between McCoy and St. Anne.
6. Ascertain the distance between Chicago and Adams.
7. How long does it take Train No. 27 to go from Guilford to Indianapolis?
8. The distance between Boston and New York on the New York, New Haven and Hartford Railroad is 232 miles. The distance between Boston and Springfield, Mass., which is between Boston and New York, is 98 miles. How far is it from Springfield to New York?
9. The distance between Chicago and Pittsburgh on the Pennsylvania railroad is 468 miles, and the distance from Fort Wayne to Chicago on the same road is 148 miles. How far is it from Fort Wayne to Pittsburgh?
10. The Overland Limited leaves Chicago at 8:10 p. m. Sunday, and arrives in San Francisco at 2:30 p. m. Wednesday. What is the exact time of this train in transit, the time being moved back one hour at North Platte and one hour at Ogden.
11. If a train leaves Portland, Oregon at 8:10 p. m. and arrives at Seattle, Washington at 1:30 p.m. the following day, what time is required for it to travel between these two cities, there being no change in time involved?
12. If Train No. 101, on the Southern Pacific, leaves New Orleans at 12:10 p. m. on Friday and arrives in El Paso at 9:20 p. m. on Saturday, what time is required to cover the distance? No change in time is involved.

BANKING

A bank is a business organized for the purpose of dealing in money and securities. One of the functions of a bank is to provide a safe place for money belonging to the bank and to others; and a convenient means of paying this money either to the owner or to others as directed by him. Money belonging to a bank and to those who have left it there for safe keeping is loaned to business men who wish to borrow and are willing to pay for the use of the money borrowed. The interest received for the use of the money loaned constitutes the principal income of a bank. Banks are operated under the supervision of the State or Federal laws; this supervision is for the protection of the depositors.

One who leaves money with a bank for safe keeping is known as a *depositor*; the money placed in a bank by a depositor is known as a *deposit*. A *deposit ticket* is a blank form provided by a bank for the convenience of its depositors when listing the cash or cash items to be deposited in the bank. A *check* is a blank form provided by the bank for the convenience of its depositors when withdrawing the funds deposited with it.

The illustration at the right shows a deposit ticket with cash and cash items deposited. Cash includes paper money, gold, silver, and small change; cash items include bank checks. It is customary to arrange the currency in the order of the denominations, with the smaller denominations on top and to place small change in coin envelopes made to contain 50 pennies, 40 nickels, 50 dimes, and 40 quarters. Each check deposited is listed separately. The name of the city is written at the left if the check is payable on a bank in another city, and the name of the bank if it is payable in the same city. There are no universal rules applicable to the preparation of a deposit ticket, the details depending upon the regulations of the bank in which the deposit is made. You are advised to consult the receiving teller regarding the listing of deposits should you have occasion to deposit money in a bank. The receiving teller gives the depositor a receipt by writing the amount of each deposit in his pass book. This pass book is a small memorandum book provided by the bank.

DEPOSITED IN		
THE PEOPLES BANK OF ERIE		
By <u>James Madison</u>		
ERIE, PA. <u>Jan. 12</u> 19		
PLEASE LIST EACH CHECK SEPARATELY		
	DOLLARS	CENTS
CURRENCY	100	
GOLD		
SILVER	56	52
CHECKS		
Chicago, Ill.	133	40
Denver, Colo.	93	60
Cincinnati, O.	66	22
Springfield, Ill.	110	20
Detroit, Mich.	127	86
First National	450	76
State National	17	62
	1156	18

The blank checks provided by a bank are usually bound in a book, each check attached to a stub. The book may contain one or more checks to the page. The illustration below shows a check book with two checks to a page. A stub is provided for each check in order that the depositor may have a permanent record of the checks he has issued. The bank account is kept on the stub by adding each deposit, and subtracting each check. The record on the check stubs should correspond with the record on the books of the bank when all the checks have been paid.

Deposit, <u>Jan 5</u> 19 <u>19</u> <u>3000</u> No. <u>1</u> Date <u>Jan 5</u> 19 <u>19</u> Pay to the order of <u>Loretta M. Willenborg</u> For the full of acct. Amount, \$ <u>53.75</u> Balance <u>2946.25</u>		<u>No. 1</u> <u>Jan 5 1919</u> MERCHANTS NATIONAL BANK Pay to the order of <u>Loretta M. Willenborg</u> <u>\$53.75</u> <u>Fifty three and 75/100 Dollars</u> <u>James Madison</u> <u>By Student</u>
Deposit, _____ No. <u>2</u> Date <u>Jan 8</u> 19 <u>19</u> Pay to the order of <u>P. A. Mortenson</u> For cash purchase Amount \$ <u>200.00</u> Bal. Carried Forward <u>2746.25</u>		<u>No. 2</u> <u>Jan 8 1919</u> MERCHANTS NATIONAL BANK Pay to the order of <u>P. A. Mortenson</u> <u>\$200.00</u> <u>Two Hundred and 00/100 Dollars</u> <u>James Madison</u> <u>By Student</u>

In the first check James Madison orders the Merchants National Bank to pay \$53.75 to Loretta M. Willenborg. The record of this check on the stub indicates that Mr. Madison owed Miss Willenborg and that the check was issued in payment of the full amount he owed. The information on the stub also indicates that before writing the check Mr. Madison had \$3,000 but after it was written, he had only \$2,946.25.

In the second check Mr. Madison orders the bank to pay \$200 to P. A. Mortenson. The record on the check stub shows that Mr. Madison bought some merchandise from Mr. Mortenson and this check is given in payment of it. The record on the check stub also shows that Mr. Madison had \$2,746.25 in the bank after the check was written.

The depositor should state the information in the check in such a manner that it cannot be changed. If the amount of the check is changed through carelessness on the part of the depositor, the bank will not be responsible. There are many machines available for inserting the amount in a check as a means of insuring that the amount to be paid when the check is presented to the bank is that which the depositor had intended to be paid.

A *Bank Statement* is a form rendered monthly by the bank to each depositor. It contains a list of the deposits made by the depositor, the amount of each check issued by the depositor and paid by the bank, and the balance the depositor has in the bank. This balance may not be the same as that shown in the depositor's record on the check stubs, because all checks issued by the depositor may not have been presented to the bank for payment. Since the bank will return to the depositor with the statement all checks it has paid, the depositor can easily reconcile his balance with the bank's balance by checking the canceled checks with the stubs. The total of the checks not paid should be the difference between the balance shown by the bank and the balance shown by the depositor's record.

The illustration on page 51 shows the form of monthly statement rendered by the bank.

EXERCISE No. 37, TRANSACTIONS WITH A BANK

1. During the month of January, the Mills Restaurant Company performed the following transactions with the Merchants National Bank:

- Jan. 2. Deposited \$1,476.52
- 4. Issued check No. 1 for \$35
- 6. Issued check No. 2 for \$47.96
- 7. Issued check No. 3 for \$74.87
- 9. Issued check No. 4 for \$13.55
- 10. Deposited \$1,231.99
- 10. Issued check No. 5 for \$206.48
- 12. Issued check No. 6 for \$198.54
- 13. Issued check No. 7 for \$54.25
- 14. Issued check No. 8 for \$75.40
- 16. Issued check No. 9 for \$146.14
- 17. Deposited \$917.45
- 19. Issued check No. 10 for \$255.80
- 23. Issued check No. 11 for \$87.87
- 24. Issued check No. 12 for \$205.75
- 24. Deposited \$1,006.38
- 25. Issued check No. 13 for \$19.20
- 26. Issued check No. 14 for \$185.95

(Concluded on page 52)

Williamson, James K., IN ACCOUNT WITH THE FIFTH-THIRD NATIONAL BANK OF CINCINNATI, OHIO.				<u>15</u> VOUCHERS RETURNED	
REPORT PROMPTLY ANY CHANGE IN YOUR ADDRESS					
PLEASE EXAMINE AT ONCE IF NO ERROR IS REPORTED WITHIN TEN DAYS THE ACCOUNT WILL BE CONSIDERED CORRECT					
DATE	CHECKS IN DETAIL			DATE	DEPOSITS
	BALANCE BROUGHT FORWARD			Sept. 1	\$613.28
Sept. 1	100.00	20.00	6.25		
9	1.00	2.08		Sept. 10	150.00
15	6.00 1.82	21.76	9.13		
20	16.19	50.00		Sept. 20	50.00
				Sept. 25	150.00
30	12.50 101.28	16.50	19.00		
				BALANCE	\$579.77
THIS BANK INVITES YOU TO OPEN A SAVINGS ACCOUNT, TO RENT A SAFE DEPOSIT BOX IN OUR SAFETY VAULTS, TO PATRONIZE OUR BOND TRAVEL AND FOREIGN DEPARTMENTS					

This statement was prepared by The Fifth-Third National Bank as a record of its transactions with James K. Williamson. The amounts shown on the left side are checks which have been paid by the bank. The entry on September 1 shows three checks paid, \$100, \$20, \$6.25; September 9 two checks, \$1, \$2.08, etc.

The amounts on the right show the money on hand at the beginning of the month and that deposited during the month. The first entry on the right shows a balance of \$613.28 on September 1. The second entry on the right is a deposit of \$150 on September 10; the third, a deposit on September 20 of \$50, and the fourth, a deposit on September 25 of \$150.

The balance, \$579.77, is ascertained by subtracting the total of the checks from the sum of the total deposits and the balance on hand at the beginning of the month. The depositor compares the canceled checks with his check stubs, indicating by a check mark on the stub those which have been returned. If one or more checks drawn on the bank have not been presented for payment, the balance on the depositor's check stub will be different from that shown by the bank, the difference being the total of the unpaid checks. Comparing the bank statement with the depositor's record is known as *reconciliation* of the bank account.

(Exercise No. 37 [1]—Continued from page 50)

- 27. Issued check No. 15 for \$3.65
- 28. Issued check No. 16 for \$125.60
- 31. Issued check No. 17 for \$210.35
- 31. Deposited \$1,158.64

You are required to show on tablet paper, (a) the balance in the bank after each deposit is added and each check written, (b) the reconciliation of the bank balance, \$4,120.01, shown by the monthly bank statement, with checks Nos. 9, 15, and 16 unpaid.

2. The C. M. Yoder Laundry Machinery Company deposited the following cash and cash items in the Lincoln National Bank of Rochester, New York on May 6:

Currency, \$122

Silver, \$7.65

Check on Dallas, Texas, \$275.55

Check on Fresno, California, \$257.40

Check on Tacoma, Washington, \$800

Check on Bangor, Maine, \$375.44

Check on Atlanta, Georgia, \$442.18

Check on Miami, Florida, \$627.86

Check on New Orleans, Louisiana, \$12.50

Check on Owensboro, Kentucky, \$562.75

The bank allows credit for the amount of this deposit, less \$2.68, the charges for collecting the out of town checks. Prepare a deposit ticket similar to the illustration, using tablet paper. Show (a) the amount of the cash and cash items, and (b) the amount of the credit after deducting the collection charges; this deduction is indicated by writing "less" on a line with the collection charge, and writing the collection charge immediately under the total and subtracting.

3. July 1, H. A. Porter received a statement from the City National Bank showing his balance to be \$1,456.85. This is \$200 more than the balance shown on the check stubs, because check No. 71, written on June 30 has not been paid by the bank. During the month of July he performed the following transactions with the bank:

July 3. Issued check No. 72 for \$50

5. Issued check No. 73 for \$17.85

(Concluded on page 53)

(Exercise No. 37 [3]—Continued from page 52)

7. Issued check No. 74 for \$846.55
9. Deposited \$256.40
11. Issued check No. 75 for \$208.62
15. Issued check No. 76 for \$13.15
17. Deposited \$144.67
18. Issued check No. 77 for \$36.55
21. Issued check No. 78 for \$175
25. Deposited \$150
28. Deposited \$18.55
28. Issued check No. 79 for \$55.60
30. Issued check No. 80 for \$98.47

August 1, he received from the bank a statement showing a balance of \$403.15 with all checks issued returned except No. 80. On tablet paper show the check stub calculations, then verify the bank balance with the balance shown on the check stubs. Report the amount of the discrepancy to the instructor for explanation.

4. L. Harrison deposited the following cash and cash items in the Cleveland National Bank, Cleveland, Ohio on August 6:

Currency, \$250

Silver, \$37.58

Check on First Nat'l Bank, \$55.40

Check on Cincinnati, Ohio, \$128.65

Check on New York City, \$827.65

Check on Chicago, Illinois, \$247.85

Check on Milwaukee, Wisconsin, \$118.72

The bank credits his account with the full amount of his deposit. Prepare a deposit ticket showing the amount with which Mr. Harrison's account was credited.

5. J. L. Miller deposited \$873 with the Farmers National Bank on October 1. During October, November and December he performed the following transactions:

- Oct. 5. Issued check for \$46.35
5. Issued check for \$75.25
11. Issued check for \$104.56
15. Deposited \$645
18. Issued check for \$45.95
25. Issued check for \$203.33

(Concluded on page 54)

(Exercise No. 37 [5]—Continued from page 52)

- Nov. 1. Deposited \$402.25
 3. Issued check for \$55.23
 7. Issued check for \$72.33
 12. Issued check for \$25.50
 15. Issued check for \$64.45
 15. Issued check for \$25.48
 20. Deposited \$105.45
 25. Issued check for \$69.93
 Dec. 1. Deposited \$809.96
 3. Issued check for \$55.25
 3. Issued check for \$25.42
 8. Issued check for \$14.15
 15. Deposited \$785.50
 18. Issued check for \$86.07
 28. Issued check for \$45.55

January 2, he received a statement from the bank showing a balance of \$2,692.84 with checks issued on November 7 and December 8 still unpaid. Verify this by showing on tablet paper the same information Mr. Miller would have on his check stubs, assuming that the two checks are unpaid.

EXERCISE No. 38, ORAL SUBTRACTION

The following advertisement setting forth bargain prices appeared in a daily newspaper. You are to state the saving to the purchaser on each article in accordance with the prices in the advertisement.

50c Jonteel Cold Cream 39c	50c Pepsodent Tooth Paste 36c
25c Cuticura Soap 18c	50c Ipana Tooth Paste 37c
25c Resinol Soap 19c	35c Ponds Vanishing Cream 23c
25c Woodbury Soap 17c	75c Stacomb 59c
\$1.25 Lilac Vegetal 86c	50c Jonteel Van. Cream 39c
75c Java Rice Powder 39c	\$1.00 Listerine 73c
50c Hinds Cream 35c	\$1.00 Rem for coughs 69c
50c Mennen's Cream 34c	75c Olive Oil 49c

CASH ACCOUNT

A record of cash received and cash paid made in the form illustrated at the top of page 55 is known as a *Cash account*. The account form may also be used for recording other facts when it is desired to indicate addition and subtraction by position.

CASH

Feb.	1		1000		Feb.	1		725	50
	2		25	50		2		50	
	5		232	65		9		42	50
	10		118	45		10		137	50
	12		87	45		16		42	50
	17		112	65		18		187	50
	20		98	40		23		42	50
	24		109	20		25		13	50
	26		98	40		26		41	75
	28		88	90		28	Balance	688	35
			1971	60				1971	60
Mar.	1	Balance	688	35					

The illustration shows a record of the following transactions:

February 1, C. H. Moore invested \$1,000 in the Grocery business.

Cash received is recorded on the left side under the date received.

February 1, paid \$725.50 for stock of merchandise.

Cash paid is recorded on the right side under the date it was paid.

February 2, received \$25.50 for merchandise sold.

Cash received is recorded on the left side under the date received with the figures arranged for addition.

February 2, paid \$50 for rent.

Cash paid is recorded on the right side under the date paid with the figures arranged for addition.

Feb. 5. Received \$232.65 for merchandise sold

9. Paid clerks \$42.50

10. Received \$118.45 for merchandise sold

10. Paid \$137.50 for merchandise purchased

12. Received \$87.45 for merchandise purchased

16. Paid \$42.50 in cash to clerks

17. Received \$112.65 for merchandise sold

18. Paid \$187.50 for merchandise purchased

20. Received \$98.40 for merchandise sold

23. Paid clerks \$42.50

24. Received for merchandise sold \$109.20

25. Paid \$13.50 for merchandise purchased

26. Received \$98.40 for merchandise sold

26. Paid \$41.75 for merchandise purchased

28. Received \$88.90 for merchandise sold

The bookkeeping term used to describe the left side of an account is *debit*, the right side, *credit*, and the difference between the two sides, *balance*. The cash on hand should be the same as the balance which is the difference between the total debits and total credits. Ascertaining this fact is known as *proving cash*.

EXERCISE No. 39, CASH ACCOUNT

The following problems provide written practice in recording transactions in account form. You are to enter the amounts on the debit or credit side of the account, as in the illustration of the Cash account, or as instructed in the exercise.

1. D. A. Wells, a retail fruit merchant, performed the following cash transactions during the first week in June:

- June 1. Balance on hand, \$126.80; paid rent, \$60; cash sales for the day as per cash register, \$75.25.
- 2. Paid for fruit and express, \$37.85; received for cash sales as per cash register, \$49.19.
- 3. Paid clerk \$10; received for cash sales as per cash register, \$39.14.
- 4. Paid occupational tax, \$2.50; paid for fruit purchased, \$22.85; received for cash sales as per cash register, \$22.19.
- 5. Paid telephone bill, \$2.75; paid for fruit purchased, \$39.16; received for cash sales as per cash register, \$41.77.
- 6. Paid gas and electric bill, \$1.57; paid for fruit purchased, \$29.65; paid balance due clerk, \$7.50; cash sales as per cash register, \$52.65.

Record these transactions in account form as illustrated on page 55. The balance on hand, each receipt, and the date received is entered on the left; each payment, and the date paid is entered on the right. The balance of cash on hand should be the difference between the total debits and the total credits.

2. A. L. Graham completed the following cash transactions during the first fifteen days in March:

- Mar. 1. Invested \$3,500 in cash
- 2. Paid for team and wagon, \$300
- 3. Paid for office and store fixtures, \$650

(Concluded on page 57)

(Exercise No. 39 [2]—Continued from page 56)

5. Received for cash sales, \$304.87; paid cash for merchandise, \$1,500
6. Received from Caleb Brothers, \$50; paid sundry expenses, \$127
7. Paid \$150 for safe; received from O. H. Arnold, \$64
8. Cash purchases, \$330.75; received for cash sales, \$204.18
9. Received from H. F. Ritter, \$15; cash purchase, \$28.65
10. Cash sales, \$264.85; received from O. B. Donaldson, \$50
12. Cash sales, \$15.79; paid Belknap Hardware Co., \$629.87
14. Paid sundry expenses, \$72.50; paid C. R. Crawford, \$75
15. Received from H. F. Ritter, \$20; borrowed from the bank, \$1,000

What is his cash balance on March 16?

3. Alvin Young performed the following transactions with his grocer during the month of March. The grocer entered the date and amount of each sale on the debit side of the account with Mr. Young; and the date and amount of cash received from Mr. Young on the credit side, also the value, date, and amount of all goods returned for credit. The difference between the sales on the debit side, and the credits for cash plus the credits for goods returned is the amount due the grocer from Mr. Young.

- Mar. 1. Purchased groceries, \$2.65
5. Purchased groceries, \$9.87
 7. Purchased groceries, \$11.42
 9. Paid \$10 in part payment for previous purchases
 12. Purchased groceries, \$7.16
 13. Purchased groceries, 88c
 14. Purchased groceries, \$9.65
 16. Paid \$20 in part payment for previous purchases
 18. Purchased groceries, \$4.19
 21. Purchased groceries, \$7.15

(Concluded on page 58)

(Exercise No. 39 [3]—Continued from page 57)

- 24. Returned 3 cans tomatoes at 35c each for credit
- 25. Purchased groceries, \$1.65
- 28. Purchased groceries, \$15.44

How much does Mr. Young owe the grocer April 1?

You will record the above transactions in account form under the name Alvin Young. The balance due April 1, will be the difference between the total of the debit side and the total of the credit side.

One who buys from a merchant on account, that is, who promises to pay on the first of each month, may know the amount he owes the merchant if he keeps a record of the sales tickets or sales invoices which the merchant renders with each sale of merchandise. It is quite evident that the customer is interested in knowing that the facts set forth in the statement are correct. He should retain these invoices so that he may check the amount of each with the amounts in the statement because there is always a possibility of the merchant's charging one customer with merchandise purchased by another.

4. The monthly sales and returned sales of the Fair Store during the year beginning with January were as follows:

Jan. 31.	Sales, \$11,642.91; returned sales, \$418.23
Feb. 28.	Sales, \$ 9,487.25; returned sales, \$325.87
Mar. 31.	Sales, \$13,587.44; returned sales, \$225.45
Apr. 30.	Sales, \$13,795.53; returned sales, \$427.45
May 31.	Sales, \$14,893.38; returned sales, \$362.25
June 30.	Sales, \$12,374.85; returned sales, \$259.95
July 31.	Sales, \$12,375.82; returned sales, \$343.73
Aug. 31.	Sales, \$11,385.82; returned sales, \$389.92
Sept. 30.	Sales, \$ 9,323.44; returned sales, \$258.45
Oct. 31.	Sales, \$10,682.25; returned sales, \$268.43
Nov. 30.	Sales, \$10,928.44; returned sales, \$284.42
Dec. 31.	Sales, \$11,483.35; returned sales, \$325.55

Record the above transactions in account form under the title "Sales". Enter the date and total sales for each month on the credit side, and the date and total returns on the debit side. The balance, which is the difference between the two totals, will be the net sales made during the year. The Fair Store keeps a record of its sales, because this information is needed in preparing its Income Tax statement.

5. Robert Browning, a clerk, receives a salary of \$300 per month. He wishes to keep a record of his income from his salary,

and his living expenses so that he may know the excess of income over expenses. During the quarter ending March 31, his income and expenses were as follows:

- Jan. 1. Received December salary, \$300; paid rent, \$52
- 2. Paid light bill, \$2.80; telephone, \$2.25
- 15. Paid \$12.50 for clothing
- 31. Paid \$38.65 for groceries and meat
- Feb. 1. Received January salary, \$300; paid rent, \$52;
paid insurance, \$113.85
- 2. Paid for light, \$2.45; telephone, \$2.25
- 28. Paid \$32.45 for groceries and meat
- Mar. 1. Received February salary, \$300; paid for clothing,
\$35.80
- 2. Paid for light, \$1.65; telephone, \$2.25
- 15. Paid insurance on household furnishings, \$18.50
- 18. Paid for house furnishings, \$55.75
- 31. Paid \$35.60 for groceries and meat

Record the above transactions in account form under the title "Income and Expenses". Income is recorded on the credit side of the account and expenses on the debit side. The difference between the total income and total expenses is Mr. Browning's savings during the three months. On the left side under date of April 1, write "Savings" and the amount and rule the account in the same form as the Cash account.

6. R. H. Norman, a farmer, wishes to keep a record of the sales of chickens and eggs and his expenses in connection with the upkeep of the poultry. During the first six months of the year, he completed the following transactions.

- Jan. 2. Purchased poultry \$126.50
- 7. Sales of chickens and eggs, \$12.65
- 12. Paid for feed \$3.50
- 15. Sales of chickens and eggs, \$18.75
- 18. Paid for feed \$4.50
- 24. Sales of chickens and eggs, \$15.50
- 27. Paid for feed \$2.25
- Feb. 6. Sales of chickens and eggs, \$16.45
- 9. Purchased poultry \$45.50
- 10. Paid for feed \$1.75

(Concluded on page 60)

(Exercise No. 39 [6]—Continued from page 59)

- 18. Sales of chickens and eggs, \$21.45
- 21. Paid for feed \$3
- 25. Sales of chickens and eggs, \$19.65
- 28. Paid for feed \$1.25
- Mar. 8. Sales of chickens and eggs, \$20.55
- 10. Paid for feed \$2.55
- 19. Sales of chickens and eggs, \$22.75
- 21. Paid for feed \$3.25
- 27. Sales of chickens and eggs, \$21.50
- 29. Paid for feed \$1.75
- April 2. Sales of chickens and eggs, \$22.55
- 3. Paid for feed \$3
- 15. Sales of chickens and eggs, \$17.50
- 18. Paid for feed \$3.25
- 27. Sales of chickens and eggs, \$18.95
- 29. Paid for feed \$2.25
- May 8. Sales of chickens and eggs, \$21.75
- 9. Paid for feed \$2.50
- 18. Sales of chickens and eggs, \$23.50
- 19. Paid for feed \$1.50
- 25. Sales of chickens and eggs, \$12.45
- 27. Paid for feed \$1.75
- June 6. Sales of chickens and eggs, \$15.50
- 19. Sales of chickens and eggs, \$14.75
- 20. Paid for feed \$2.15
- 26. Sales of chickens and eggs, \$12.95
- 27. Paid for feed \$1.25
- 30. Sold all chickens and eggs on hand for \$129.50

You are to record in account form under the title "Poultry Income and Expense" the transactions outlined above. The cost of the chickens and feed should be recorded on the debit side, and the amounts received from sales recorded on the credit side. The balance will be the net income; that is, the cash received in excess of the cash paid. When you have ascertained the profit, enter this on the debit side under date of June 30 and rule the account.

CHAPTER V

MULTIPLICATION

The purpose of this chapter is to provide practice in multiplication, short-cut methods for applying the process, and proof of the product. The information is given under the assumption that you understand the mathematical principles involved in the multiplication process.

Multiplication is a short method of addition. The *product* is the result obtained by multiplication, and is the same as the sum, if the process were by addition.

The *multiplicand* and the *multiplier* are the factors of the product. In other words, the multiplicand is the number to be added, and the multiplier the number of times it is to be added. If the number to be added refers to some specific article, it is quite evident that the product obtained by adding the desired number of times would refer to the same article.

Illustration. A farmer sells five bushels of wheat at \$1.60 per bushel. Five is the multiplier, and \$1.60 the multiplicand. The sum of \$1.60 repeated five times in a column is \$8.00. If this problem had been 160 pounds of rice at 5c per pound, the multiplier would have been 160 and the multiplicand would have been five. The sum of five repeated 160 times in a column and added is \$8.00. This illustration will show you that it is necessary to consider the nature of the problem and the information desired, rather than to try to memorize any definite rules regarding the multiplier and multiplicand.

The *process of multiplication* when applied to business problems is usually indicated by writing the word "at" or the symbol "@" between the multiplier and the multiplicand. Sometimes in invoices where a statement of the facts indicates multiplication, the word "at" and the symbol may be omitted.

Illustration. "10 lbs. of coffee at 35c" is equal to 35 multiplied by 10; the product is \$3.50 because 35c is added ten times. Where the price applies to a specific quantity the word "at" is omitted; "10 lbs. salt, 75c", indicates that the selling price of the entire quantity is 75c.

Accuracy in multiplication is of great importance especially in the solution of those business problems which relate to the

amount of a sale. If the multiplication is incorrect, either the buyer or the seller loses if the error is not discovered.

Illustration. You purchased from the butcher 9 lbs. of meat at 29c per lb., and gave him a five-dollar bill in payment. He made an error of 10c in his calculations, recording the amount as \$2.51 instead of \$2.61 and gave you change accordingly. At the end of the day he proved his record of cash by counting the cash in the cash register. This error of 10c would not be detected by his proof, because the amount recorded and the amount of cash received, which serve as a proof of his cash record, were the same.

EXERCISE No. 40, THE MULTIPLICATION TABLE

The purpose of the multiplication table given below is to provide oral practice in the multiplication of numbers up to and including 15 multiplied by 15.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

EXERCISE No. 41, MULTIPLICATION

The purpose of the following problems is to provide further oral drill in multiplication.

1	2	3	4	5	6
a. 15×9	a. 14×13	a. 15×12	a. 15×13	a. 12×11	a. 15×8
b. 13×11	b. 7×15	b. 14×8	b. 12×11	b. 13×12	b. 7×11
c. 14×7	c. 12×13	c. 13×11	c. 13×14	c. 14×13	c. 13×15
d. 11×15	d. 14×12	d. 9×14	d. 14×12	d. 13×13	d. 14×9
e. 15×7	e. 12×15	e. 14×11	e. 13×7	e. 11×15	e. 9×15
f. 9×12	f. 13×11	f. 13×9	f. 12×11	f. 13×11	f. 12×11

EXERCISE No. 42, PROCESS AND PROOF OF MULTIPLICATION

Multiplication is a short-cut process of addition, hence the product can be proved in the same manner as the sum in addition, that is, by reverse process and by check figures. The illustration below shows the process of multiplication and proof by reverse multiplication applied to the problem 671 multiplied by 196.

671		196
196	You will observe that the multiplier and	671
<u> </u>	multiplicand are reversed in order to prove	<u> </u>
4026	by reverse multiplication. It is better when	196
6039	proving by this method to make the calculations	1372
671	on a separate sheet of paper, then	1176
<u> </u>	compare the results.	<u> </u>
131516		131516

The following problems provide written practice in multiplication. You are to show on tablet paper the process of multiplication with the multiplicand above the multiplier and the process by the reverse method in which the multiplicand is below the multiplier.

1	2	3	4
a. $2,792 \times 347$	a. $6,453 \times 999$	a. $3,765 \times 2,839$	a. $7,837 \times 4,723$
b. $3,806 \times 342$	b. $4,700 \times 387$	b. $4,274 \times 3,746$	b. $1,238 \times 3,746$
c. $4,785 \times 617$	c. $8,876 \times 427$	c. $3,989 \times 1,894$	c. $8,132 \times 7,325$
d. $8,293 \times 723$	d. $9,423 \times 471$	d. $4,563 \times 8,139$	d. $2,736 \times 4,635$

EXERCISE No. 43, PROOF OF MULTIPLICATION BY A CHECK FIGURE

The product in multiplication can be proved by a check figure the same as the sum in addition. The illustration below shows the process of multiplication and the proof by the check figure 9 applied to the problem 671 multiplied by 196.

$$671 = 6 + 7 + 1 = 14 = 5$$

$$196 = 1 + 9 + 6 = 16 = 7$$

$$4026$$

$$6039$$

$$671$$

$$131516 = 1 + 3 + 1 + 5 + 1 + 6 = 17 = 8$$

$$35 = 8$$

Eliminating the nines from the multiplicand gives a remainder of 5 and from the multiplier a remainder of 7; 5 multiplied by 7 equals 35; eliminating the nines from 35 gives the proof figure 8; eliminating

the nines from the product gives the proof figure 8 which indicates the results are correct because the two proof figures are the same.

The following problems provide written practice in multiplication and proof by the check figures 9 and 11. You will show on tablet paper the multiplication process in each problem and prove problems 1 and 3 with the check figure 9, and problems 2 and 4 with the check figure 11. Arrange your work the same as in the illustration.

1	2	3	4
<i>a.</i> $4,875 \times 329$	<i>a.</i> $2,938 \times 643$	<i>a.</i> $5,907 \times 335$	<i>a.</i> $1,475 \times 317$
<i>b.</i> $9,802 \times 678$	<i>b.</i> $7,214 \times 521$	<i>b.</i> $6,218 \times 865$	<i>b.</i> $3,918 \times 594$
<i>c.</i> $7,743 \times 237$	<i>c.</i> $8,347 \times 423$	<i>c.</i> $4,253 \times 789$	<i>c.</i> $6,714 \times 845$
<i>d.</i> $6,584 \times 567$	<i>d.</i> $1,987 \times 925$	<i>d.</i> $9,216 \times 415$	<i>d.</i> $8,231 \times 953$

EXERCISE No. 44, ADDITION, SUBTRACTION, AND MULTIPLICATION

The following problems provide oral practice in adding, subtracting, and multiplying. You will complete the operations in the order given as indicated by the signs.

The mental process of completing problem 1a would be "3 times 4 equals 12, plus 8 equals 20, minus 10 equals 10, times 4 equals 40, times 2 equals 80," or expressed more briefly "12, 20, 10, 40, 80".

- | 1 | 2 |
|---|--|
| <i>a.</i> $3 \times 4 + 8 - 10 \times 4 \times 2 =$ | <i>a.</i> $6 \times 6 - 6 \times 3 + 8 - 48 =$ |
| <i>b.</i> $6 \times 7 - 2 \times 3 + 8 - 60 =$ | <i>b.</i> $8 + 7 \times 3 - 5 + 9 \times 4 =$ |
| <i>c.</i> $9 + 6 - 5 \times 3 + 7 + 3 =$ | <i>c.</i> $4 \times 5 - 9 + 9 \times 3 - 40 =$ |
| <i>d.</i> $9 - 3 \times 8 - 8 + 2 \times 4 =$ | <i>d.</i> $3 \times 8 - 4 \times 3 + 6 - 36 =$ |
| 3 | 4 |
| <i>a.</i> $5 \times 4 - 4 \times 3 + 8 - 16 =$ | <i>a.</i> $4 - 2 \times 8 - 6 \times 4 \times 3 =$ |
| <i>b.</i> $9 + 1 \times 5 - 8 + 2 + 40 =$ | <i>b.</i> $8 + 2 \times 5 + 8 - 3 \times 2 =$ |
| <i>c.</i> $8 \times 5 - 8 - 2 \times 3 - 25 =$ | <i>c.</i> $4 \times 5 + 8 \times 2 + 9 - 35 =$ |
| <i>d.</i> $8 - 2 \times 5 - 8 + 2 \times 4 =$ | <i>d.</i> $9 - 3 \times 5 + 6 - 3 \times 3 =$ |
5. $6 \times 5 - 20 \times 4 + 8 - 18 + 2 - 16 + 4 - 5 \times 3 + 5 \times 2 - 50 - 25 =$
6. $9 + 1 \times 4 - 5 + 9 \times 1 - 4 + 20 - 40 \times 2 + 8 + 7 - 5 \times 2 - 60 + 5 =$

SHORT-CUTS IN MULTIPLICATION

There are no short-cuts in the process of multiplication which are applicable in all cases, but, there are conditions under which short-cut methods may be used to advantage. Some of these are illustrated in the following problems. Give careful attention to each process so that you may become more proficient in multiplication.

EXERCISE No. 45, MULTIPLYING BY A NUMBER ENDING IN ONE

When the digit in the units column of the multiplier is one, time can be saved by proceeding as illustrated below.

$$\begin{array}{r}
 735 \times 31 = 735 \\
 \underline{2205} \\
 22785
 \end{array}
 \qquad
 \begin{array}{r}
 \$7.35 \times 31 = 735 \\
 \underline{2205} \\
 \$227.85
 \end{array}
 \qquad
 \begin{array}{r}
 \$4.67 \times 111 = 467 \\
 \underline{5137} \\
 \$518.37
 \end{array}$$

You will observe that the product of the multiplication by the digit in the tens column of the multiplier is written underneath the multiplicand, and that when dollars and cents are involved, the cents are indicated in the product.

The following problems provide written practice in multiplying when the multiplier ends in one. Arrange your work as in the illustration.

1	2	3	4
a. 845×21	a. $\$38.64 \times 121$	a. 745×161	a. $485 \times 51c$
b. 607×91	b. $\$45.25 \times 151$	b. 869×181	b. $839 \times 21c$
c. 184×31	c. $\$93.34 \times 131$	c. 723×131	c. $784 \times 41c$
d. 726×51	d. $\$67.45 \times 171$	d. 398×141	d. $287 \times 31c$
e. 815×81	e. $\$92.89 \times 141$	e. 757×121	e. $948 \times 81c$
f. 526×41	f. $\$78.26 \times 161$	f. 485×151	f. $746 \times 71c$

EXERCISE No. 46, SPECIAL

When the sum of the digits in the units column of both multiplier and multiplicand is ten, and the digits in the tens column are alike, the process below may be applied with a saving of time. In this problem it is desired to multiply 32 by 38.

$$\begin{array}{r}
 32 \\
 38 \\
 \hline
 1216
 \end{array}
 \qquad
 \begin{array}{l}
 \text{Write the product of the unit digits, } 8 \times 2 = 16. \\
 \text{Add 1 to the digit in the tens column of the multiplier,} \\
 \text{multiply by the digit in the tens column of the multiplicand,} \\
 \text{and bring down the product, } 3 + 1 \times 3 = 12.
 \end{array}$$

The following problems provide written practice in the application of the short-cut method described above.

1	2	3	4
a. $25 \times 25 =$	a. $75 \times 75 =$	a. $52 \times 58 =$	a. $73 \times 77 =$
b. $27 \times 23 =$	b. $45 \times 45 =$	b. $59 \times 51 = *$	b. $99 \times 91 =$
c. $82 \times 88 =$	c. $33 \times 37 =$	c. $65 \times 65 =$	c. $36 \times 34 =$

*NOTE: When the product of the digits in the units column is less than 10, place a cipher in the tens place of the product as illustrated at the right.

$$\begin{array}{r}
 59 \\
 51 \\
 \hline
 3009
 \end{array}$$

EXERCISE No. 47, SPECIAL

When the units digits in the multiplier and multiplicand are the same and the sum of the tens digits is ten, the solution below may be used. This shows the multiplication of 76 by 36.

$$\begin{array}{r} 76 \\ 36 \\ \hline 2736 \end{array}$$

The illustration at the left shows the solution of 76×36 . The product of the units digits, 36, is written below the line and the units digit is added to the product of the tens digits and this amount written at the left of the product of the units digits. $6 \times 6 = 36$; $3 \times 7 = 21$; $21 + 6 = 27$; $2,736 =$ the product.

The following problems provide oral practice in applying the process of multiplication explained in the above illustration.

1	2	3	4
a. $28 \times 88 =$	a. $61 \times 41 =$	a. $34 \times 74 =$	a. $54 \times 54 =$
b. $45 \times 65 =$	b. $24 \times 84 =$	b. $36 \times 76 =$	b. $48 \times 68 =$
c. $73 \times 33 =$	c. $77 \times 37 =$	c. $62 \times 42 =$	c. $23 \times 83 =$
d. $29 \times 89 =$	d. $32 \times 72 =$	d. $16 \times 96 =$	d. $71 \times 31 =$

EXERCISE No. 48, CROSS MULTIPLICATION

Cross multiplication may be applied to an advantage when neither of the combinations illustrated in the two preceding exercises are applicable. The illustration below shows the process of multiplying 37 by 42, and the information at the right explains the process.

$$\begin{array}{r} 37 \\ 42 \\ \hline 1554 \end{array}$$

$2 \times 7 = 14$, place the 4 and carry the 1.

$4 \times 7 = 28$ and $2 \times 3 = 6$.

$28 + 6 + 1 = 35$, place the 5 and carry the 3.

$4 \times 3 = 12 + 3 = 15$.

The following problems provide written practice in applying cross multiplication. Show the information at the right of the illustration in connection with each problem in the first column; and the multiplicand, multiplier, and product only for those in the other columns.

1	2	3	4
a. $24 \times 37 =$	a. $81 \times 24 =$	a. $32 \times 17 =$	a. $63 \times 24 =$
b. $55 \times 28 =$	b. $56 \times 45 =$	b. $72 \times 19 =$	b. $82 \times 45 =$
c. $73 \times 24 =$	c. $59 \times 28 =$	c. $35 \times 58 =$	c. $21 \times 36 =$
d. $59 \times 73 =$	d. $29 \times 16 =$	d. $24 \times 31 =$	d. $37 \times 42 =$

EXERCISE No. 49, NUMBERS ENDING WITH FIVE

When the units digits in both the multiplicand and multiplier are five, and the sum of the tens digits is even, the product will end in 25, and the number at the left of this will be the product of the tens digits plus one half of their sum. The illustration below shows this process applied to the multiplication of 85 by 45.

$\begin{array}{r} 85 \\ 45 \\ \hline 3825 \end{array}$	The mental process of multiplying 85 by 45 is 4 times 8 is 32, plus 6 ($\frac{1}{2}$ of [4 plus 8]) is 38. No mental process is necessary in obtaining the two numbers at the right in the product, because they are always 25 when the sum of the tens digits is even.
--	---

When the units digits in both the multiplicand and multiplier are five, and the sum of the tens digits is odd, the product will end in 75, and the number at the left of this will be the product of the tens digits, plus one half of the sum of the tens digits with the fraction discarded.

$\begin{array}{r} 65 \\ 35 \\ \hline 2275 \end{array}$	The mental process of multiplying 65 by 35 is 3 times 6 is 18, plus 4 ($\frac{1}{2}$ of [3 plus 6] with the fraction discarded) is 22. There is no mental process in obtaining the two numbers at the right in the product, because they are always 75 when the sum of the tens digits is odd.
--	--

This method is satisfactory when there are only two or not more than three digits in either the multiplier or multiplicand. When dollars and cents are involved, the two right-hand digits in the product, will be cents, and the digits at the left of the cents will be dollars.

The following problems provide oral practice in applying these short-cut methods of multiplication. Keep in mind the fact that the product of the units digits is known to be either 25 or 75, hence you must concentrate on arriving at the product of the tens digits, then call the entire product beginning with the product of the tens digits.

- | 1 | 2 | 3 |
|--------------------|-----------------------|---------------------|
| a. 95×35 | a. 145×75 | a. $155 \times 35c$ |
| b. 65×45 | b. $\$1.25 \times 35$ | b. $125 \times 25c$ |
| c. 55×35 | c. $\$1.35 \times 25$ | c. 135×45 |
| d. 75×65 | d. $\$1.55 \times 55$ | d. $135 \times 15c$ |
| e. 135×25 | e. $\$1.45 \times 45$ | e. 25×85 |
| f. 142×65 | f. $145 \times 25c$ | f. 165×25 |

EXERCISE No. 50, TEN OR A MULTIPLE OF TEN

When the multiplier is ten or a power of ten, the product is ascertained by adding as many ciphers to the multiplicand as there are ciphers in the multiplier. If dollars and cents are involved, the last two figures in the product will be cents and the figures at the left of the cents will be dollars. The following problems provide oral practice in multiplying by ten or a multiple of ten.

1	2	3
a. \$67,719 $\times 10$	a. \$37.94 $\times 10$	a. \$132.40 $\times 1,000$
b. \$58,836 $\times 10$	b. \$57.78 $\times 10$	b. \$105.55 $\times 1,000$
c. \$47,537 $\times 10$	c. \$67.25 $\times 100$	c. \$156.25 $\times 1,000$
d. \$58,375 $\times 10$	d. \$375.89 $\times 100$	d. \$178.27 $\times 1,000$
e. \$45.35 $\times 10$	e. \$928.38 $\times 100$	e. 3,725 $\times 100$
f. \$78.87 $\times 10$	f. \$747.38 $\times 100$	f. 8,740 $\times 1,000$

EXERCISE No. 51, NINETY-NINE OR ONE HUNDRED ONE

When either the multiplicand or multiplier is 99 or 101, the product may be ascertained in the first case by multiplying by 100 and subtracting the amount of the multiplicand, or in the second case by multiplying by 100 and adding the amount of the multiplicand. $786 \times 99 = 77,814$; $786 \times 101 = 79,386$.

$$786 \times 99 = 78,600 - 786 = 77,814; \quad 786 \times 101 = 78,600 + 786 = 79,386.$$

The same process will apply when the multiplier is 999 or 1,001, but in business problems, the 99 and 101 will be used more frequently. This process might be extended to include 102, 103, 104, etc., but it is better to follow the usual process, because the application of the short-cut method would be more difficult.

When multiplying by 99 or 101 where dollars and cents are involved, the two right-hand figures in the product will be cents, and the figures at the left of the cents will be dollars. The following problems provide written practice in applying this procedure.

1	2	3
a. 48,572 $\times 99c$	a. 8,275 $\times 101$	a. 3,465 $\times \$1.01$
b. 67,392 $\times 99c$	b. 4,738 $\times 101$	b. 7,836 $\times \$1.01$
c. 78,473 $\times 99c$	c. 5,647 $\times 101$	c. 6,927 $\times \$1.01$
d. 92,837 $\times 99c$	d. 4,738 $\times 101$	d. 8,947 $\times \$1.01$
e. 47,384 $\times 99c$	e. 2,938 $\times 101$	e. 7,725 $\times \$1.01$
f. 92,837 $\times 99c$	f. 9,384 $\times 101$	f. 5,427 $\times \$1.01$

EXERCISE No. 52, APPLICATION OF SHORT-CUTS

The following problems provide written practice in applying all of the short-cut methods discussed in the preceding exercises. You are required to write the product of each item, and the total of all items in each problem. Indicate immediately after the total, the exercise number in which the short-cut method used was explained. Prove the addition in each problem by the elimination of nines in the first two, by the elimination of elevens in the second two, and by reverse multiplication in the last one.

1. Find the cost of the following:
 - 89 lbs. bacon at 29c per pound
 - 62 pounds of butter at 42c per pound
 - 37 pounds of prunes at 16c per pound
 - 75 pounds of lard at 15c per pound
 - 145 pounds of tea at 99c per pound
2. Find the cost of:
 - 165 bushels of corn at 85c per bushel
 - 85 bushels of wheat at \$1.25 per bushel
 - 122 bushels of wheat at \$1.28 per bushel
 - 55 bushels of oats at 55c per bushel
3. Find the cost of:
 - 25 dozen pocket knives at \$3.75
 - 36 Gillette razors at 96c each
 - 48 Eveready Flashlights at 67c
 - 22 children's skates at 88c
 - 86 vacuum bottles at 84c
4. Find the cost of:
 - 35 boys' blouses at \$1.95
 - 36 men's socks at 76c
 - 45 men's initialed handkerchiefs at 35c
 - 25 men's socks at 85c
 - 345 children's mufflers at \$1.01
5. Find the cost of:
 - 75 Orchard games at 95c
 - 32 boxes of "Tru-Life Paper Dolls" at 38c
 - 31 "Knock-Out Prize Fighters" at 39c
 - 15 scooters at \$4.75
 - 55 Ham and Sam at \$1.45

BILLING

When merchandise is sold, the buyer receives in addition to the merchandise a list of the merchandise he has purchased. This list is known as an *invoice*; in the retail business it is sometimes referred to as a *sales ticket*. The buyer regards the list of goods which he has bought as a purchase invoice, and the seller regards it as a sales invoice. There is always a buyer and a seller in connection with every sale of merchandise.

There is no standard form of blank to be used for the preparation of invoices. Every business concern provides its own form to meet the desire of the owner, and the conditions under which the business is operating. The illustration below shows one form of an invoice. In addition to the list of merchandise which changed hands as evidenced by the invoice, it contains the date, the name and address of the buyer, the name and address of the seller, the terms, and the method of delivery. The invoice may also contain any other information which the seller may wish it to show. This is explained more fully in connection with the study of bookkeeping where this additional information has a bearing on the bookkeeping records.

H. L. RICHESON & CO. CASH WHOLESALE GROCERS 621 EAST FIRST STREET  SUGAR KINGS DAYTON, OHIO October 15, 192				
SOLD TO W. R. Swanson Grocery Co. 3612 Creighton Ave., Dayton, Ohio.			DATE	
TERMS Net 10 days OUR ORDER NO. S 11161			YOUR ORDER NO. 112	
			DELIVERED VIA truck	
600 lbs. 300 lbs. 120 lbs. 6 doz. 10 doz.	Granulated Sugar (6 seks. 100 lbs. each) Domino Sugar (2 pound cartons) Confectionery Sugar (1 pound cartons) cans Peaches (3 cases) cans Corn (5 cases)	cwt. 5.75 cwt. 6.25 cwt. 7.50 doz. 2.50 doz. 2.00	34 18 9 15 20	50 75 00 00 00
			97	25

Accuracy is essential in ascertaining the value of merchandise sold, because an error in the invoice means a loss to either the buyer or seller. As explained in previous discussions, the recording of business transactions in accordance with the standard bookkeeping

process will not detect errors in the invoice. The usual process of multiplication is applied when it is necessary to ascertain the value of the merchandise sold. Ascertaining the value of merchandise sold as listed on the invoice is known as *making extensions*. Short-cut methods in multiplication may be applied to advantage when making these extensions, thus insuring speed and accuracy.

EXERCISE No. 53, BILLING

The following problems provide written practice in billing in six different lines of merchandise. The information is not given in the form of an invoice, but this is not necessary in order to make the extensions. Write the amount of each extension in the order in which it is given in the problem, and ascertain the total of all items sold. Prove the correctness of the multiplication of each extension, and the addition of the total.

1. Wholesale dry goods:

- 15 bolts of percale at \$4.85*
- 18 yards of velvet at \$3.75
- 8 bolts of domestic cotton at \$4.40
- 39 bolts of binding tape at 31c
- 25 bolts of sheeting at \$15
- 80 yards of flannel at \$1.98

2. Wholesale groceries:

- 12 dozen bars of Palmolive soap at 53c
- 15 dozen cans of tomatoes at \$1.35
- 6 dozen cans of California peaches at \$2.42
- 5 dozen cans of asparagus at \$3.05
- 9 dozen cans of cherries at \$2.25
- 25 dozen Delmonte Alaska salmon at \$2.05

3. Wholesale sugar:

- 1263 pounds (3 barrels) granulated sugar at 7c
- 656 pounds (2 barrels) brown sugar at 6c
- 400 pounds (16 sacks) granulated sugar at 8c
- 225 boxes Domino sugar at 21c
- 82 boxes powdered sugar at 88c
- 35 boxes of powdered sugar at 55c

*NOTE: The price given applies to the quantity mentioned. Thus, if the quantity is given in dozens, the price stated will be per dozen and not per article.

4. Wholesale provisions:

- 855 pounds of leaf lard at 19c
- 125 pounds of breakfast bacon at 35c
- 416 pounds of ham at 27c
- 125 pounds of premium butter at 45c
- 227 pounds of chicken at 43c
- 516 pounds of bacon sides at 16c

5. Mechanical toys:

- 25 Racing cars at 25c
- 35 Crazy cars at 79c
- 42 Automobiles at 62c
- 35 "Spark Plugs" at 95c
- 24 "Bucking Bronchos" at 84c
- 15 "Tut-Tuts" at \$1.05
- 24 Dancing Dolls at 24c

6. Wholesale furniture:

- 16 mahogany smoking cabinets at \$2.48
- 8 buffet mirrors at \$7.65
- 15 reed rockers at \$11.25
- 35 oak chairs at \$1.35
- 52 oak rockers at \$2.25
- 24 kitchen tables at 98c
- 12 bentwood chairs at \$4.20

EXERCISE No. 54, AUDITING

The seller should retain a copy of each invoice which he renders, to facilitate adjustments in case there is a misunderstanding, or in case the buyer should want to return some of the merchandise. This copy is usually made by placing a sheet of carbon paper between two invoices, or between an invoice and a sheet of blank paper. Writing on the invoice produces a copy of the writing, thus providing the seller with an exact copy of the invoice.

The following problems represent the seller's carbon copy of extensions in invoices rendered. A variety of merchandise is represented to familiarize you with merchandise sold by different lines of business. You are to verify each extension, prepare a written solution of those which are not extended correctly, and

show the correct total of all the items. Show your method of proof as well as the process of multiplication.

	<i>Items</i>	<i>Price</i>	<i>Amount</i>
a.	38 pocket knives.....	\$.32	\$ 12.16
b.	66 pocket knives.....	.46	30.36
c.	42 Gillette razors.....	.88	36.06
d.	15 barrels of flour.....	7.25	108.25
e.	9 family tool chests.....	8.69	78.21
f.	12 dozen boys' tan shoes.....	12.65	153.80
g.	21 unbreakable "Mama Dolls" ..	1.69	35.49
h.	48 dozen Ivory soap.....	.68	32.64
i.	8 Crosley radio amplifiers.....	8.85	70.40
j.	15 kiddy scooters.....	1.65	24.25
k.	165 pounds of Red Ribbon butter.	.45	74.25
l.	16 dozen pairs men's silk socks..	7.65	132.40
m.	48 dozen dolls.....	9.99	479.52
n.	21 one-tube radios.....	6.81	133.01
o.	468 pounds of breakfast bacon..	.32	149.76
p.	45 cases Safe Home matches....	5.75	258.75
q.	35 dozen pairs ladies' hose.....	6.55	239.25
r.	655 pounds (25 kegs) 10d nails..	.07	45.85
s.	25 pairs kid gloves.....	2.75	68.25
t.	15 smoking jackets.....	4.95	74.25
u.	31 Baldwin type "C" speakers..	1.85	56.35
v.	7 "Recreator" radio horns.....	21.95	153.65
w.	525 sacks of salt.....	.05	26.25
x.	45 boxes of Red Cross soap.....	2.95	163.25
y.	28 kiddies road rippers.....	7.35	205.80
z.	56 men's ribbed sport coats.....	2.98	166.88
aa.	125 wooden cases.....	.85	106.25
bb.	48 pounds of Jones' sausage.....	.48	23.04
cc.	8 Aragon battery chargers.....	12.35	96.60
dd.	8 crystal radio sets.....	.88	7.04
ee.	48 bath towels.....	.79	35.92
ff.	92 kitchen tables.....	.98	90.16
gg.	250 natural "Mama Dolls"	.99	247.50
hh.	42 girls' moccasin style slippers..	1.01	42.42
ii.	250 boxes Iodized Salt.....	.12	3.00
jj.	31 boxes Crane's Stationery #16	.57	17.67
	Total.....		?

EXERCISE No. 55, INVENTORY

The merchant buys merchandise and places it in stock for the convenience of his customers who wish to purchase it. When operating a business it is necessary for the merchant to maintain a stock of merchandise at all times. At various times, usually at the end of each year, he will want to ascertain the value of all merchandise he has in stock. This is known as *taking stock* and the list of merchandise resulting therefrom, as an *inventory*. The inventory contains a list of the merchandise and the price at which it is invoiced, usually cost, or market price if less than cost.

Problem 1 is an inventory of a retail merchant, taken on January 31; the other problems provide practice in preparing the inventory list and in making the extensions. The process of obtaining the value of each item, and the total of all items is the same as that of extending and adding an invoice.

1. R. T. Foreman, a retail grocer, had in stock on January 31, the merchandise given in the inventory below. You are to write the amount of each extension in a column, add, and show the total.

INVENTORY, JANUARY 31

173 lbs. Granulated Sugar.....	\$.06	
196 lbs. Brown Sugar.....	.05	
2 doz. Ivory Soap.....	.35	
2 doz. Blacking.....	.32	
26 lbs. Arbuckle Coffee.....	.18	
15 lbs. Tea.....	.27	
41 gal. Coal Oil.....	.10	
656 lbs. Bacon.....	.20	
500 lbs. Ham.....	.24	
1,388 lbs. Shoulder.....	.21	
117 lbs. Lard.....	.19	
24 doz. Cans Tomatoes.....	2.00	
35 gal. Syrup.....	.80	
5 bags Irish Potatoes.....	1.95	
25 bbls. White Lily Flour.....	8.85	
37 bbls. Fancy Flour.....	7.30	
78 lbs. Red Ribbon Butter.....	.35	
96 doz. Eggs.....	.40	
Total.....		

2. The Divine Piano Company had the following merchandise in stock on December 31: 187 music rolls @ 65c; 37 music rolls @ 80c; 29 music rolls @ \$1; 67 music rolls @ \$1.25; 215 pieces of music @ 16c; 118 pieces of music @ 18c; 27 pieces of music @ 21c; 2 pianos @ \$272.50; 3 pianos @ \$315.50; 4 pianos @ \$450; 2 pianos @ \$550. Show on tablet paper (a) the description of each item of merchandise and its value, and (b) the total value of all the merchandise.

3. The Jones Confectionery Company had in stock June 30, the following merchandise: 12 1-lb. boxes Johnson's candy @ 72c per lb.; 14 2-lb. boxes Johnson's candy @ 72c per lb.; 16 5-lb. boxes Johnson's candy @ 72c per lb.; 15 boxes A1 Cream Chocolates @ 85c per box; 215 lbs. mixed candy @ 17c; 15 lbs. mints @ 42c. Find the total value.

4. The Regan Radio Company had the following supplies in stock on September 30: 3 Crosley Head Phones @ \$4.25; 8 Navy Head Phones @ \$7.60; 14 spools No. 26 Green Silk Wire @ 88c per spool; 48 Porcelain Socket Tubes @ \$26.50 per doz.; 3 DeForest Amplifiers @ \$12.40; 5 DeForest Detectors @ \$21.50; 12 Crystal Cups @ 48c each; 6 "A" Storage Batteries @ \$15.50; 14 Silvertone Jr. Loud Speakers @ \$17.50; 5 Music Box Crystal Sets @ \$6. Find the value of the inventory.

In the two problems below, the figures in the first column represent the units, and in the second column, the price per unit. You are to ascertain the value of each quantity and the total.

5		6	
a.	88 @ \$.62	a.	347 @ \$.57
b.	269 @ .48	b.	714 @ .88
c.	478 @ .92	c.	279 @ .45
d.	527 @ .44	d.	826 @ .96
e.	686 @ .32	e.	559 @ 1.13
f.	541 @ .14	f.	673 @ 1.22
g.	722 @ .13	g.	821 @ .73
h.	361 @ 1.25	h.	87 @ 1.36
i.	462 @ 1.18	i.	129 @ 1.55

EXERCISE No. 56, AUDITING INVENTORY

You are to audit the extensions and addition of the inventory given on page 76 and report the errors. Find the total after you

have corrected the extensions, and show your figures in the extensions of those items which are incorrect.

1. Merchandise Inventory, February 28, 19..

350	lbs. Arbuckle Coffee.....	\$.15	56	50	
3,012	lbs. Granulated Sugar, per cwt.....	4.80	144	58	
181	bxs. Domino Sugar.....	.21	38	81	
$\frac{1}{2}$	doz. bars Ivory Soap.....	.60		30	
81	doz. bars Star Soap.....	.39	27	59	
8	doz. cans Eagle Milk.....	1.35	40	80	
33	pkgs. Sun Maid Seedless Raisins....	.16	5	28	
52	bxs. Crackers.....	.20	10	40	
528	lbs. Silver Leaf Lard.....	.19	106	32	
287	lbs. Breakfast Bacon.....	.20	57	40	
525	lbs. Premium Hams.....	.24	126		
$5\frac{1}{2}$	bbls. Roller King Flour.....	7.20	39	80	
3	bbls. Superior Flour.....	8.15	24	45	
60	lbs. American Cheese.....	.28 $\frac{1}{2}$	15	35	
124	lbs. Red Ribbon Butter.....	.35	47	40	
22	cans Good Luck Baking Powder....	.30	8	60	
2	doz. cans California Peaches.....	2.45	4	90	
15	doz. cans Del Monte Alaska Salmon	2.00	30		
$1\frac{1}{2}$	doz. cans Imperial Corn.....	2.00	3	50	
21	lbs. Sunset Prunes.....	.30	8	30	
86	bxs. Rice.....	.38	32	68	
62 $\frac{1}{2}$	gal. Golden Glow Maple Syrup.....	.80	50		
82	doz. Eggs.....	.40	32	80	
5	doz. Lemons.....	.28	1	40	
65	bxs. Jello.....	.07	4	55	
$2\frac{1}{2}$	doz. Canned Asparagus.....	3.00	7	50	
7	doz. Canned Pineapple.....	2.25	15	75	
6	bxs. Cornflakes.....	1.00	6		
3	doz. Canned Cherries.....	2.15	16	45	
8	bbls. White Lily Flour.....	8.85	70	85	
4	bbls. Fancy Flour.....	7.30	29	20	
6	cases Pearl Soap.....	3.05	18	30	
16	bricks Badger Brick Cheese.....	.68	16	88	
2	buckets Candy.....	4.00	8		
3	bags Michigan Potatoes.....	1.95	5	85	
51	doz. bxs. Safe Home Matches.....	.47	24	72	
21	lbs. Powdered Sugar.....	.08	2	68	
49	gal. New Orleans Molasses.....	.69 $\frac{1}{2}$	34	46	
18	bxs. Animal Cakes.....	.30	6	40	
Value of Merchandise on hand					1,417 08

The figures in the first column of each of the following four problems represent the quantity; in the second, the price; and in the third, the extension. You are to (a) audit these extensions; (b) show in detail the calculations necessary to ascertain the correct amount in those which are wrong and (c), show the correct total of each problem.*

	2	
a. 315 bu.	\$1.10	\$346.50
b. 439 gal.	.59	249.01
c. 221 lbs.	.17	37.57
d. 17 lbs.	.22	3.74
e. 345 gal.	.78	279.10
f. 718 doz.	.88	632.84
g. 954 bu.	1.09	939.86
h. 87 gal.	.65	56.55
i. 231 doz.	.98	226.38
j. 652 bu.	1.50	978.00
k. 82 lbs.	.29	24.78
l. 110 lbs.	.15	16.50

	4	
a. 217 bu.	\$1.10	\$238.70
b. 67 bu.	1.15	77.10
c. 28 bu.	1.50	43.00
d. 13 bu.	1.75	22.75
e. 123 lbs.	.16	19.68
f. 435 lbs.	.27	116.35
g. 513 lbs.	.11	55.43
h. 249 lbs.	.06	14.94
i. 587 lbs.	.05	29.35
j. 389 lbs.	.10	37.80

	3	
a. 225 lbs.	\$.11	\$ 23.75
b. 13 bu.	1.50	19.50
c. 371 lbs.	.15	55.65
d. 413 lbs.	.10	42.20
e. 516 doz.	.25	128.00
f. 176 lbs.	.09	16.74
g. 654 bu.	1.10	719.40
h. 315 gal.	.79	248.85
i. 710 doz.	.35	258.60
j. 823 lbs.	.16	141.58
k. 238 lbs.	.13	30.94
l. 75 lbs.	.22	16.50

	5	
a. 115 lbs.	\$.11	\$ 12.65
b. 375 lbs.	.08	40.00
c. 237 doz.	.68	151.16
d. 451 gal.	.29	130.79
e. 151 bu.	1.05	158.55
f. 138 doz.	.15	30.70
g. 561 doz.	.17	96.27
h. 135 lbs.	.21	28.35
i. 650 doz.	.20	130.00
j. 548 lbs.	.19	104.12

EXERCISE No. 57, MAKING CHANGE

Making change in connection with a cash sale involves both multiplication and subtraction by addition as explained in Ex-

*NOTE: Unless the calculations are correct when a sale is made, the buyer or the seller will lose through the error. It is only fair that each should make the calculations and verify the amount of the sale. It is unfair to the buyer, if the seller observes an error in his favor and does not report it. It is just as unfair for the seller to take advantage of his customer in the same way.

ercise No. 32. Multiplication is necessary to ascertain the amount of the sale, and subtraction, to ascertain the amount of money due the customer, unless the amount tendered in payment is the exact amount of the purchase. The process is illustrated below.

You purchase six pounds of breakfast bacon at 42c per pound, and give the clerk three one-dollar bills in payment. The amount of the sale as indicated by correct multiplication is \$2.52. He accepts the \$3, places them in the cash register, and removes therefrom three cents, two dimes, and a quarter, and hands it to you counting 252, 253, 254, 255, 265, 275, \$3. He arrives at the amount to take out of the cash register by the same mental process. In other words, instead of subtracting 252 from \$3, and returning to you 48c in change, he arrives at the amount due you by counting the money beginning with the amount of the sale.

When the proper change is available, it is customary to make it with the least number of pieces. This plan was followed in the foregoing illustration. If the merchant had not had sufficient change to make it in this manner, he might have given you two nickels in place of one of the dimes, or a nickel and four dimes instead of the quarter and the dimes. In either case he would count the coins as he gives them to you. Thus, if he gives you three cents, a nickel, and four dimes, he would say 252, 253, 254, 255, 260, 270, 280, 290, \$3.

The following problems provide oral practice in ascertaining the amount of a sale and making change:

1. 3 boxes of linen note paper at 50c per box;
paid for with \$2 bill.
2. 5 Eversharp pencils at 50c;
paid for with three \$1 bills.
3. 1 Icy-hot bottle for \$2.19;
paid for with a \$5 bill. No quarters are available.
4. 5 lbs. of hard candy at 59c;
paid for with a \$5 bill.
5. 8 lbs. of stuffed dates at 59c;
paid for with a \$20 bill.
6. 1 five-lb. box of Johnson's chocolates, \$3.75;
paid for with a \$10 bill.
7. 1 rubber-set brush at 98c;
paid for with a \$2 bill.
8. 2 military brushes at \$3.78;
paid for with a \$10 bill.
9. 1 shaving mirror at 69c;
paid for with a \$1 bill.
10. 6 men's handkerchiefs at 11c;
paid for with a quarter and a 50c piece.

CHAPTER VI

DIVISION

The purpose of the information in this chapter is to provide practice in division and in proving the results. The statements are made under the assumption that you understand the mathematical principles involved in the division process.

Division is the process of finding how many times one number is contained in another number. The number to be divided is the dividend; the number by which it is divided is the divisor; the number of times the divisor is contained in the dividend is called the quotient; and if the divisor is not contained an exact number of times in the dividend, the number left over is the remainder.

The process of division, when applied to business problems, is usually indicated by the nature of the problem, and not by the usual sign of division, ($\div$).

PROBLEM NO. 25, SHORT DIVISION

The process of division may be by one of two methods: (a) short division, and (b) long division. The short division process for dividing 24276 by 12 is illustrated below on the left, and the process for dividing 44742 by 15 is illustrated on the right; in the one process there is no remainder and in the other there is a remainder.

$$\begin{array}{r} 2023 \\ 12 \overline{) 24276} \end{array}$$

$$\begin{array}{r} 2996 \\ 12 \overline{) 35952} \end{array}$$

Referring to the illustration at the left you will observe that 12 is contained in 24, 2 times, 4 and 2 times in 27, and 6 times in 76, but is contained in 27

$$\begin{array}{r} 2982 \\ 15 \overline{) 44742} \end{array}$$

$$\begin{array}{r} 2982 \\ 15 \overline{) 44742} \end{array}$$

2 times with a remainder of 3, and in 27 it goes with no remainder. The quotient can be proved by multiplying 2996 by 12; the product should be the dividend.

In the process on the right 15 is contained in 44, 2 times with a remainder of 14, in 47, 3 times with a remainder of 2, and in 42, 2 times with a remainder of 12; the remainder of 12 is read as the word "do". The quotient can be proved by multiplying 2982 by 15 and adding the 2; the sum of the product and the remainder should be the dividend.

The following problems provide written practice in short division, and illustrate the divisibility of numbers. Show all figures in your quotient as in the illustrations.

1	2	3
a. $3,486 \div 2$	a. $3,264 \div 4$	a. $7,626 \div 6$
b. $1,278 \div 2$	b. $9,244 \div 4$	b. $4,837 \div 6$
c. $9,832 \div 2$	c. $1,484 \div 4$	c. $8,634 \div 6$
d. $4,782 \div 3$	d. $3,405 \div 5$	d. $4,837 \div 7$
e. $2,346 \div 3$	e. $3,290 \div 5$	e. $8,323 \div 7$
f. $6,846 \div 3$	f. $6,380 \div 5$	f. $5,734 \div 7$
4	5	6
a. $3,819 \div 8$	a. $4,653 \div 11$	a. $3,262 \div 14$
b. $4,653 \div 8$	b. $5,786 \div 11$	b. $4,637 \div 14$
c. $2,781 \div 8$	c. $5,748 \div 11$	c. $5,642 \div 14$
d. $4,689 \div 9$	d. $2,784 \div 13$	d. $5,735 \div 15$
e. $3,798 \div 9$	e. $6,344 \div 13$	e. $1,905 \div 15$
f. $8,392 \div 9$	f. $8,151 \div 13$	f. $7,290 \div 15$

EXERCISE No. 59, LONG DIVISION

The process of long division requires the application of multiplication and subtraction, as indicated in the following problems; in short division these processes are required, but are performed mentally. The illustration below at the left shows the long division process for dividing 9,085 by 79, and the illustration at the right, the long division process for dividing 17,334 by 84.

$$\begin{array}{r}
 115 \\
 79 \overline{) 9085} \\
 \underline{79} \\
 118 \\
 \underline{79} \\
 395 \\
 \underline{395} \\
 0
 \end{array}$$

The illustration at the left shows the process of long division when there is no remainder, and the one at the right, when there is a remainder. The divisor is written at the left of the dividend and the quotient on a line above the dividend.

When there is a remainder, it is written at the right of the quotient, as in the illustration at the right.

$$\begin{array}{r}
 206-30R \\
 84 \overline{) 17334} \\
 \underline{168} \\
 534 \\
 \underline{504} \\
 30
 \end{array}$$

The problems at the top of page 81 provide written practice in long division. Your work on tablet paper should be arranged in the same manner as in the illustrations.

1	2	3	4
a. $1,568 \div 84$	a. $4,268 \div 49$	a. $8,165 \div 71$	a. $844,806 \div 196$
b. $3,978 \div 17$	b. $4,275 \div 25$	b. $8,125 \div 23$	b. $527,843 \div 829$
c. $7,280 \div 35$	c. $3,384 \div 62$	c. $7,776 \div 98$	c. $115,972 \div 158$
d. $7,668 \div 42$	d. $4,368 \div 28$	d. $5,460 \div 16$	d. $961,053 \div 258$
e. $4,784 \div 26$	e. $1,586 \div 26$	e. $3,625 \div 25$	e. $135,280 \div 356$
f. $6,120 \div 45$	f. $4,608 \div 36$	f. $8,658 \div 37$	f. $126,624 \div 487$

EXERCISE No. 60, PROOF OF DIVISION

The quotient in a division problem may be proved by multiplication or by check figures. When there is a remainder, it is necessary to add this to the product of the quotient and the divisor to prove the result.

The use of the check figure 9 to prove division (a) when there is no remainder, and (b) when there is a remainder is illustrated below. The check figure 11 can also be used for proving division. The illustration below shows the method of proving the division of 3,751 by 121 through the elimination of 9's.

	The excess of 9's in 121 is 4.	31-103R
<u>31</u>	The excess of 9's in 31 is 4.	
121)3751	The product of 4×4 is 16.	121)3854
363	The excess of 9's in 16 is 7.	363
<u>121</u>	The excess of 9's in 3,751 is 7.	<u>224</u>
121	This proves the division in the	121
<u>121</u>	solution at the left because the key	<u>103</u>
	figure resulting from eliminating the	
	9's from the product of the quotient	

and divisor is the same as the key figure resulting from eliminating the 9's from the dividend.

When there is a remainder, the key figure resulting from the product of the key figures in the divisor and quotient, added to the key figure resulting from eliminating the 9's from the remainder should equal the key figure of the dividend. Applying this to the solution at the right, $4 \times 4 = 16 =$ key figure 7; eliminating 9's from 103 gives a key figure of 4; $7 + 4 = 11$; eliminating the 9's gives the key figure 2; eliminating the 9's from 3,854 gives the key figure 2.

The problems at the top of page 82 provide written practice in division and the proof of division. You are to prove the first column by multiplication and addition, if there is a remainder; and those in the remaining columns by the check figure 9.

1	2	3	4
a. $4,268 \div 39$	a. $685,525 \div 425$	a. $473,827 \div 921$	a. $622,080 \div 324$
b. $4,275 \div 25$	b. $738,472 \div 147$	b. $684,990 \div 129$	b. $573,841 \div 425$
c. $3,384 \div 12$	c. $362,746 \div 323$	c. $473,823 \div 352$	c. $473,821 \div 378$
d. $4,637 \div 14$	d. $263,120 \div 236$	d. $948,371 \div 237$	d. $948,573 \div 467$

EXERCISE No. 61, DIVISION BY SUBTRACTION

One number may be divided by another by subtracting the divisor from the dividend. Continue the subtraction until the remainder is less than the subtrahend. The number of times it is necessary to subtract is the quotient.

$$\begin{array}{r}
 400 \\
 129 \\
 \hline
 271 \\
 129 \\
 \hline
 142 \\
 129 \\
 \hline
 13
 \end{array}$$

The illustration at the left shows the process of division by subtraction when the problem is $400 \div 129$. You will observe that it is necessary to subtract three times, and after the third subtraction, there is a remainder of 13. This illustrates the fact that division is a short-cut method of subtraction.

The following problems provide written practice in division by subtraction. Show on tablet paper the subtraction in each problem arranged as in the illustration, and prove by long division.

1	2	3	4
a. $702 \div 56$	a. $2,768 \div 346$	a. $11,830 \div 2346$	a. $143,764 \div 47,856$
b. $675 \div 45$	b. $5,124 \div 854$	b. $54,391 \div 6785$	b. $182,275 \div 36,455$
c. $653 \div 36$	c. $5,791 \div 643$	c. $23,432 \div 5783$	c. $111,544 \div 27,886$
d. $450 \div 75$	d. $2,315 \div 385$	d. $24,988 \div 3568$	d. $350,592 \div 58,432$

EXERCISE No. 62, MULTIPLICATION BY DIVISION

Division may be used to advantage in multiplication when two, four, or eight times either the multiplier or multiplicand equals 1,000 or a multiple of 1,000. The illustration below shows the application of this method to ascertaining the value of 125 bushels of wheat at \$1.44 per bushel.

$$\begin{array}{r}
 144 \\
 125 \\
 \hline
 720 \\
 288 \\
 144 \\
 \hline
 \$180.00
 \end{array}$$

The value of 125 bushels of wheat at \$1.44 is \$180. This is ascertained by multiplication, as shown at the left, or by multiplication and division, as shown at the right.

125 is contained in 1,000, eight times, hence 144 multiplied by 1,000 and divided by eight gives the same result as \$1.44 multiplied by 125.

$$\begin{array}{r}
 1000 \div 125 = 8 \\
 144 \\
 \hline
 1000 \\
 8)144000 \\
 \hline
 \$180.00
 \end{array}$$

The following problems provide oral practice in multiplication by division.

- 1**
- a. 17,648 bu. @ \$1.25
 b. 464 bu. @ \$2.50
 c. 1,250 lbs. @ 96c
 d. 1,464 bu. @ \$1.25
 e. 500 lbs. @ \$1.45
 f. 564 yds. @ \$12.50
 g. 2,500 yds. @ 14c
 h. 64,486 lbs. @ \$1.25
 i. 67,384 lbs. @ \$2.50
 j. 452 bu. @ \$1.25

- 2**
- a. 798 bu. @ \$5.00
 b. 12,464 bu. @ \$1.25
 c. 2,500 lbs. @ 75c
 d. 4,866 yds. @ \$1.25
 e. 892 yds. @ \$1.25
 f. 4,132 lbs. @ \$2.50
 g. 654 bu. @ \$1.25
 h. 2,500 lbs. @ 57c
 i. 1,250 yds. @ \$1.25
 j. 4,236 bu. @ \$1.25

EXERCISE No. 63, BILLING

The nature of the product and the method of packing often require division, as well as multiplication and addition in making the extensions of an invoice. Grain is usually shipped in sacks, and priced per bushel; hay, in bales, and priced per ton; coal, in cars or trucks, and priced per ton; lumber is cut in standard lengths, and priced per thousand board feet. When billing grain it is customary to state the quantity in pounds, and the price per bushel; hay, the quantity in pounds and the price per ton; and lumber, the quantity in board feet, and the price per thousand board feet.

The illustrations below show two processes of ascertaining the value of 2240 pounds of corn at 87c per bushel of 56 pounds.

$$\begin{array}{r} 40 \\ 56 \overline{)2240} \\ \underline{224} \\ .87 \\ 40 \\ \hline \$34.80 \end{array}$$

In the illustration at the left, the quantity is divided by the number of pounds in a bushel, and the price multiplied by the quotient.

In the illustration at the right, the number of

$$\begin{array}{r} 2240 \\ .87 \\ \hline 15680 \\ 17920 \\ \hline 1948.80 \end{array}$$

$$\begin{array}{r} \$34.80 \\ 56 \overline{)1948.80} \\ \underline{168} \\ 268 \\ \underline{224} \\ 448 \\ \underline{448} \end{array}$$

pounds is multiplied by the price per bushel, and the product divided by the number of pounds in a bushel. The quotient is \$34.80 because the price per bushel is 87c.

The following problems provide written practice in making extensions and ascertaining the total of an invoice. When the price is per bushel or ton and the quantity stated in pounds, you are to show on tablet paper, both processes as given in the illustrations, one as the method of solution, and the other as a proof.

1. The Central Grain Exchange sold W. S. Fedders:

100 sacks (11,480 lbs.) No. 1 corn	97c
100 sacks (11,424 lbs.) No. 2 corn	93c
100 sacks (6,624 lbs.) oats	57c

NOTE: The price of corn is per bushel of 56 pounds and the price of oats is per bushel of 32 pounds.

2. The Shelley Grain Company sold the Jameson Grain Company:

50 sacks (5,824 lbs.) No. 1 corn	95c
50 sacks (5,936 lbs.) No. 2 corn	93c
50 sacks (3,296 lbs.) oats	57c

3. The Martin Feed Company sold L. J. DeVine:

25 bales (1,975 lbs.) No. 1 hay	\$22.75
25 sacks (2,968 lbs.) No. 1 corn	.95
25 sacks (2,856 lbs.) No. 2 corn	.93

NOTE: The price of hay is per ton of 2,000 pounds. Multiply the price by the number of pounds and divide the product by 2; the product will be dollars and cents, the five figures at the right representing cents and fractions of a cent; $1,975 \times 22.75 = 4,493,125$; $4,493,125 \div 2 = \$22.46562 = \22.47 .

4. The Martin Feed Company sold T. L. Dicken:

50 bales (4,827 lbs.) No. 1 hay	\$22.25
40 sacks (4,480 lbs.) No. 1 corn	.97
20 sacks (2,408 lbs.) No. 2 corn	.91
25 sacks (1,728 lbs.) oats	.53

5. The Martin Feed Company sold C. H. Madison:

100 bales (9,427 lbs.) hay	\$22.25
100 sacks (3,296 lbs.) oats	.55

6. The Central Coal Company sold J. L. Daly:

1 load (2,265 lbs.) lump coal	\$7.45
1 load (2,487 lbs.) nut coal	7.15

NOTE: The price of coal is per ton of 2,000 pounds.

EXERCISE No. 64, AVERAGES

An average is obtained by dividing the sum of two or more items by the number of items added. Thus, if it is desired to ascertain the average sales made by a salesman during a week, the sales for each day will be added, and the sum divided by six; the quotient will be the average sales. If it is desired to ascertain the average number of hours worked by forty employees during a given period, divide the total number of hours all the employees worked, by forty.

Mon.	\$156.50
Tues.	218.75
Wed.	74.65
Thurs.	128.80
Fri.	185.90
Sat.	91.60
	6)856.20
	\$142.70

The illustration at the left shows the method of ascertaining the average sales made by James Kirk, a salesman, during a given week. Adding the daily sales and dividing by 6, gives \$142.70, his average daily sales. If it is desired to compare the daily sales with the average, this is done by subtracting the daily sales from the average or the average from the daily sales. Thus, if the average is \$142.70, and the sales on Monday are \$156.60, the sales for this day are \$13.80 above the average. On Wednesday the sales were only \$74.65, so the sales for this day are \$68.05 below the average.

The following problems provide written practice in determining averages, and in making comparisons.

1. Monday morning, Robert Dixon and family started on an automobile trip. The speedometer registered 9,674 miles before starting. On Monday night it registered 9,948 miles; Tuesday night, 10,207 miles; Wednesday night, 10,512 miles; Thursday night, 10,804 miles; Friday night, 11,078 miles; and Saturday night, 11,396 miles. Mr. Dixon wishes to know: (a) the actual number of miles traveled each day; and (b) the average number of miles traveled each day.

2. The sales for the Central Drug Company by months for the year were as follows: January, \$2,345.02; February, \$3,247.02; March, \$2,749.54; April, \$2,957.40; May, \$4,123.95; June, \$3,574.21; July, \$2,450.73; August, \$3,927.45; September, \$3,547.20; October, \$3,354.10; November, \$3,752.30; December, \$2,947.56. You are to find the average monthly sales.

3. A class of 16 students in arithmetic took a test and made the following grades: 86; 95; 73; 92; 89; 85; 82; 78; 86; 92; 98; 84; 72; 69; 78; 85. What was the average grade of the class?

4. J. A. Ross, an author, received royalty from his publishers over a period of six years. The first year the royalty was \$786.45; the second year, \$806.48; the third year, \$882.25; the fourth year, \$986.65; the fifth year, \$1,035.68; and the sixth year, \$1,129.41. You are to ascertain: (a) the total royalty received by Mr. Ross in the six years; and (b) the average yearly royalty.

5. The circulation for an afternoon paper in a certain city during one week was as follows: Monday, 11,462; Tuesday, 10,987; Wednesday, 12,455; Thursday, 11,009; Friday, 9,087; and Saturday, 16,214. You are required to show: (a) the average daily circulation; and (b) a comparison of circulation for each day with the average. Mark the amounts which are below the average by a minus sign, and the amounts which are above the average by a plus sign.

6. Department A of the Brown Manufacturing Company had twelve employees. During one week the time worked by all the employees was as follows: Monday, 103 hours; Tuesday, 102 hours; Wednesday, 97 hours; Thursday, 94 hours; Friday, 101 hours; and Saturday, 67 hours. You are to ascertain: (a) the average number of hours worked; and (b) compare the time of the following employees with this average; No. 1, 51 hours; No. 2, 48 hours; No. 3, 40 hours; and No. 4, 44 hours.

NOTE: The management is interested in each employee working the maximum number of hours, because the expense of maintaining space, heat, light, etc. for each employee goes on just the same whether he works or not. An employee who cannot work approximately the average time, should be replaced by one who can do so.

7. The weight of each player on two college football teams was as follows:

Eastern College: 178 lbs., 162 lbs., 159 lbs., 154 lbs., 146 lbs., 137 lbs., 149 lbs., 156 lbs., 148 lbs., 144 lbs., 139 lbs.

Western College: 158 lbs., 152 lbs., 148 lbs., 141 lbs., 142 lbs., 140 lbs., 134 lbs., 136 lbs., 137 lbs., 133 lbs., 174 lbs.

Find (a) the total weight of each team, (b) the average weight of each team, and (c) the difference between the average weights of the teams.

8. According to the bookkeeping records, the cost of operating the business of J. L. Holland during a certain year was as follows: January, \$854.25; February, \$783.55; March, \$825.47; April,

\$906.78; May, \$867.66; June, \$798.83; July, \$678.42; August, \$754.45; September, \$834.67; October, \$805.26; November, \$756.55; December, \$782.47. It is desired to know (a) the average monthly expenses, (b) the comparison of each month with this average and, (c) the comparison showing the increase or decrease, indicated by a plus or minus sign.

EXERCISE No. 65, DISTRIBUTION OF OVERHEAD EXPENSE

A manufacturer must know the cost of the products of his factory in order to fix a selling price which will provide sufficient profit to justify the operation of the factory. There are three principal elements in the cost: (a) raw material; (b) labor; and (c) overhead, or manufacturing expense. *Raw material* refers to the articles needed in the manufacture of the product; *labor* refers to the wages paid to the employees to operate the machines or do hand work; *overhead expense* includes, the superintendent's salary, the foreman's salary, rent, heat, light, and other expenses which are a part of the direct manufacturing cost.

The following problems provide written practice in distributing monthly overhead expenses. No. 1, requires the distribution of the rent in accordance with the number of square feet in each department, as given in the five columns at the right. No. 2, requires the distribution of the superintendent's salary on a basis of the number of hours produced in each department. No. 3, requires the distribution of the bill for lighting on a basis of the number of lamps in each department. No. 4, requires the distribution of the power used for the machines on a basis of the horse power used in each department. No. 5, requires the distribution of the towel cost on a basis of the number of employees.

		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
1.	Rent, \$500.00. Square feet	1,650	1,040	950	960	400
2.	Salary, \$300.00. Prod. hours	1,450	1,590	1,620	950	390
3.	Light, \$88.00. No. lamps	29	27	21	16	17
4.	Power, \$490.56. Horse power	262	256	250	240	160
5.	Towels, \$12.80. No. employees	30	25	25	22	26

To distribute the \$500 rent, ascertain the total number of square feet by adding the number of square feet in each department; divide this into the amount paid for rent to ascertain the cost per square foot; and multiply the number of square feet in each department by the rate per square foot. Follow the same plan for distributing the superintendent's salary, cost of light, cost of horse power, and the cost of the towels.

EXERCISE No. 66, FACTORS

An *integer* is a whole number such as 1, 4, 9, etc. A *prime number* is one that is divisible only by 1 and itself, as 7, 11, 13, 37, etc. A *composite number* is one that is divisible by other numbers in addition to one and itself, as 4, 18, 48, etc. A *factor* of a number is a divisor of that number. A *prime factor* is a prime number. An integer that is divisible by two is an *even number*; all other integers are *odd numbers*.

When it is desired to separate a number into its factors, it is necessary to divide by the factors. The following facts regarding the divisibility of numbers will be of assistance in factoring: (a) 2 is contained in all even numbers; (b) 3 is contained in a number if the sum of the digits is divisible by 3; (c) 4 is contained in a number if the number expressed by the last two digits is divisible by 4; (d) 5 is contained in any number ending with 0 or 5; (e) 6 is contained in any number that is divisible by both 2 and 3; (f) 8 is contained in a number if the number expressed by the last three digits is divisible by 8; (g) 9 is contained in a number, if the sum of the digits is divisible by 9.

$$\begin{array}{r} 2 \overline{)1716} \end{array}$$

$$\begin{array}{r} 2 \overline{)858} \end{array}$$

$$\begin{array}{r} 3 \overline{)429} \end{array}$$

$$\begin{array}{r} 11 \overline{)143} \end{array}$$

$$\begin{array}{r} 13 \end{array}$$

The illustration at the left shows the process of separating 1,716 into its prime factors. You will observe that to do this it is necessary to divide by a prime factor and continue dividing by a prime factor until the last quotient is a prime number. Completing the division, you ascertain the prime factors of 1,716, which are 2, 2, 3, 11, 13.

The following problems provide written practice in factoring, and applying the divisibility of numbers. You are to show on tablet paper the prime factors of each number, arranging your work in the same order as in the illustration.

1	2	3	4	5
a. 2,048	a. 68,544	a. 73,260	a. 378,000	a. 874,874
b. 6,561	b. 38,625	b. 27,930	b. 599,676	b. 356,250
c. 9,375	c. 28,980	c. 30,030	c. 498,498	c. 273,564
d. 6,930	d. 25,344	d. 26,100	d. 470,475	d. 327,978

A factor or divisor common to two or more integers is called a *common factor* or *divisor*. The greatest common divisor (g. c. d.) is the largest factor that is common to two or more given numbers; thus, 8 is the greatest common divisor of 16, 24, and 32, because it is the largest number that will exactly divide all the numbers.

EXERCISE No. 67, MULTIPLES

A *multiple* of a number is an integral number of times that number. A *common multiple* of two or more numbers is a number that is a multiple of each. The *least common multiple* (l. c. m.) of two or more numbers is the least number that is a multiple of each.

22 is a multiple of 11, because 2, one integer, multiplied by 11, another integer, equals 22. 16 is a common multiple of 8 and 4, because either of these integers multiplied by some other integer will equal 16. 30 is the least common multiple of 5, 6, 10, 15, because any of these integers multiplied by some integer will equal 30, and 30 is the least number which will contain each of them without a remainder.

When it is desired to ascertain the least common multiple of two or more numbers, it is necessary to separate each number into its prime factors, and ascertain the product of the factors.

$$16 = 2 \times 2 \times 2 \times 2$$

$$30 = 2 \times 3 \times 5$$

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 =$$

$$480, \text{ the l. c. m.}$$

The illustration at the left shows one of the processes of finding the least common multiple of 16, 30, and 32. Each number is first separated into its prime factors. The product of these factors is then found by using each factor the greatest number of times it occurs in any one of the given numbers.

The illustration at the right shows another method of securing the least common multiple of 16, 30, and 32. The numbers are divided by any prime number that will exactly divide two or more of them. 16, 30 and 32 are divisible by 2. 15 is not divisible by 2, so this is brought down. The least common multiple is the product of all the divisors and the last quotients.

$$2)16-30-32$$

$$2)8-15-16$$

$$2)4-15-8$$

$$2)2-15-4$$

$$1-15-2$$

$$2 \times 2 \times 2 \times 2 \times 15 \times 2 =$$

$$480, \text{ the l. c. m.}$$

The following problems provide written practice in finding the l. c. m. of a series of numbers.

1

$$a. 12, 15, 18$$

$$b. 16, 24, 28$$

$$c. 25, 28, 30$$

$$d. 36, 42, 48$$

2

$$a. 12, 25, 27, 32$$

$$b. 14, 81, 35, 15$$

$$c. 24, 36, 60, 48$$

$$d. 36, 75, 48, 24$$

3

$$a. 14, 28, 21, 49$$

$$b. 16, 32, 48, 28$$

$$c. 32, 48, 35, 70$$

$$d. 40, 30, 65, 48$$

EXERCISE No. 68, CANCELLATION

Cancellation refers to the elimination of factors in the divisor and dividend before dividing. The quotient is not affected by the elimination of factors which are common to both dividend and divisor. The illustrations below show the solution of $20,338 \div 5,648$ with and without cancellation.

$$\begin{array}{r}
 36 \\
 \hline
 5648 \overline{) 203328} \\
 \underline{16944} \\
 33888 \\
 \underline{33888} \\
 0
 \end{array}$$

The solution at the left shows long division without cancellation. The process is the same as in previous exercises given in long division.

The solution (a) at the right shows the dividend factored; (b) shows the divisor factored; and (c) the division by cancellation. It is not necessary to separate the numbers into their prime factors, hence the divisibility of numbers, explained in a previous exercise may be applied. After factoring, the factors of the dividend are written above the line, and the factors of the divisor below the line. Since 8 and 353 occur both above and below the line, these may be cancelled. 2 is contained in 8, 4 times. 9 times 4 are 36, the same as the answer obtained by long division.

$$(a) \quad 20338 = 9 \times 8 \times 8 \times 353$$

$$(b) \quad 5648 = 2 \times 8 \times 353$$

$$(c) \quad \begin{array}{r}
 4 \\
 9 \times \cancel{8} \times \cancel{8} \times \cancel{353} \\
 \hline
 \cancel{8} \times \cancel{2} \times \cancel{353} = 36
 \end{array}$$

The following problems provide written practice in division by cancellation. Prove the problems in Nos. 1 and 3 by long division, and the problems in Nos. 2 and 4 by multiplication.

1

- a. $49,248 \div 2,736$
- b. $38,100 \div 2,540$
- c. $58,368 \div 3,648$
- d. $38,486 \div 2,749$
- e. $76,125 \div 3,045$

3

- a. $52,800 \div 3,520$
- b. $36,864 \div 1,152$
- c. $32,768 \div 2,048$
- d. $38,367 \div 1,827$
- e. $90,384 \div 3,228$

2

- a. $(8 \times 8 \times 907) \div (2 \times 2 \times 907) =$
- b. $(7 \times 7 \times 7 \times 7 \times 7 \times 4) \div (7 \times 7 \times 7 \times 7 \times 2) =$
- c. $(6 \times 6 \times 4 \times 4 \times 4 \times 19) \div (6 \times 6 \times 4 \times 19) =$
- d. $(7 \times 8 \times 4 \times 3 \times 37) \div (7 \times 4 \times 3 \times 37) =$
- e. $(5 \times 6 \times 6 \times 8 \times 24) \div (6 \times 5 \times 3 \times 8) =$

4

- a. $(4 \times 3 \times 8 \times 32) \div (2 \times 2 \times 16 \times 3) =$
- b. $(81 \times 16 \times 10 \times 12) \div (9 \times 27 \times 2 \times 5) =$
- c. $(31 \times 28 \times 19 \times 80) \div (7 \times 19 \times 2 \times 10) =$
- d. $(63 \times 49 \times 21 \times 13) \div (3 \times 7 \times 9 \times 7) =$
- e. $(42 \times 36 \times 88 \times 22) \div (11 \times 6 \times 22 \times 21) =$

CHAPTER VII

PRACTICAL PROBLEMS

The purpose of the following problems is to provide additional practice in applying the four fundamental operations of arithmetic to the solution of business problems.

1. The distance from New York to Chicago is 998 miles; from Chicago to Omaha, 488 miles; Omaha to Cheyenne, 507 miles; Cheyenne to Ogden, 483 miles; Ogden to San Francisco, 783 miles. What is the distance from New York to San Francisco?

2. Omaha is 1,033 feet above sea level; Chicago is 590 feet above sea level; and Denver is 4,149 feet higher above sea level than Omaha. How much higher is Denver than Chicago?

3. Denver is 5,182 feet above sea level; Death Valley, California, is 271 feet below sea level. How much higher is Denver than Death Valley?

4. The following is the income for the United States Government for two consecutive years. You are to ascertain the total receipts for each year, and the increase or decrease for the second year.

	First Year	Second Year
Distilled spirits	\$ 25,902,820.28	\$ 27,580,380.64
Fermented liquors	1,954.44	5,373.50
Tobacco manufactures . . .	345,247,210.96	325,638,931.14
Oleomargarine	3,038,927.84	2,914,104.14
Capital stock tax	95,814,152.60	95,286,105.44
Miscellaneous	344,738,257.14	490,676,910.89
Internal Revenue Stamps	7,738,895.47	12,418,180.25
Income and profit taxes..	1,761,695,099.51	1,841,759,316.80

5. Robert Fisher sold his home for \$11,764, his store for \$16,145, his stock at the inventory price, for \$26,381, and received \$20,000 as first payment. For the remainder he received farm land at \$45 per acre. How many acres did he receive?

6. According to the fundamental principles of bookkeeping, the total resources should equal the sum of the total capital and the liabilities. Add and prove the following bank statement.

BROTHERHOOD NATIONAL BANK

Report of condition at the close, on December 31, 19...

RESOURCES

Loans and discounts.....	\$1,400,241.60
Overdrafts, secured \$200.06, unsecured \$459.32.....	659.38
U. S. Government securities owned:	
Deposited to secure circulation.....	200,000.00
All other Government securities.....	13,043.00
Other bonds, stocks, securities, &c.....	1,658,890.15
Furniture and fixtures.....	49,019.59
Lawful reserve with Federal Reserve Bank.....	127,313.24
Cash in vault and due from national banks.....	168,569.65
Amount due from banks.....	4,457.26
Exchanges for clearing houses.....	46,562.94
Miscellaneous cash items.....	3,081.00
Redemption fund deposited with U. S. Treasurer.....	10,000.00
Other assets.....	38,593.12

Total resources.....

LIABILITIES

Capital stock paid in.....	\$ 200,000.00
Surplus fund.....	50,000.00
Reserved for taxes, interest, &c. accrued.....	5,765.43
Circulating notes outstanding.....	200,000.00
Amount due to national banks.....	37,826.28
Certified checks outstanding.....	9,569.27
Cashier's checks outstanding.....	16,066.41
Demand deposits.....	577,600.34
Certificates of deposit due in less than 30 days.....	9,596.66
Other Certificates of deposit.....	37,036.00
State, county or other municipal deposits.....	300,000.00
Other time deposits.....	2,272,689.82
Liabilities other than those above stated.....	4,280.72

Total Capital and Liabilities.....

7. The daily sales of a high school cafeteria for the week ending March 17 were as follows: Monday, \$92.65; Tuesday, \$91.87; Wednesday, \$93.44; Thursday, \$95.14; Friday, \$89.50. What was the average sales for the five days?

8. Ascertain the total amount necessary to purchase the following items. Show on tablet paper the amount of each purchase and the total.

2 doz. eggs at 49c	1 bx. aspirin at 39c
3 lbs. coffee at 48c	1 bottle listerine at 79c
5 lbs. meat at 39c	2 pkgs. figs at 18c
11 yds. percale at 29c	15 lbs. parsnips at 5c
25 lbs. sugar at 9c	12 lbs. apples at 4c
24 lbs. sweet potatoes at 6c	20 lbs. potatoes at 6c

9. The Dry Fuel Co. bought 500 cords of wood at \$11 per cord. Sold 200 cords at \$14, 100 cords at \$14.75, and the remainder at \$10 per cord. (a) Did they gain or lose? (b) How much? (c) If the expense of delivery was 25c per cord, what was the net gain or loss?

10. A truck loaded with coal weighed 14,025 pounds. The truck weighed 4,985 when it was empty. At 80 pounds per bushel, how many bushels of coal did it contain?

11. A. L. Davis paid \$22,500 for a farm and sold it for \$25,675. What was his profit after paying \$770.25 commission to the agent making the sale?

12. The following sales were made during the first six months of the year by a department store. Ascertain the information that should be in the blank spaces and prove the results by cross addition.

Month	Books	Shoes	Hats	Silks	Notions	Total
Jan.	\$1,275.47	\$1,581.65	\$985.68	\$1,845.69	\$825.15	6,613.64
Feb.	1,389.05	1,421.79	925.42	2,021.45	782.28	
March	1,299.75	1,507.31	896.25	2,124.22	806.21	
April	1,403.88	1,487.64	947.72	1,732.48	847.31	
May	1,348.78	1,757.46	963.13	1,921.42	356.25	
June	1,357.21	1,645.25	894.50	1,876.42	825.42	
Total	8,074.14	8,971.04	5,612.71	10,521.68	4,039.81	37,199.38

13. The following pay roll sheet shows the time for the twelve employees of the Crown Manufacturing Company during a certain week. Each employee is paid 78c per hour with time and a half for overtime; that is, for time in excess of eight hours for the first five days in the week, and four hours on Saturday.

PAY ROLL SHEET

Emp.	Mon.		Tues.		Wed.		Thurs.		Fri.		Sat.		Reg. Time		Over-time	
	hrs.	min.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.				
# 1	9	30	8	00	10	00	9	30	8	30	7	30				
2	9	30	8	30	11	00	10	30	9	30	6	00				
3	10	30	10	30	9	30	8	30	10	00	5	00				
4	9	30	10	00	9	30	11	00	8	00	7	00				
5	9	30	8	30	10	30	8	00	11	00	7	30				
6	9	00	9	30	11	00	10	00	9	30	4	00				
7	8	00	10	30	10	00	9	30	11	00	6	00				
8	10	00	9	30	8	30	11	00	9	30	6	30				
9	9	30	11	00	10	00	9	00	8	30	4	00				
10	9	30	9	30	10	30	9	30	11	00	5	00				
11	10	30	8	00	9	30	11	30	9	00	7	30				
12	10	00	10	30	11	00	8	30	9	30	7	30				
Total																

You are required to find: (a) the amount due each employee, allowing overtime for time worked in excess of eight hours each day, except Saturday, and in excess of four hours on Saturday—show regular time and overtime in separate amounts; (b) the amount due all the employees for the week—show the regular time and overtime in separate amounts; (c) the average time of each employee for the week; (d) a comparison of each employee's

time with the average, indicating those above the average with a plus sign and those below the average with a minus sign; (e) the average overtime for the week with a comparison of each employee's overtime with this; (f) the average earnings; and (g) prepare a change sheet showing the number of pieces of the various denominations of currency which the paymaster must secure from the bank in order to have sufficient change to place the amount due each employee in envelopes.

14. An automobile which cost \$1,260 was traded in at the end of three years for a new automobile, and the owner was allowed \$510 for it. What was (a) the total loss on the car due to depreciation during the three years, (b) the loss per year?

15. The Busy Bee Department store has four departments with space as follows: Department A, 1,250 square feet; Department B, 1,800 square feet; Department C, 3,750 square feet; and Department D, 2,200 square feet. It is desired to know the departmental charge for the monthly rent of \$720 per month.

16. You are given a \$5 bill to purchase from a grocery store the following items: 6 lbs. of sugar at 9c; 1 loaf of bread at 15c; 2 lbs. of prunes at 45c per lb.; 2 lbs. of butter at 42c per lb.; and 2 cans of peaches at 38c each. What amount of change should you receive, and in what denominations?

17. The report of a restaurant for one day is given below. Find the average price spent for a meal.

12 meals at 20c; 18 meals at 25c; 24 meals at 30c;
25 meals at 35c; 56 meals at 40c; 55 meals at 45c;
10 meals at 50c; 14 meals at 55c; 13 meals at 65c;
12 meals at 70c; 10 meals at 80c; 9 meals at 95c.

18. The Cambria Manufacturing Company had raw material to the value of \$14,467.75 in stock on January 2. During the year, the raw material account showed the total purchases as \$42,561.99; the cost of labor during the year was \$14,461.90; and the overhead expenses were \$14,378.55. At the end of the year the inventory of raw material was \$18,261.54. Ascertain from these figures the cost of the manufactured goods sold during the year.

NOTE: Add the first four amounts and subtract the fifth amount from their sum.

The prices in the two problems below are quoted from an after Christmas bargain sale. You are to ascertain (a) the saving on each article, and (b) the total saving if one customer bought one each of the articles in problem 19, and problem 20.

19.

\$3.75 Sheets at \$2.90
\$4.75 Sheets at \$3.89
\$5.00 Sheets at \$3.98
\$5.50 Sheets at \$4.29
\$1.25 Sheets at \$1.05
\$1.65 Sheets at \$1.29
\$1.79 Sheets at \$1.39
\$5.00 Napkins at \$3.98
\$1.25 Damask at 89c
\$4.50 Napkins at \$3.59
\$6.50 Napkins at \$4.98
\$7.50 Napkins at \$5.98
\$10.00 Napkins at \$6.98
\$7.00 Bed Sets at \$4.98
\$4.50 Bed Sets at \$3.25

20.

25c Irish crash, yd. 19c
50c Irish crash, yd. 39c
29c Linen crash, yd. 22c
\$2.50 Damask, yd. \$1.69
\$1.50 Table padding, yd. 98c
35c Turkish towels 25c
42c Turkish towels 33c
50c Turkish towels 39c
69c Turkish towels 49c
\$6.00 Napkins at \$4.59
\$4.25 Sheets at \$3.39
\$1.85 Sheets at \$1.59
\$10.00 Bed Sets at \$7.59
\$5.25 Table Cloths at \$4.45
\$7.35 Table Cloths at \$6.58

21. A. Y. Burroughs lives at the corner of Main and Oak Streets. His lot fronts 100 feet on Main Street and 200 feet on Oak Street on the inside of the walk. What will it cost to erect a wire fence between him and his neighbors on the side and on the back at \$1.05 per foot?

22. What will it cost him to put down a new cement sidewalk 6 feet wide on both Oak and Main Streets at \$2 per square yard?

23. He has a back porch which is 15 feet long and 5 feet wide. What will the lumber to refloor this cost at \$42.50 per thousand? Each piece of flooring is six inches wide, but being tongued and grooved, only five inches of surface is obtained out of each six inch width. (See note under problem 48 on page 104.)

24. That portion of Main Street on which his house faces has been made an improvement district, and is to be repaved. The cost of paving the 100 feet owned by Mr. Burroughs is \$2,761.50. The amount of Mr. Burrough's share is one half of that to be paid by the city. He is to have the option of paying his share in five equal installments. What is (a) his total share, and (b) the amount of each installment?

25. The total valuation of public utilities in Ohio during two consecutive years is shown in the following table:

	First Year	Second Year
Steam railroad companies.....	\$839,465,430	\$826,683,420
Street, suburban and interurban.....	179,506,770	183,440,700
Natural gas companies.....	113,572,550	118,552,250
Artificial gas companies.....	1,481,100	845,630
Telephone companies.....	127,047,580	119,512,590
Electric light companies.....	208,592,280	178,035,090
Waterworks companies.....	6,164,370	6,076,570
Pipe line companies.....	26,250,480	28,306,650
Messenger and signal companies.....	1,088,750	1,069,270
Heating and cooling companies.....	321,920	303,570
Express companies.....	1,943,370	1,931,530
Telegraph companies.....	9,933,950	9,325,600
Water transportation companies.....	396,490	2,472,930
Union depot companies.....	4,552,190	3,950,560

You are to ascertain (a) the total valuation for the first year, (b) the total valuation for the second year, and (c) the increase or decrease in the total valuation.

26. During a certain business year the sales of the Fox Mercantile Company were \$24,721.40. The merchandise in stock at the beginning of the year was valued at \$6,241.82, the purchases during the year were \$15,265.92, and the merchandise remaining in stock at the end of the year was valued at \$4,427.35. How much did the Fox Mercantile Company make on the sale of its merchandise?

27. J. H. Grusen accepted bids on the construction of a home as follows: foundation, \$2,181.45; lumber, \$4,216.50; brick, \$955.15; construction of house including roof, \$3,917; plastering, \$1,845.50; painting, \$965.05; plumbing, \$1,842.50; electric wiring, \$722.95. You are to ascertain the total cost of the house based on these accepted bids.

28. J. B. Reed reported the following facts regarding an automobile trip: speedometer reading at the beginning of the trip, 21,121; speedometer reading at the end of the trip, 29,507; number of gallons of gasoline in tank at the beginning of the trip, 18; number of gallons of gasoline purchased, 592; number of gallons of gasoline in tank at the conclusion of the trip, 11. You are to ascertain (a) total number of miles traveled, (b) number of gallons of gasoline used, and (c) the average number of miles per gallon.

29. The information given in the table below is from a report published by the Federal Reserve Bank in New York City at the close of a certain year. You are to ascertain (a) the total New Capital, (b) the total Refunding, and (c) the difference between the two.

	New Capital	Refunding
Railroads.....	\$ 369,745,730	\$124,853,000
Public utilities.....	1,449,510,304	226,266,300
Industrials—		
Iron, steel, coal, copper.....	194,992,500	21,496,000
Equipment manufacturers.....	13,840,000	
Motors and accessories.....	191,136,721	22,241,389
Other industrials and manufacturing.	453,435,108	79,281,400
Oil.....	147,857,528	114,180,810
Land, buildings, etc.....	700,056,050	38,173,000
Rubber.....	49,485,000	800,000
Shipping.....	33,604,895	4,315,225
Miscellaneous.....	336,152,776	21,497,000
Municipal.....	1,271,779,485	46,156,232
Canadian brought out in U. S.....	38,658,000	96,797,000
U. S. Territories and possessions.....	8,815,000	
Foreign governments.....	601,781,000	134,600,000
Farm loan issues.....	127,353,100	19,527,900
Canadian corporations.....	85,195,000	21,016,000
Other foreign corporations.....	388,765,000	
Total.....		

30. The total advertising cost of the Ideal Store for the month of August was \$7,500. It is desired to distribute this on a \$100 unit basis over five departments of the store in accordance with the sales in each department which are as follows: Department No. 1, \$12,408.53; Department No. 2, \$14,923.25; Department No. 3, \$14,001.54; Department No. 4, \$16,045.30; Department No. 5, \$17,689.50.

To distribute the cost of advertising over the various departments on a one hundred dollar unit basis means that instead of using the actual amount of the sales in making the distribution, the nearest even hundred dollars will be used. Thus, the sales in department No. 1, will be regarded as \$12,400, or 124 hundred dollar units; the sales in department No. 5, as \$17,700, or 177 hundred dollar units. The total sales for all departments on this unit basis is 750 hundred dollar units. $\$7,500 \div 750$ equals \$10 to be charged to each department on each hundred dollars of sales for the department.

31. On March 7, Brown Brothers, manufacturers of men's clothing, received an order from A. L. Day, one of their customers.

The order was referred to the shipping clerk with instructions to pack and ship, and give the office a statement of the weights so that proper instructions could be given the transportation company, also so that Mr. Day can be sent a copy to check against the shipment when received. After packing the order the shipping clerk reported that it required twenty cases to contain the merchandise shipped. The weights in pounds of these cases were as follows: 256, 319, 408, 271, 400, 356, 445, 485, 316, 389, 412, 487, 365, 358, 427, 475, 247, 268, 349, and 285. The cases were numbered in the order given, beginning with one, and the number and weight written on the outside of each case. You are to prepare a list showing the number of each case, the weight, and the total weight of all the cases to be attached to the invoice sent to Mr. Day. When the merchandise is received, he will check the cases by number and by weight as given on this list, and the total weight with the freight bill which he receives from the railroad.

32. The distance between Denver and Washington, D. C., by one route, is 1810 miles, between Denver and Chicago is 1,022 miles; and between Denver and Pittsburgh is 1,490 miles. What is the distance between: (a) Pittsburgh and Chicago; (b) Pittsburgh and Washington, D. C.; and (c) Chicago and Washington?

33. Below is given in the first two columns the Treasury receipts of the United States Government for two consecutive months, and in the other two columns the receipts for two consecutive years. You are to ascertain the total of each of the four columns and prove by reverse addition.

Receipts	a	b	c	d
Customs.....	\$31,035,094.56	\$25,909,177.05	\$229,157,479.87	\$215,123,708.16
Income and profit taxes...	18,914,893.85	16,716,628.22	486,687,239.35	448,030,222.79
Misc. internal revenue....	47,630,967.35	44,344,105.06	384,581,260.63	353,462,360.18
Foreign principal.....		3,544.63	178,742.75	150,738.94
Foreign interest.....	164,169.06	171,441.10	10,424,974.58	10,883,202.62
Railroad securities.....	1,924,460.53	11,814,380.68	13,663,282.74	100,036,791.13
All others.....	1,851,918.90	2,421,420.41	13,182,253.11	3,366,352.61
Trust fund receipts.....	1,211,880.89	1,651,102.71	14,371,253.09	12,508,296.28
Sale surplus property.....	583,549.35	347,814.30	6,349,104.31	9,882,508.89
Panama Canal tolls.....	2,059,964.42	1,667,121.64	9,053,661.71	9,752,386.78
Rec. from misc. sources...	1,496,755.92	1,080,796.22	8,882,811.44	9,900,746.20
Other miscellaneous.....	8,465,616.77	8,460,300.31	60,985,079.60	65,670,236.14
TOTALS.....				

34. The Trial Balance below was prepared from a ledger which was in balance, but the Trial Balance is not in balance, because the bookkeeper failed to transfer from the ledger the amount of the Sales Returns; that is, the total of the various amounts of merchandise returned by customers. Copy the Trial Balance writing Sales Returns between the Sales and Inventory, leaving the amount column blank. Add each column and take the difference. Write this difference in the column that is smaller and on a line with Sales Returns. The totals of the two columns of the Trial Balance should now be the same, thus proving that the ledger is in balance.

Trial Balance, December 31, 19..

Ledger Page		Debits	Credits
1	Cash.....	1532 17	
1	Notes Receivable.....	1345 65	
2	Accounts Receivable.....	2353 01	
2	Reserve for Doubtful Accounts.....	10 01	19 19
4	Furniture and Fixtures.....	684	
4	Reserve for Depreciation of Furniture.....		34 20
6	Buildings.....	6000	
6	Reserve for Depreciation of Buildings.....		345
7	Land.....	4000	
8	Office Supplies.....	301 25	
8	Insurance.....	190 96	
9	Notes Payable.....		1795 82
9	Accounts Payable.....		2081 39
11	Charles Cook, Capital.....		5000
11	Thomas Duggan, Capital.....		5000
12	Sales.....		26783 12
14	Inventory.....	2198 71	
15	Purchases.....	20280 51	
15	Purchases Returns and Allowances....		411 85
16	Selling Expense.....	1520 65	
17	Administrative Expense.....	1325 83	
18	Interest Earned.....		151 11
18	Purchases Discount.....		520 36
19	Interest Cost.....	91 72	
19	Sales Discount.....	64 75	

35. The population of Chicago is 2,701,705; of the State of Illinois is 6,485,280; of Springfield is 59,183; Rockford is 65,651; and Peoria is 76,121. What is the difference between (a) the

population of Chicago and the sum of the population of the other three cities; (b) the population of the State of Illinois and Chicago; (c) the population of Peoria and Springfield; and (d) the population of Chicago and Springfield?

36. J. L. Dix operated a retail hardware store for a year and at the end of that time wished to know his profit or loss from the operation of the store. During the year he kept a record of: (a) each purchase of merchandise; (b) each sale of merchandise; (c) each expense in connection with the operation of the store. This record showed the total purchases as \$14,562.75, total sales as \$20,904.78, and total expenses as \$6,752.06. At the end of the year, December 31, he had merchandise on hand that had not been sold valued at \$3,487.55. What was his net profit from the operation of the store?

The total purchases, minus the goods not sold, equal the cost of goods sold; the sales, minus the cost of the goods sold, equal the profit on sales; and the profit on sales, minus the expenses, equals the net profit.

37. During the following year Mr. Dix maintained a record of purchases, sales and expenses, the same as the first year. At the end of the second year his records showed: purchases, \$16,786.49; sales, \$23,907.18; and expenses, \$7,409.85. The merchandise in stock at the end of the year was valued at \$3,892.76. (a) What was his net profit the second year? (b) How did it compare with the preceding year?

You will observe that he had some merchandise on hand at the beginning of the year which should be added to the cost of the goods bought during the year to ascertain the total purchases.

38. A. F. Allen earned \$356.50 during August. During September he earned \$39.75 more than in August, and in October he earned as much as he did during August and November. If he earned \$150.00 in November, how much did he earn in: (a) September, (b) October, and (c) during the four months?

39. A. L. Downing has a lot 250 feet long and 120 feet wide. William Dale contracts to dig a ditch 4 feet wide and 3 feet deep around the lot, the outer edge of the ditch being inside the lot 1 foot from the division line. The contract price is 3c per cubic foot. How much will Mr. Dale receive for his work?

40. The following advertisement of bargains appeared in a daily newspaper of a certain large city.

Quantity	ARTICLE	Marked Price	Sale Price
1,415 Yds.	Plain and Fancy Satin Taffetas, Tub Silks and Rayon Fabrics, per yard.....	\$1.89	\$1.10
975 Yds.	Crepe de Chine, Taffeta, Satin Foulards, tub silk, etc. Special, per yard.....	\$2.25	\$1.25
795 Yds.	Knitted Rayon Tricolet, black and navy, for dresses, slips, per yard.....	\$1.25	65c
375 Yds.	50- and 54-inch Satin Crepe and Flat Crepes, plain and novelty borders, fine quality, per yard.....	\$3.75	\$2.95
575 Yds.	Satin Crepe, Flat Crepe, Canton Crepe, etc., per yard.....	\$3.50	\$1.85
425 Yds.	39-inch Black Satin Charmeuse, supple quality. Special, per yard.....	\$1.98	\$1.45
1,000 Yds.	Japanese Pongee Silk, 33 inches wide, 12 momme. Special, per yard.....	79c	57c
2,300 Yds.	Lining Remnants—a vast accumulation of odds and ends, per yard.....	69c	25c
400 Yds.	50-inch All-Wool Chiffon Broadcloth—light and dark colors, per yard.....	\$3.50	\$2.48
350 Yds.	54-inch All-Wool Scotch Tweeds and border cloths, per yard.....	\$3.00	\$1.89
535 Yds.	27- to 30-inch All-Wool Dress Flannels, good colors, per yard.....	\$1.50	89c
650 Yds.	72-inch Beacon Robe Flannel, pretty colors. Special, per yard.....	\$2.00	\$1.39

You are to ascertain: (a) the amount received for this merchandise, assuming that all of it was sold during the sale; and (b) how much more would have been received if the merchandise had been sold at the marked price?

41. The cost of operating a bus line between two cities during August was as follows: labor, \$327.65; gasoline, \$299.40; repairs, \$22.50; depreciation, \$250. The income from passenger fares during the period was \$1,146.83. (a) Did the bus company lose or gain through their operations? (b) How much?

42. Samuel May paid \$60 for four automobile tires. The speedometer reading when the new tires were put on was 8,641 miles, and when they were replaced with new ones, 23,641. What was the tire cost per hundred miles?

43. A grocer bought five bags of coffee weighing: 123 lbs., 132 lbs., 129 lbs., 127 lbs., 133 lbs., at 14c per pound, and sold it at 17c per pound. You are required to show: (a) the cost of the coffee; (b) the selling price of the coffee; and (c) the profit.

44. H. M. Clark purchased a farm containing 640 acres at \$60 an acre. After paying \$3,250 for improvements, he sold the farm for \$45,382. You are to show (a) the cost of the farm plus the improvements, and (b) the profit.

45.. Solve the following problems by cancellation:

- a. $(9 \times 8 \times 6 \times 5 \times 4 \times 2) \div (8 \times 5 \times 4 \times 3 \times 2) =$
- b. $(8 \times 8 \times 7 \times 6 \times 7 \times 2) \div (4 \times 4 \times 3 \times 2 \times 2) =$
- c. $(7 \times 6 \times 6 \times 6 \times 3 \times 3) \div (7 \times 6 \times 3 \times 3 \times 3) =$
- d. $(9 \times 9 \times 8 \times 8 \times 5 \times 5) \div (9 \times 8 \times 3 \times 3 \times 4) =$
- e. $(9 \times 6 \times 6 \times 4 \times 3 \times 2) \div (9 \times 8 \times 3 \times 3 \times 2) =$
- f. $(8 \times 7 \times 7 \times 6 \times 3 \times 2) \div (7 \times 7 \times 4 \times 3 \times 3) =$
- g. $(6 \times 6 \times 3 \times 3 \times 393) \div (3 \times 3 \times 2 \times 131 \times 2) =$
- h. $(8 \times 8 \times 6 \times 3 \times 257) \div (4 \times 4 \times 3 \times 257 \times 4) =$
- i. $(5 \times 6 \times 5 \times 2 \times 151) \div (5 \times 5 \times 2 \times 151 \times 3) =$
- j. $(7 \times 6 \times 6 \times 3 \times 363) \div (7 \times 3 \times 2 \times 121 \times 9) =$
- k. $(8 \times 7 \times 4 \times 4 \times 494) \div (7 \times 2 \times 2 \times 247 \times 2) =$
- l. $(9 \times 8 \times 8 \times 4 \times 756) \div (8 \times 3 \times 4 \times 189 \times 4) =$

46. The average yield of a field of wheat was 38 bushel per acre. What was the total yield if the field contained 48 acres?

47. What is the average weight per bushel of the following: 2 bu. potatoes (60 pounds); 3 bu. timothy seed, (45 pounds); 6 bu. wheat (60 pounds); 6 bu. oats (32 pounds); 3 bu. corn (56 pounds).

The weights given in parenthesis are the weights per bushel.

48. The Murphy Lumber Company sold the Hazen Construction Company the following bill of lumber: 125 pieces, 2"

x 4" x 14' at \$42.50; 165 pieces, 2" x 6" x 16' at \$42.50; 150 pieces, 1" x 10" x 12' at \$41; 225 pieces, 1" x 3" x 12' at \$39.50; 145 pieces, 4" x 4" x 12' at \$41.50; 189 pieces, 3" x 3" x 10' at \$44; and 175 pieces, 3" x 4" x 14' at \$46.50. Ascertain the value of this lumber the price being per thousand board feet.

NOTE: A piece of lumber 2" x 4" x 14' is 14 feet long, 2 inches thick, and 4 inches wide; (") means inches, (') means feet when placed after a number. The number of pieces multiplied by the length in feet, by the width in inches, by the thickness in inches, divided by 12 gives the number of board feet; a board foot is one inch or less thick, 12 inches wide and 12 inches long. Multiply the number of board feet by the price per thousand to ascertain the value. Cancellation can be used for ascertaining the value of the lumber. The product, if the price is expressed in dollars and cents, is dollars and cents, with the five figures at the right expressing the cents and fraction of a cent; if the price is expressed in dollars only, the three figures at the right are cents and fraction of a cent.

49. J. O. McKinsey leased an apartment on July 1 for three years from the following October 1, at a rental of \$3,000 per year. Mr. McKinsey was allowed to occupy the apartment immediately, but was not required to pay rent until October 1, at which time he was to pay the October rent; the rent was to be paid in advance on the first of each month thereafter. What was his average monthly rental at the expiration of the lease?

50. The cash receipts of the Stanley Department store for the first week in March, were as follows: Monday, \$883.40; Tuesday, \$986.52; Wednesday, \$1,087.63; Thursday, \$988.43; Friday, \$1,046.25; and Saturday, \$742.51. What was the average daily sales?

CHAPTER VIII

COMMON FRACTIONS

A *fraction* is one or more of the equal parts of a unit. When it is desired to express a value which is less than a unit, it is necessary to show the number of equal parts into which the unit is divided, and the number of these parts that are expressed. Thus, if a unit is divided into four equal parts, and it is desired to express the value of three of these equal parts, the value will be expressed as three fourths.

There are two methods of indicating a fractional value with figures. When the number of equal parts into which the unit is divided is written below a line, and the number of parts expressed, above the line, the expression is known as a *common fraction*; another expression known as a decimal fraction is discussed in a subsequent chapter. The *denominator* of a common fraction is the number which indicates the equal number of parts into which the unit is divided, and the *numerator* is the number which indicates the number of parts expressed. The numerator of a fraction is written above a horizontal line, or at the left of an oblique line, and the denominator is written below the horizontal line or at the right of the oblique line. Thus, if it is desired to express the fraction "three fourths", it is written " $\frac{3}{4}$ " or " $\frac{3}{4}$ "; 3 is the numerator, and 4 is the denominator.

When the denominator is larger than the numerator, the fraction is a *proper* fraction; when the numerator is equal to or greater than the denominator, it is an *improper* fraction. Thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{9}{10}$, $\frac{7}{11}$, etc., are proper fractions, and $\frac{5}{4}$, $\frac{11}{7}$, $\frac{25}{15}$, etc., are improper fractions. The value of a proper fraction is *less* than a unit; the value of an improper fraction is equal to or *more* than a unit.

EXERCISE No. 69, WRITING FRACTIONS

The following problems provide oral practice in: (a) explaining the method of writing a fraction; and (b) stating whether it is a proper or improper fraction. You are to describe the

method of writing the fraction and state the reason for its being a proper or improper fraction.

1	2	3
a. Three fourths	a. Nine sevenths	a. Sixteen nineteenthths
b. Nine eighths	b. Six sevenths	b. Fifteen fourteenthths
c. Seven sixths	c. Eleven fourteenthths	c. Six seventeenthths
d. Five sixths	d. Twenty fifthths	d. Fifteen twelfthths
e. Three thirds	e. Nineteen twentiethths	e. Ten tenthths

EXERCISE No. 70, REDUCTION OF FRACTIONS

The numbers used to express a fraction are referred to as its *terms*. When the numerator and the denominator are prime to each other, the fraction is expressed in its lowest terms. Factors which are common to both the numerator and the denominator may be eliminated without changing the value of the fraction. Dividing both the numerator and the denominator of a fraction by the same number eliminates a factor common to both; this is referred to as reducing the fraction to *lower* terms; if the division process eliminates all the factors which are common to both the numerator and denominator, the fraction is reduced to its *lowest* terms. Multiplying both the numerator and the denominator of a fraction by the same number, changes the terms, but does not alter the value; this process is known as reducing the fraction to *higher* terms.

The fraction, $\frac{5}{6}$, is expressed in its lowest terms, because 5 and 6 are prime to each other. The fraction, $\frac{6}{10}$, is not expressed in its lowest terms, because 2 is a factor common to both the numerator and denominator; hence, to reduce it to its lowest terms, it will be necessary to divide by 2, changing the fraction to $\frac{3}{5}$. If it is desired to express the fraction, $\frac{5}{6}$, in twelfthths, it is necessary to multiply both the numerator and denominator by 2, changing the fraction to $\frac{10}{12}$; the value of the fraction, $\frac{10}{12}$, is the same as the value of the fraction, $\frac{5}{6}$.

The following problems provide oral practice in the reduction of fractions.

1. Reduce to lowest terms: $\frac{6}{10}$, $\frac{7}{21}$, $\frac{5}{20}$, $\frac{27}{33}$, $\frac{10}{15}$, $\frac{15}{18}$, $\frac{20}{24}$, $\frac{18}{27}$.
2. Express each of the following fractions with a denominator of twelve: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

3. Reduce to lowest terms: $\frac{9}{12}, \frac{14}{21}, \frac{5}{15}, \frac{6}{12}, \frac{27}{36}, \frac{12}{18}, \frac{8}{16}, \frac{9}{21}$.
4. Express each of the following with a denominator of thirty-six: $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}, \frac{4}{9}, \frac{5}{12}, \frac{7}{18}$.
5. Reduce to lowest terms: $\frac{18}{20}, \frac{6}{18}, \frac{5}{25}, \frac{25}{30}, \frac{13}{26}, \frac{19}{28}, \frac{5}{20}, \frac{14}{28}$.
6. Express each of the following with a denominator of forty-eight: $\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}, \frac{3}{8}, \frac{11}{16}, \frac{7}{24}$.
7. Reduce to lowest terms: $\frac{24}{48}, \frac{36}{54}, \frac{17}{51}, \frac{63}{72}, \frac{27}{81}, \frac{60}{70}$.
8. $\frac{1}{2}$ = how many 4ths, 10ths, 6ths, 12ths, 8ths, 24ths, 16ths, 14ths, 20ths, 32ds, 26ths, 40ths, 72ds, 18ths, 22ds, 34ths, 30ths, 88ths, 56ths, 38ths, 76ths, 64ths?
9. $\frac{1}{3}$ = how many 9ths, 30ths, 6ths, 15ths, 21sts, 18ths, 45ths, 12ths, 27ths, 72ds, 51sts, 33ds, 42ds, 39ths, 54ths, 60ths, 36ths, 57ths, 48ths, 69ths, 75ths, 93ds?
10. $\frac{1}{4}$ = how many 8ths, 16ths, 24ths, 44ths, 12ths, 56ths, 64ths, 32ds, 40ths, 52ds, 84ths, 96ths, 76ths, 36ths, 48ths, 28ths, 60ths, 68ths, 80ths, 88ths, 92ds?
11. $\frac{1}{5}$ = how many 5ths, 30ths, 45ths, 20ths, 80ths, 65ths, 15ths, 35ths, 75ths, 90ths, 55ths, 70ths, 25ths, 50ths, 10ths, 60ths, 95ths, 125ths, 100ths, 110ths?

EXERCISE No. 71, CHANGING THE VALUE OF A FRACTION

The value of a fraction is increased (a) by multiplying the numerator by a number greater than one; (b) by dividing the denominator by a number greater than one. Thus, multiplying the numerator of the fraction $\frac{3}{4}$ by 2 gives the fraction $\frac{6}{4}$; dividing the denominator of the fraction $\frac{3}{4}$ by 2 gives $\frac{3}{2}$. The value of the fraction has been doubled in each case.

The value of a fraction is decreased (a) by dividing the numerator by a number greater than one; (b) by multiplying the de-

nominator by a number greater than one. Thus, dividing the numerator of the fraction $\frac{4}{5}$ by 2 gives $\frac{2}{5}$; multiplying the denominator of the fraction $\frac{4}{5}$ by 2 gives $\frac{4}{10}$. The value of the fraction has been reduced one half in each case.

The reason for this increase and decrease in value of fractions is because the denominator indicates the number of equal parts into which a unit is divided, and the numerator the number of these parts that are expressed. If the number of parts into which a fraction is divided is increased, the value of the fraction is decreased; and if the number of parts expressed are increased, the value of the fraction is increased.

The following problems provide oral practice in increasing or decreasing the value of a fraction by the multiplication or division of the numerator or denominator. You are required to state in each case the value of the fraction after multiplication or division, and whether it is greater or less.

1. Divide the numerator or multiply the denominator of each of the following fractions by 3: $\frac{3}{4}$, $\frac{6}{7}$, $\frac{12}{13}$, $\frac{18}{19}$, $\frac{24}{25}$, $\frac{30}{31}$.
2. Multiply the numerator or divide the denominator of each of the following fractions by 4: $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{12}$, $\frac{11}{16}$, $\frac{19}{20}$, $\frac{23}{24}$.
3. Divide the numerator or multiply the denominator of each of the following fractions by 6: $\frac{6}{7}$, $\frac{12}{17}$, $\frac{18}{19}$, $\frac{24}{25}$, $\frac{30}{31}$, $\frac{36}{37}$.
4. Multiply the numerator or divide the denominator of each of the following fractions by 8: $\frac{3}{8}$, $\frac{5}{16}$, $\frac{7}{24}$, $\frac{5}{32}$, $\frac{9}{40}$, $\frac{11}{48}$.

EXERCISE No. 72, MIXED NUMBERS

A mixed number is one which expresses a value involving a whole number and a fraction. Thus, if the value is five and three fourths, it is expressed in figures as $5\frac{3}{4}$; the value of a mixed number may be represented by an improper fraction, as $\frac{23}{4}$.

The following problems provide written practice in expressing the value in figures in the form of a mixed number. You are to express the following values with figures.

1. Ninety-nine and seven sixteenths
2. Twelve and nine elevenths
3. Two hundred forty-five and four twenty-fifths
4. Ninety-one and three fourths
5. Seventy-six and five eighteenths

6. Fifty-four and four ninths
7. Sixteen and five eighths
8. Twenty-seven and five thirty-seconds
9. Thirty-six and four twenty-fifths
10. Forty-six and fifteen sixteenths

EXERCISE No. 73, MIXED NUMBERS

The following problems provide written practice in changing an improper fraction to a mixed number, and a mixed number to an improper fraction.

When it is desired to change a mixed number to an improper fraction, it is necessary to multiply the integer by the denominator of the fraction, and add the numerator to ascertain the numerator of the improper fraction; the denominator will be the same as the denominator in the fraction of the mixed number. When a value is expressed in the form of an improper fraction and it is desired to change it to a mixed number, it is necessary to divide the numerator of the improper fraction by the denominator to ascertain the integer; the fractional value in the mixed number will be the remainder expressed as the numerator of a fraction, with the denominator the same as that in the improper fraction.

The illustrations below show the calculations necessary to change the mixed number $37\frac{17}{18}$ to an improper fraction, and the improper fraction $\frac{683}{18}$ to a mixed number.

$$37\frac{17}{18} = \frac{683}{18}$$

18

296

37

666

17

683

Following instructions, 37 is multiplied by 18, and 17 is added to the product, producing a fraction with a numerator, 683, and the denominator, 18.

Following instructions for reversing the process, the numerator, 683, is divided by the denominator, 18; the quotient, 37, is the integer; the numerator of

$$\frac{683}{18} = 683 \div 18 = 37\frac{17}{18}$$

37 $\frac{17}{18}$

18)683

54

143

126

17

the fraction is 17, and the denominator, 18, which is the same denominator as that in the improper fraction.

Change to mixed numbers:

1. $a. \frac{465}{16}$

b. $\frac{641}{85}$

c. $\frac{523}{41}$

d. $\frac{625}{26}$

e. $\frac{330}{43}$

f. $\frac{448}{221}$

2. $a. \frac{234}{87}$

b. $\frac{563}{55}$

c. $\frac{479}{76}$

d. $\frac{809}{87}$

e. $\frac{688}{24}$

f. $\frac{222}{112}$

3. $a. \frac{125}{68}$

b. $\frac{827}{88}$

c. $\frac{734}{71}$

d. $\frac{426}{42}$

e. $\frac{522}{86}$

f. $\frac{643}{333}$

4. $a. \frac{654}{89}$

b. $\frac{236}{28}$

c. $\frac{128}{36}$

d. $\frac{689}{42}$

e. $\frac{956}{634}$

f. $\frac{996}{122}$

Change to improper fractions:

1	2	3	4	5	6
a. $66\frac{3}{5}$	a. $47\frac{2}{5}$	a. $88\frac{4}{5}$	a. $183\frac{4}{5}$	a. $476\frac{5}{12}$	a. $588\frac{1}{4}\frac{5}{6}$
b. $47\frac{5}{8}$	b. $55\frac{4}{9}$	b. $91\frac{2}{3}$	b. $198\frac{6}{7}$	b. $684\frac{9}{11}$	b. $226\frac{9}{4}$
c. $86\frac{2}{5}$	c. $68\frac{3}{8}$	c. $72\frac{4}{7}$	c. $234\frac{7}{8}$	c. $782\frac{1}{1}\frac{3}{5}$	c. $325\frac{2}{3}\frac{3}{9}$
d. $27\frac{7}{9}$	d. $48\frac{3}{4}$	d. $89\frac{2}{9}$	d. $681\frac{2}{3}$	d. $575\frac{6}{11}$	d. $609\frac{4}{7}\frac{1}{1}$
e. $48\frac{2}{3}$	e. $19\frac{1}{6}$	e. $16\frac{1}{8}$	e. $181\frac{1}{2}$	e. $431\frac{1}{11}$	e. $189\frac{1}{7}$
f. $13\frac{5}{6}$	f. $11\frac{5}{8}$	f. $17\frac{2}{3}$	f. $168\frac{1}{3}$	f. $134\frac{8}{9}$	f. $300\frac{1}{9}$
g. $24\frac{2}{7}$	g. $9\frac{1}{11}$	g. $81\frac{3}{8}$	g. $99\frac{9}{10}$	g. $101\frac{1}{10}$	g. $201\frac{1}{11}$

ADDITION OF COMMON FRACTIONS

As explained in Chapter II, the purpose of addition is to ascertain the sum of two or more numbers. This same purpose applies to the addition of fractions. The sum obtained from the addition of two or more fractions will be of greater value than any one of the fractions, just the same as the sum of two or more whole numbers is greater than any of the numbers.

Since a fraction is one or more of the equal parts of a unit, and the equal division of the units is indicated by the denominator and the value expressed by the numerator, it is quite evident that fractions cannot be added, unless the denominators are the same. When the denominators of two fractions are expressed by the same number, the process of addition is to add the numerators, and write this sum as the numerator over the same number as the denominator. Thus, $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$.

When the denominators of each of the fractions to be added are expressed by different numbers, it is necessary to reduce the fractions to a common denominator. Thus, if it is desired to add $\frac{5}{6}$ and $\frac{1}{3}$, the third may be changed to sixths by reducing the fraction to higher terms, as explained in Exercise No. 70. Following this process, three is contained in six two times; multiplying the numerator of one third by two changes the form of the fraction to $\frac{2}{6}$, hence the problem would be: $\frac{5}{6} + \frac{2}{6} = \frac{7}{6} = 1\frac{1}{6}$.

Common fractions are used extensively in business, and in some lines such as dry goods, many problems require the addition of fractional values expressed in common fraction form. Usually in business problems, fractional values are expressed with small denominators; such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, and sometimes $\frac{1}{32}$.

EXERCISE No. 74, ADDITION OF FRACTIONS

The following problems provide oral practice in adding fractions with a small denominator, when the denominators are the same, or can be made the same without written calculations.

The answer should be stated as a fraction if the value is less than one, or a mixed number with the fraction in its lowest terms, if the value is more than one.

1	2	3	4
$a. \frac{1}{5} + \frac{2}{5} =$	$a. \frac{2}{3} + \frac{3}{4} =$	$a. \frac{3}{5} + \frac{2}{5} + \frac{4}{5} =$	$a. \frac{1}{3} + \frac{1}{4} + \frac{1}{2} =$
$b. \frac{7}{10} + \frac{3}{10} =$	$b. \frac{3}{5} + \frac{7}{10} =$	$b. \frac{3}{7} + \frac{5}{7} + \frac{2}{7} =$	$b. \frac{4}{5} + \frac{1}{3} + \frac{3}{4} =$
$c. \frac{4}{5} + \frac{3}{5} =$	$c. \frac{4}{7} + \frac{3}{4} =$	$c. \frac{4}{9} + \frac{2}{9} + \frac{5}{9} =$	$c. \frac{2}{9} + \frac{3}{6} + \frac{1}{3} =$
$d. \frac{5}{11} + \frac{4}{11} =$	$d. \frac{5}{6} + \frac{1}{3} =$	$d. \frac{3}{10} + \frac{5}{10} + \frac{7}{10} =$	$d. \frac{3}{7} + \frac{5}{14} + \frac{2}{21} =$
$e. \frac{7}{12} + \frac{7}{12} =$	$e. \frac{4}{9} + \frac{5}{9} =$	$e. \frac{3}{14} + \frac{5}{14} + \frac{9}{14} =$	$e. \frac{7}{16} + \frac{3}{4} + \frac{3}{8} =$

EXERCISE No. 75, LEAST COMMON DENOMINATOR

When the denominators of the fractions to be added are expressed as large numbers, or there are a number of small denominators, thus making it inconvenient to determine the common denominator mentally, it is necessary to obtain the common denominators of the fractions by factoring as explained in Exercise No. 67. Although a common denominator is satisfactory, it is better to secure the least common denominator, and thus save time in reducing and adding the fractions.

The illustrations below show a solution of the problem: $\frac{5}{36} + \frac{7}{24} + \frac{17}{18} = 1\frac{3}{8}$

$$a. \begin{array}{r} 2) 36 - 24 - 18 \\ \hline \end{array}$$

$$2) 18 - 12 - 9$$

$$3) 9 - 6 - 9$$

$$3) 3 - 2 - 3$$

$$1 - 2 - 1$$

$$2 \times 2 \times 3 \times 3 \times 2 = 72$$

$$l. c. m. = 72$$

$$b. \frac{5}{36} = 72 \div 36 = 2 \times 5 = 10$$

$$\frac{7}{24} = 72 \div 24 = 3 \times 7 = 21$$

$$\frac{17}{18} = 72 \div 18 = 4 \times 17 = 68$$

$$d. \frac{9}{72} = 1\frac{27}{72} = 1\frac{3}{8}$$

$$c. \frac{10}{72} + \frac{21}{72} + \frac{68}{72} = \frac{99}{72}$$

When factoring to secure the least common multiple and reduce the fractions to their least common denominator, it is necessary to (a) divide the denominators of the fractions by prime numbers which will exactly divide two or more of them. The l. c. m. which is to be the denominator of the fractions is obtained by multiplying together the divisors and remainders. The next process (b) is to find the value of each fraction in seventy-seconds. In (c) the values found in (b) are placed over the l. c. d., 72, and added. The sum (d) is an improper fraction which is reduced to a mixed number with the fraction in its lowest terms.

The following problems provide written practice in the addition of two or more fractions. Show on tablet paper: (a) the factoring necessary to secure the l. c. m.; (b) the calculations necessary to reduce each fraction so that its value may be expressed in that of the l. c. d.; (c) the process of addition and reduction of the sum to a fraction, or a mixed number with the fraction in its lowest terms.

1	2	3	4
$a. \frac{9}{14} + \frac{17}{28} =$	$a. \frac{17}{17} + \frac{8}{34} =$	$a. \frac{5}{28} + \frac{7}{16} + \frac{3}{14} =$	$a. \frac{5}{13} + \frac{3}{26} + \frac{7}{8} =$
$b. \frac{13}{15} + \frac{13}{20} =$	$b. \frac{5}{72} + \frac{53}{56} =$	$b. \frac{4}{15} + \frac{13}{25} + \frac{29}{30} =$	$b. \frac{3}{11} + \frac{8}{99} + \frac{7}{66} =$
$c. \frac{7}{24} + \frac{19}{36} =$	$c. \frac{31}{32} + \frac{49}{96} =$	$c. \frac{11}{12} + \frac{29}{32} + \frac{13}{16} =$	$c. \frac{13}{45} + \frac{23}{36} + \frac{13}{18} =$
$d. \frac{25}{36} + \frac{19}{48} =$	$d. \frac{43}{48} + \frac{35}{36} =$	$d. \frac{13}{30} + \frac{47}{54} + \frac{23}{27} =$	$d. \frac{25}{56} + \frac{13}{28} + \frac{7}{12} =$
$e. \frac{41}{80} + \frac{12}{25} =$	$e. \frac{19}{20} + \frac{7}{40} =$	$e. \frac{1}{10} + \frac{3}{20} + \frac{7}{30} =$	$e. \frac{4}{7} + \frac{3}{28} + \frac{2}{42} =$

EXERCISE No. 76, ADDITION OF FRACTIONS

The following problems provide written practice in adding a number of fractions.

1	2	3
$a. \frac{1}{6} + \frac{2}{3} + \frac{3}{4} + \frac{1}{2} =$	$a. \frac{2}{5} + \frac{3}{10} + \frac{7}{10} + \frac{3}{5} =$	$a. \frac{5}{24} + \frac{3}{8} + \frac{5}{16} + \frac{8}{9} =$
$b. \frac{3}{5} + \frac{1}{10} + \frac{1}{2} + \frac{5}{6} =$	$b. \frac{1}{8} + \frac{3}{4} + \frac{1}{2} + \frac{7}{8} =$	$b. \frac{3}{20} + \frac{3}{5} + \frac{5}{10} + \frac{2}{15} =$
$c. \frac{2}{3} + \frac{5}{8} + \frac{7}{12} + \frac{1}{6} =$	$c. \frac{2}{3} + \frac{5}{6} + \frac{1}{2} + \frac{1}{3} =$	$c. \frac{7}{72} + \frac{9}{24} + \frac{7}{8} + \frac{5}{12} =$
$d. \frac{5}{7} + \frac{2}{3} + \frac{7}{9} + \frac{1}{3} =$	$d. \frac{1}{4} + \frac{7}{12} + \frac{2}{3} + \frac{2}{6} =$	$d. \frac{24}{80} + \frac{2}{5} + \frac{7}{16} + \frac{1}{8} =$
$e. \frac{1}{9} + \frac{3}{7} + \frac{2}{3} + \frac{6}{7} =$	$e. \frac{2}{3} + \frac{4}{9} + \frac{1}{18} + \frac{5}{6} =$	$e. \frac{32}{65} + \frac{3}{13} + \frac{4}{5} + \frac{9}{10} =$
$f. \frac{1}{6} + \frac{1}{8} + \frac{1}{3} + \frac{1}{4} =$	$f. \frac{2}{9} + \frac{3}{7} + \frac{1}{18} + \frac{1}{14} =$	$f. \frac{22}{31} + \frac{1}{62} + \frac{1}{4} + \frac{1}{12} =$

EXERCISE No. 77, ADDITION OF MIXED NUMBERS

When a problem requires the addition of mixed numbers, the process is the same as the addition of whole numbers, and the addition of fractions. The illustration below shows the process of adding $49\frac{2}{3}$ and $16\frac{2}{3}$.

$$\begin{array}{r} 49\frac{2}{3} \\ 16\frac{2}{3} \\ \hline 66\frac{1}{3} \end{array}$$

Add the fractions first, $\frac{2}{3} + \frac{2}{3} = \frac{4}{3} = 1\frac{1}{3}$. Place the $\frac{1}{3}$ and carry the 1. Add the unit column of the whole numbers, $1 + 6 + 9 = 16$. Place the 6 and carry the 1. Add the tens column, $1 + 1 + 4 = 6$, which is placed under the tens column, making the sum of the two mixed numbers $66\frac{1}{3}$.

The following problems provide oral practice in the addition of mixed numbers

- | | | |
|---------------------------------------|---|--|
| 1. $3\frac{1}{2} + 18\frac{1}{2} =$ | 11. $35\frac{5}{6} + 33\frac{1}{3} =$ | 21. $45\frac{5}{6} + 13\frac{5}{12} =$ |
| 2. $16\frac{2}{3} + 24\frac{1}{3} =$ | 12. $17\frac{1}{5} + 18\frac{3}{10} =$ | 22. $43\frac{1}{3} + 17\frac{4}{5} =$ |
| 3. $37\frac{1}{8} + 12\frac{1}{4} =$ | 13. $37\frac{5}{8} + 19\frac{1}{4} =$ | 23. $58\frac{5}{12} + 16\frac{5}{6} =$ |
| 4. $77\frac{1}{5} + 18\frac{2}{5} =$ | 14. $17\frac{1}{2} + 45\frac{3}{4} =$ | 24. $39\frac{3}{8} + 15\frac{1}{6} =$ |
| 5. $19\frac{3}{4} + 17\frac{1}{8} =$ | 15. $18\frac{1}{6} + 14\frac{2}{3} =$ | 25. $27\frac{5}{6} + 17\frac{2}{3} =$ |
| 6. $64\frac{1}{7} + 46\frac{5}{7} =$ | 16. $22\frac{3}{8} + 19\frac{3}{4} =$ | 26. $15\frac{1}{2} + 15\frac{2}{3} =$ |
| 7. $49\frac{2}{3} + 16\frac{2}{3} =$ | 17. $19\frac{3}{10} + 75\frac{4}{5} =$ | 27. $71\frac{1}{9} + 12\frac{5}{6} =$ |
| 8. $19\frac{3}{8} + 27\frac{7}{8} =$ | 18. $77\frac{3}{7} + 15\frac{9}{14} =$ | 28. $45\frac{2}{3} + 11\frac{1}{4} =$ |
| 9. $14\frac{6}{7} + 21\frac{5}{7} =$ | 19. $78\frac{1}{10} + 12\frac{1}{5} =$ | 29. $39\frac{4}{7} + 93\frac{5}{7} =$ |
| 10. $78\frac{5}{9} + 12\frac{2}{3} =$ | 20. $68\frac{7}{10} + 11\frac{9}{10} =$ | 30. $75\frac{1}{8} + 45\frac{3}{4} =$ |

EXERCISE No. 78, ADDITION OF MIXED NUMBERS

The following problems provide written practice in the addition of mixed numbers.

- | | | |
|--|--|---|
| 1. $192\frac{3}{4} + 17\frac{5}{6} =$ | 14. $214\frac{9}{32} + 69\frac{3}{8} =$ | 27. $137\frac{5}{6} + 78\frac{6}{7} =$ |
| 2. $187\frac{1}{6} + 96\frac{2}{3} =$ | 15. $163\frac{1}{31} + 724\frac{2}{3} =$ | 28. $194\frac{1}{21} + 183\frac{6}{7} =$ |
| 3. $98\frac{3}{5} + 85\frac{3}{8} =$ | 16. $147\frac{3}{10} + 127\frac{2}{7} =$ | 29. $186\frac{1}{16} + 75\frac{9}{108} =$ |
| 4. $126\frac{6}{7} + 74\frac{3}{5} =$ | 17. $193\frac{1}{5} + 67\frac{1}{25} =$ | 30. $87\frac{3}{4} + 67\frac{7}{45} =$ |
| 5. $79\frac{6}{11} + 81\frac{1}{22} =$ | 18. $98\frac{1}{8} + 77\frac{3}{5} =$ | 31. $128\frac{1}{7} + 76\frac{1}{63} =$ |
| 6. $96\frac{1}{4} + 77\frac{9}{20} =$ | 19. $714\frac{4}{9} + 69\frac{1}{2} =$ | 32. $126\frac{1}{12} + 47\frac{1}{18} =$ |
| 7. $73\frac{1}{45} + 106\frac{8}{9} =$ | 20. $215\frac{9}{8} + 75\frac{3}{4} =$ | 33. $195\frac{8}{10} + 726\frac{5}{11} =$ |
| 8. $915\frac{6}{7} + 94\frac{1}{15} =$ | 21. $89\frac{7}{8} + 45\frac{1}{5} =$ | 34. $624\frac{9}{24} + 115\frac{1}{2} =$ |
| 9. $726\frac{1}{12} + 86\frac{4}{5} =$ | 22. $215\frac{1}{2} + 230\frac{1}{6} =$ | 35. $312\frac{3}{2} + 65\frac{1}{8} =$ |
| 10. $123\frac{7}{8} + 49\frac{6}{7} =$ | 23. $318\frac{9}{10} + 64\frac{7}{8} =$ | 36. $163\frac{8}{11} + 83\frac{3}{10} =$ |
| 11. $215\frac{9}{54} + 67\frac{3}{40} =$ | 24. $64\frac{2}{30} + 86\frac{2}{48} =$ | 37. $134\frac{2}{33} + 526\frac{4}{63} =$ |
| 12. $945\frac{3}{8} + 212\frac{3}{16} =$ | 25. $86\frac{1}{25} + 76\frac{3}{45} =$ | 38. $915\frac{1}{21} + 49\frac{7}{105} =$ |
| 13. $712\frac{7}{5} + 127\frac{2}{36} =$ | 26. $126\frac{7}{2} + 815\frac{2}{24} =$ | 39. $715\frac{6}{72} + 146\frac{7}{12} =$ |

SUBTRACTION OF COMMON FRACTIONS

As explained in Chapter III, the purpose of subtraction is to ascertain the difference between two numbers. This same purpose applies to the subtraction of fractions. The subtraction of fractions involves: (a) the subtraction of one common fraction

from another; (b) the subtraction of a fraction from a whole number; (c) the subtraction of a fraction from a mixed number; (d) and the subtraction of one mixed number from another. The process requires the denominators of the fractions to be the same, and one numerator is subtracted from the other.

EXERCISE No. 79, SUBTRACTION OF COMMON FRACTIONS

The following problems provide oral practice in subtracting fractions, when the denominators are the same, or the denominators are such that the least common denominator may be determined by observation. Accuracy and speed in solving problems involving fractions are just as important as in solving problems involving whole numbers.

1	2	3	4
a. $\frac{7}{8} - \frac{2}{8}$	a. $\frac{7}{9} - \frac{3}{9}$	a. $\frac{5}{6} - \frac{1}{6}$	a. $\frac{4}{5} - \frac{1}{10}$
b. $\frac{3}{4} - \frac{1}{4}$	b. $\frac{4}{7} - \frac{1}{6}$	b. $\frac{5}{8} - \frac{1}{6}$	b. $\frac{7}{9} - \frac{4}{18}$
c. $\frac{5}{6} - \frac{3}{8}$	c. $\frac{8}{9} - \frac{3}{4}$	c. $\frac{3}{4} - \frac{2}{7}$	c. $\frac{2}{3} - \frac{1}{10}$
d. $\frac{2}{3} - \frac{1}{3}$	d. $\frac{2}{5} - \frac{2}{7}$	d. $\frac{7}{8} - \frac{1}{2}$	d. $\frac{6}{7} - \frac{3}{14}$
e. $\frac{4}{9} - \frac{2}{9}$	e. $\frac{11}{27} - \frac{1}{9}$	e. $\frac{5}{6} - \frac{1}{3}$	e. $\frac{5}{6} - \frac{5}{12}$
f. $\frac{2}{3} - \frac{1}{4}$	f. $\frac{13}{15} - \frac{3}{5}$	f. $\frac{3}{4} - \frac{1}{2}$	f. $\frac{4}{5} - \frac{3}{10}$
g. $\frac{7}{12} - \frac{1}{6}$	g. $\frac{5}{6} - \frac{2}{9}$	g. $\frac{5}{8} - \frac{1}{2}$	g. $\frac{1}{2} - \frac{2}{9}$
h. $\frac{4}{15} - \frac{1}{5}$	h. $\frac{8}{15} - \frac{1}{3}$	h. $\frac{3}{4} - \frac{1}{5}$	h. $\frac{2}{3} - \frac{1}{6}$

EXERCISE No. 80, SUBTRACTION OF COMMON FRACTIONS

When the denominators are not the same, and a common denominator cannot be ascertained mentally, it is necessary to factor the same as in addition.

The following problems provide written practice in subtracting one fraction from another. Show on tablet paper all your calculations necessary to find the difference in each problem.

1	2	3	4
a. $\frac{11}{15} - \frac{3}{5}$	a. $\frac{25}{56} - \frac{2}{7}$	a. $\frac{14}{25} - \frac{3}{5}$	a. $\frac{14}{39} - \frac{3}{13}$
b. $\frac{10}{11} - \frac{3}{22}$	b. $\frac{23}{28} - \frac{5}{14}$	b. $\frac{27}{32} - \frac{5}{64}$	b. $\frac{13}{15} - \frac{13}{21}$
c. $\frac{11}{12} - \frac{7}{60}$	c. $\frac{37}{42} - \frac{5}{18}$	c. $\frac{8}{27} - \frac{5}{54}$	c. $\frac{17}{28} - \frac{13}{42}$
d. $\frac{22}{27} - \frac{2}{9}$	d. $\frac{7}{18} - \frac{3}{64}$	d. $\frac{9}{10} - \frac{11}{15}$	d. $\frac{21}{64} - \frac{13}{56}$
e. $\frac{13}{15} - \frac{8}{45}$	e. $\frac{16}{36} - \frac{1}{9}$	e. $\frac{17}{24} - \frac{5}{18}$	e. $\frac{18}{24} - \frac{11}{26}$
f. $\frac{27}{32} - \frac{13}{24}$	f. $\frac{49}{72} - \frac{3}{8}$	f. $\frac{7}{18} - \frac{5}{36}$	f. $\frac{32}{36} - \frac{13}{48}$

EXERCISE No. 81, A FRACTION FROM A WHOLE NUMBER

When it is desired to subtract a fraction from a whole number, the difference will be expressed as a fraction if the value of the whole number is one, or as a mixed number if the value is more than one.

The following problems provide oral practice in subtracting a fraction from a whole number.

1	2	3	4
a. $1 - \frac{1}{2}$	a. $5 - \frac{5}{9}$	a. $7 - \frac{7}{11}$	a. $16 - \frac{2}{3}$
b. $3 - \frac{3}{4}$	b. $6 - \frac{7}{8}$	b. $8 - \frac{8}{13}$	b. $4 - \frac{3}{4}$
c. $4 - \frac{5}{8}$	c. $5 - \frac{4}{5}$	c. $16 - \frac{7}{16}$	c. $2 - \frac{2}{3}$
d. $1 - \frac{7}{8}$	d. $1 - \frac{1}{3}$	d. $18 - \frac{11}{8}$	d. $1 - \frac{9}{10}$
e. $2 - \frac{3}{7}$	e. $2 - \frac{5}{6}$	e. $12 - \frac{7}{12}$	e. $15 - \frac{4}{5}$
f. $3 - \frac{2}{3}$	f. $8 - \frac{2}{5}$	f. $15 - \frac{3}{4}$	f. $18 - \frac{1}{6}$

EXERCISE No. 82, A MIXED NUMBER FROM A WHOLE NUMBER

When it is desired to subtract a mixed number from a whole number, or a whole number from a mixed number, the fractional value should be subtracted first, and then the whole number. The illustration at the left shows the process of subtracting 55 from $77\frac{4}{5}$, and the one at the right the process of subtracting $37\frac{5}{8}$ from 88 .

$\begin{array}{r} 77\frac{4}{5} \\ 55 \\ \hline 22\frac{4}{5} \end{array}$	In the illustration at the left the fraction $\frac{4}{5}$ is brought down, because there is nothing to subtract from it. The whole numbers are then subtracted in the usual way.	$\begin{array}{r} 88 \\ 37\frac{5}{8} \\ \hline 50\frac{11}{8} \end{array}$
--	---	---

In the illustration at the right, one whole number, $\frac{1}{8}$, is borrowed from 88 in order to subtract $\frac{5}{8}$ from it. This leaves a remainder of $\frac{1}{8}$. 88 is now 87 , because 1 has been borrowed. $87 - 37 = 50$.

The following problems provide written practice in subtracting a mixed number from a whole number, and a whole number from a mixed number.

1	2	3	4
a. $49 - 32\frac{3}{8}$	a. $27\frac{5}{8} - 19$	a. $39 - 27\frac{5}{8}$	a. $95\frac{1}{5} - 77$
b. $97 - 38\frac{2}{5}$	b. $92\frac{2}{5} - 63$	b. $42 - 33\frac{1}{5}$	b. $79\frac{2}{10} - 36$
c. $55 - 27\frac{1}{3}$	c. $52\frac{1}{3} - 27$	c. $86 - 57\frac{1}{4}$	c. $85\frac{4}{3} - 67$
d. $63 - 32\frac{2}{3}$	d. $49\frac{1}{3} - 41$	d. $74 - 35\frac{2}{3}$	d. $68\frac{3}{2} - 35$

EXERCISE No. 83 A MIXED NUMBER FROM A MIXED NUMBER

When a problem requires the subtraction of one mixed number from another, the process is the same as when one of the numbers involved is an integer; that is, the fraction is subtracted first, and then the whole number. When the fraction in the minuend is of less value than the fraction in the subtrahend, it is necessary to borrow before subtracting.

$$\begin{array}{r} 47\frac{7}{16} \\ 25\frac{3}{16} \\ \hline \end{array}$$

$$22\frac{4}{16} = 22\frac{1}{4}$$

The illustration at the left shows the solution of $47\frac{7}{16} - 25\frac{3}{16}$. Since $\frac{3}{16}$ is smaller than $\frac{7}{16}$, it is not necessary to borrow.

The illustration at the right shows the solution of $37\frac{1}{6} - 21\frac{3}{4}$. Since $\frac{1}{6}$

$$37\frac{1}{6} = \frac{2}{12} = \frac{14}{12}$$

$$21\frac{3}{4} = \frac{9}{12} = \frac{9}{12}$$

$$\begin{array}{r} 15 \\ \frac{5}{12} \end{array}$$

is smaller than $\frac{3}{4}$, it is necessary to borrow 1, or $\frac{12}{12}$, making the difference between the fractions, $\frac{5}{12}$; 21 subtracted from 36 leaves 15.

The following problems provide written practice in the subtraction of mixed numbers. Show on tablet paper all calculations necessary in solving these problems.

1

$$a. 18\frac{5}{12} - 16\frac{3}{5} =$$

$$b. 92\frac{1}{8} - 36\frac{4}{5} =$$

$$c. 71\frac{1}{5} - 47\frac{3}{7} =$$

$$d. 18\frac{2}{7} - 13\frac{1}{5} =$$

2

$$a. 197\frac{1}{4} - 88\frac{1}{12} =$$

$$b. 279\frac{5}{7} - 93\frac{3}{4} =$$

$$c. 261\frac{7}{8} - 39\frac{7}{12} =$$

$$d. 198\frac{6}{7} - 96\frac{3}{8} =$$

3

$$a. 712\frac{1}{6} - 639\frac{3}{40} =$$

$$b. 212\frac{1}{5} - 138\frac{6}{45} =$$

$$c. 176\frac{3}{32} - 132\frac{7}{96} =$$

$$d. 169\frac{7}{11} - 142\frac{8}{22} =$$

EXERCISE No. 84, ORAL PROBLEMS

1. A merchant bought a piece of cloth containing $30\frac{1}{2}$ yards, from which he has cut two pieces of $8\frac{3}{4}$ and $12\frac{1}{2}$ yards. How many yards remained?

2. From the number 34 continue to subtract $4\frac{1}{4}$ until nothing remains.

3. You buy $14\frac{1}{2}$ yards of cloth and make two dresses from the same pattern. If you have $\frac{3}{4}$ yard left and allow $\frac{1}{2}$ yard for waste, how many yards in each dress?

4. Add 6 to each term of $\frac{2}{3}$ and state how much the fraction is increased or decreased.

5. Add 2 to the numerator of the fraction $\frac{3}{5}$ and subtract 3 from the denominator; show the effect in fractional form by subtraction.

6. A clerk sold $6\frac{1}{2}$ yards of cloth from a piece containing $20\frac{1}{4}$ yds. How many yards remained?

CHAPTER IX

COMMON FRACTIONS (Continued)

MULTIPLICATION OF COMMON FRACTIONS

The multiplication of common fractions involves: (a) the multiplication of one fraction by another; (b) the multiplication of an integer by a fraction, or a fraction by an integer; (c) the multiplication of two mixed numbers; and (d) the multiplication of a mixed number by an integer or an integer by a mixed number. Multiplication of fractions is indicated by (a) the multiplication sign, " $\times$ ", or (b) by the word, "of", between the two fractions to be multiplied.

$\frac{3}{4} \times \frac{5}{6}$, indicates that it is desired to ascertain the product of the two fractions. $\frac{5}{6}$ of $\frac{3}{4}$, indicates that it is desired to ascertain the product of these two fractions. The product is the same with either form of expression.

EXERCISE No. 85, MULTIPLICATION OF FRACTIONS

The following problems provide oral practice in multiplying one fraction by another. The product is obtained by multiplying the numerators together to form the numerator of the product, and multiplying the denominators together to form the denominator of the product. When the numerator and denominator in the product are not prime to each other, express in lowest terms.

1	2	3	4	5
a. $\frac{2}{3} \times \frac{3}{4}$	a. $\frac{3}{5}$ of $\frac{2}{3}$	a. $\frac{4}{9} \times \frac{5}{6}$	a. $\frac{1}{2}$ of $\frac{7}{9}$	a. $\frac{3}{4}$ of $\frac{3}{5}$
b. $\frac{3}{8} \times \frac{3}{4}$	b. $\frac{3}{4}$ of $\frac{2}{3}$	b. $\frac{2}{5} \times \frac{5}{6}$	b. $\frac{2}{5}$ of $\frac{3}{7}$	b. $\frac{4}{7}$ of $\frac{3}{7}$
c. $\frac{1}{3} \times \frac{3}{8}$	c. $\frac{2}{7}$ of $\frac{1}{3}$	c. $\frac{3}{5} \times \frac{5}{6}$	c. $\frac{3}{5}$ of $\frac{2}{3}$	c. $\frac{5}{8}$ of $\frac{2}{4}$
d. $\frac{2}{3} \times \frac{1}{2}$	d. $\frac{4}{5}$ of $\frac{2}{3}$	d. $\frac{3}{5} \times \frac{5}{9}$	d. $\frac{1}{2}$ of $\frac{1}{8}$	d. $\frac{7}{8}$ of $\frac{3}{4}$
e. $\frac{5}{8} \times \frac{2}{3}$	e. $\frac{1}{5}$ of $\frac{3}{4}$	e. $\frac{1}{2} \times \frac{1}{2}$	e. $\frac{5}{6}$ of $\frac{1}{2}$	e. $\frac{5}{6}$ of $\frac{7}{8}$

EXERCISE No. 86, MULTIPLICATION OF FRACTIONS

The following problems provide written practice in the multiplication of fractions. You are to show on tablet paper: (a) the multiplication process to ascertain the numerator; (b) the multi-

plication process to ascertain the denominator; and (c) the reduction process necessary to express the fraction in its lowest terms.

$$\frac{11}{36} \times \frac{9}{16} = \frac{99}{576}$$

$$\frac{99}{576} \div \frac{9}{9} = \frac{11}{64}$$

The illustration at the left shows the solution of $\frac{11}{36} \times \frac{9}{16}$. The product of the numerators (99) is placed over the product of the denominators (576).

Since 99 and 576 are not prime to each other, the fraction is reduced to its lowest terms, $\frac{11}{64}$.

1	2	3	4	5
a. $\frac{16}{35} \times \frac{13}{22}$	a. $\frac{12}{41}$ of $\frac{15}{56}$	a. $\frac{72}{91} \times \frac{53}{64}$	a. $\frac{34}{49}$ of $\frac{28}{39}$	a. $\frac{45}{84} \times \frac{72}{93}$
b. $\frac{9}{40} \times \frac{21}{32}$	b. $\frac{27}{35}$ of $\frac{24}{31}$	b. $\frac{48}{53} \times \frac{23}{26}$	b. $\frac{36}{85}$ of $\frac{34}{45}$	b. $\frac{39}{62} \times \frac{42}{53}$
c. $\frac{15}{42} \times \frac{27}{35}$	c. $\frac{24}{29}$ of $\frac{37}{38}$	c. $\frac{45}{64} \times \frac{56}{63}$	c. $\frac{37}{48}$ of $\frac{72}{83}$	c. $\frac{35}{86} \times \frac{27}{43}$
d. $\frac{29}{42} \times \frac{7}{36}$	d. $\frac{18}{65}$ of $\frac{26}{27}$	d. $\frac{72}{83} \times \frac{45}{64}$	d. $\frac{56}{63}$ of $\frac{36}{49}$	d. $\frac{48}{53} \times \frac{57}{64}$
e. $\frac{24}{45} \times \frac{17}{22}$	e. $\frac{25}{44}$ of $\frac{48}{55}$	e. $\frac{27}{35} \times \frac{31}{50}$	e. $\frac{27}{56}$ of $\frac{64}{81}$	e. $\frac{63}{64} \times \frac{65}{72}$

EXERCISE No. 87, MULTIPLICATION BY CANCELLATION

When two or more fractions are to be multiplied together and there are factors common to the numerators and denominators, cancellation may be used before multiplying, thus reducing the calculations necessary to obtain the product. The cancellation process is as follows:

$$\frac{5}{6} \text{ of } \frac{12}{35} \text{ of } \frac{7}{9} = \frac{5}{6} \times \frac{12}{35} \times \frac{7}{9} = \frac{2}{9}.$$

1	2	3
a. $\frac{5}{8}$ of $\frac{12}{25}$	a. $\frac{1}{2} \times \frac{4}{9} \times \frac{3}{4}$	a. $\frac{16}{35} \times \frac{27}{36} \times \frac{25}{32} \times \frac{18}{13}$
b. $\frac{3}{4}$ of $\frac{12}{15}$	b. $\frac{7}{8} \times \frac{2}{3} \times \frac{5}{7}$	b. $\frac{45}{56} \times \frac{35}{54} \times \frac{39}{64} \times \frac{24}{25}$
c. $\frac{3}{4}$ of 16	c. $\frac{5}{8} \times \frac{4}{5} \times \frac{5}{7}$	c. $\frac{27}{50} \times \frac{20}{63} \times 48 \times \frac{35}{24}$
d. $\frac{5}{8}$ of 24	d. $\frac{8}{9} \times \frac{5}{8} \times \frac{3}{5}$	d. $36 \times \frac{19}{24} \times \frac{15}{38} \times \frac{16}{45}$

EXERCISE No. 88, MULTIPLICATION OF MIXED NUMBERS

When a fraction is involved in either the multiplier or multiplicand, it is best to multiply by the fraction first, and then by the whole numbers. The method is illustrated below.

$$\begin{array}{r} 468 \\ 44\frac{3}{4} \\ \hline 351 \\ 1872 \\ 1872 \\ \hline 20943 \end{array}$$

The solution at the left shows the process of multiplying a whole number by a mixed number. $\frac{3}{4}$ of 468 is 351. This amount is written as the first partial product in the solution. The whole numbers are then multiplied in the usual manner.

The solution at the right shows the process of multiplying a mixed number by a whole number. $\frac{3}{4}$ of 65 is 26. This amount

$$\begin{array}{r} 385\frac{3}{4} \\ 65 \\ \hline 26 \\ 1925 \\ 2310 \\ \hline 25051 \end{array}$$

is written as the first partial product in the solution. The whole numbers are then multiplied in the usual manner.

The following problems provide written practice in multiplication of mixed numbers when a fraction is in either the multiplier or multiplicand. Solve each problem on tablet paper as illustrated, and prove the product by reverse multiplication.

- | | | |
|-------------------------------|--------------------------------|--------------------------------|
| 1. $145\frac{1}{5} \times 85$ | 6. $623 \times 55\frac{3}{7}$ | 11. $784 \times 16\frac{3}{4}$ |
| 2. $318\frac{1}{3} \times 99$ | 7. $560 \times 25\frac{3}{8}$ | 12. $272 \times 27\frac{1}{8}$ |
| 3. $423\frac{1}{2} \times 74$ | 8. $728 \times 57\frac{7}{8}$ | 13. $565 \times 32\frac{3}{8}$ |
| 4. $438\frac{5}{6} \times 36$ | 9. $372 \times 45\frac{3}{4}$ | 14. $459 \times 27\frac{4}{9}$ |
| 5. $523\frac{1}{8} \times 48$ | 10. $256 \times 88\frac{5}{8}$ | 15. $875 \times 42\frac{5}{6}$ |
16. 19 gallons of gasoline at $22\frac{1}{2}$ c per gallon
 17. 85 yards of gingham at $36\frac{3}{4}$ c per yard
 18. 506 pounds of sugar at $9\frac{1}{2}$ c per pound
 19. $58\frac{1}{2}$ pounds of rice at 12c per pound
 20. 56 pounds of coffee at $47\frac{1}{2}$ c per pound

EXERCISE No. 89, MULTIPLICATION OF MIXED NUMBERS

When both the multiplier and the multiplicand are mixed numbers, there are two methods of solution: (a) multiply the fractions first, and then multiply the whole numbers; (b) change the mixed numbers to improper fractions, multiply, and reduce. The illustrations below show both methods applied to the problem, $228\frac{1}{2} \times 36\frac{2}{3}$.

$228\frac{1}{2}$
 $36\frac{2}{3}$

 $\frac{1}{3}$
 152
 18
 1368
 684

 $8378\frac{1}{3}$

The solution at the left shows $228\frac{1}{2} = \frac{457}{2}$, $36\frac{2}{3} = \frac{110}{3}$ the multiplication without changing to an improper fraction. The product of the two fractions is $\frac{1}{3}$; the product of $228 \times \frac{2}{3}$ is 152; $\frac{457}{2} \times \frac{110}{3} = \frac{50270}{6} = \frac{25135}{3} = 8,378\frac{1}{3}$ the product of $36 \times \frac{1}{2}$ is 18; the product of 228×6 is 1,368; the product of 228×3 is 684. The sum of the 5 partial products is $8,378\frac{1}{3}$.

The solution at the right shows the multiplication after changing to improper fractions. The multiplicand is equivalent to the fraction, $\frac{457}{2}$, and the multiplier, the fraction, $\frac{110}{3}$. The product of these two fractions will give the desired answer. In this case the 2 and 110 have a factor which is common, hence cancellation is employed before multiplication.

It is suggested that you use the method at the left when the denominator of the fraction in the multiplier is exactly contained in the whole number of the multiplicand, and the denominator of the fraction in the multiplicand is exactly contained in the whole number of the multiplier. When the above is not the case, the solution at the right should be used.

The following problems provide written practice in multiplying mixed numbers. Show your results on tablet paper in the same form as the illustrations, and prove by reverse multiplication.

- | | | |
|--|--|---|
| 1. $368\frac{2}{7} \times 42\frac{5}{8}$ | 6. $785\frac{7}{9} \times 235\frac{3}{8}$ | 11. $348\frac{2}{3} \times 27\frac{5}{8}$ |
| 2. $425\frac{1}{7} \times 27\frac{2}{5}$ | 7. $496\frac{1}{5} \times 312\frac{2}{5}$ | 12. $573\frac{2}{5} \times 45\frac{1}{5}$ |
| 3. $436\frac{1}{6} \times 25\frac{3}{5}$ | 8. $425\frac{2}{5} \times 345\frac{3}{5}$ | 13. $549\frac{5}{6} \times 72\frac{4}{9}$ |
| 4. $336\frac{3}{5} \times 48\frac{3}{4}$ | 9. $878\frac{4}{9} \times 256\frac{3}{7}$ | 14. $785\frac{4}{7} \times 35\frac{3}{5}$ |
| 5. $880\frac{5}{6} \times 48\frac{1}{4}$ | 10. $327\frac{1}{2} \times 124\frac{2}{3}$ | 15. $924\frac{2}{3} \times 46\frac{3}{5}$ |
16. $384\frac{1}{4}$ yards of cotton sheeting at $68\frac{1}{2}$ c per yard*
 17. $246\frac{1}{4}$ yards of tissue gingham at $24\frac{1}{2}$ c per yard
 18. $35\frac{1}{2}$ tons of coal at $\$5.62\frac{1}{2}$ per ton
 19. $128\frac{1}{4}$ pounds of sugar at $7\frac{1}{2}$ c per pound
 20. $56\frac{1}{2}$ pounds of lard at $11\frac{1}{2}$ c per pound

EXERCISE No. 90, WRITTEN PROBLEMS

(Problems with unnecessary facts given)

1. If a tank $12\frac{1}{2}$ feet wide and $16\frac{1}{8}$ feet long contains $820\frac{1}{3}$ gallons when $\frac{2}{3}$ full, how many gallons will it hold when full?

2. A cube is $7\frac{1}{3}$ inches each way and weighs $125\frac{2}{3}$ lbs. What will it cost to put a brass edging on all edges at $4\frac{1}{2}$ c a foot, and a brass ball on each corner at $7\frac{1}{2}$ c each?

3. A square garden $4\frac{1}{3}$ rods each way and a square garden $8\frac{2}{3}$ rods each way were planted to cabbage. The first produced 2,610 lbs. of cabbage; the second 4,350 lbs. The cabbage was sold at $3\frac{1}{2}$ c a lb. The cabbage produced on the first garden is what fractional part of that produced on the second?

4. J. L. Simpson, a farmer, sold 138 watermelons weighing 2,055 pounds. He sold one half of the melons at three for a dollar, and the remainder at 25c each. How much did he receive?

5. T. C. Moore sold 125 sacks of wheat containing two and one fourth bushels per sack. He received $\$2.37\frac{1}{2}$ per sack for 54 sacks and $\$2.12\frac{1}{2}$ per sack for the remainder. How much did he receive for the wheat?

*NOTE. It is the custom in business, when solving problems involving dollars and cents to drop the fraction in the result if less than one half cent, and to call the fraction an extra cent if one half or more. You should follow this method unless otherwise instructed.

EXERCISE No. 91, BILLING

Fractions are used frequently in business in marking the extensions of invoices and inventories. It is not always possible to have the unit value of the merchandise expressed as a whole number, and market conditions often make it necessary to use fractions in the prices.

The following problems provide written practice in making extensions and ascertaining the total of an invoice, when common fractions are involved in the quantity, price, or both.

1. The Creasey Wholesale Grocery Company sold the T. F. Cook Company the following merchandise on April 1:

50 bbls. sugar (15,539½ lbs.)	@	8½c per lb.
1 bbl. molasses (51¼ gals.)	@	64½c per gal.
102 lbs. cheese	@	29½c per lb.
15 doz. lemons	@	36½c per doz.
8½ bags of potatoes	@	\$1.95 per bag
28½ lbs. lard	@	28¼c per lb.
10½ doz. cans asparagus	@	\$2.80 per doz.
12 doz. bars Grandma's soap	@	61¼c per doz.

2. Ascertain the value of the following sales made by the Fair Store, May 18, to Ronald Smith:

29½ yards of fancy outing flannel	@	19⅓c per yard
14½ yards dress gingham	@	18¼c " "
27¾ yards of windsor crepe	@	22⅓c " "
35¼ yards of challie	@	16½c " "
36½ yards of white dimity	@	19¼c " "

3. On May 5, Donaldson Brothers, wholesale dry goods merchants, sold P. W. Smith the following merchandise:

6 Pieces Flannel	@	75⅔c per yard
39³, 37², 40, 38³, 39¹, 40¹		
6 Pieces of Gingham	@	19c per yard
58³, 59¹, 57³, 56², 59¹, 58¹		
8 Pieces of Prints	@	16⅔c per yard
42¹, 43³, 44¹, 44³, 45¹, 44², 43³, 45		
12 Pieces of Linings	@	39¼c per yard
44¹, 44³, 45², 46¹, 47, 48, 44³, 43³, 45¹, 46², 48³, 47		

4.

SIMON, TUCKER & COMPANY

Wholesale Dry Goods Merchants

New York, May 10, 1926

SOLD TO: Messrs. B. T. Frank & Co.,

Albany, New York

Terms: 30 days

8	pieces Printz				13 $\frac{1}{2}$ c				
	41 ¹	42 ²	39 ³	40 ¹					
	41 ³	41 ²	40 ²	39 ³					
8	pieces Chiffon				1.85				
	37 ²	38 ³	40 ²	42 ¹					
	41 ³	37 ¹	39 ²	41 ³					
6	pieces Percale				16 $\frac{1}{2}$ c				
	56 ¹	54 ²	53 ¹						
	53 ²	54 ³	52 ³						
4	pieces Taffeta				1.47				
	49 ³	53 ¹	54 ²	51 ²					

NOTE: The figures under the descriptions of the goods refer to the number of yards in each piece. The small figures indicate fourths of a yard. It is customary in the dry goods business to omit the denominator four, because practically all pieces are an exact number of yards or fourths of a yard. To complete the extensions copy the invoice on tablet paper, add the number of yards in each item, and write the total after the number of pieces given. Multiply the total by the price and write the result in the first money column at the right. The total of all the amounts written in this column will be the amount of the invoice.

5. Ascertain the value of the following drugs in the stock of the Dow Drug Company, December 31:

126 $\frac{1}{4}$ doz. Mulsified Coconut Oil	@	24 $\frac{1}{2}$ c each
151 doz. D. & R. Cold Cream	@	26 $\frac{3}{4}$ c "
125 doz. Mavis Talcum Powder	@	13 $\frac{1}{2}$ c "
49 doz. Pinaud's Lilac Vegetal	@	62 $\frac{1}{2}$ c "
46 $\frac{3}{4}$ doz. Pepsodent Tooth Paste	@	25 $\frac{1}{2}$ c "
58 doz. Lyons' Tooth Paste	@	32 $\frac{3}{4}$ c "
56 $\frac{1}{4}$ doz. Palmolive Shampoo	@	28 $\frac{3}{4}$ c "
62 $\frac{1}{2}$ doz. Vick's Salve	@	41 $\frac{1}{4}$ c "

6. The Chatfield Paper Company sold the Goodwill Printing Company the following merchandise on November 16:

12 $\frac{1}{2}$ rms. Snowdrift Writing, 17×22, 24 lbs. per rm. @ 37 $\frac{1}{2}$ c

25 $\frac{2}{5}$ rms. Foldwell Writing, 22×34, 48 lbs. per rm. @ 18 $\frac{1}{2}$ c

57 $\frac{4}{5}$ rms. Paramount Bond, 17×28, 20 $\frac{1}{2}$ lbs. per rm., @ 13 $\frac{3}{4}$ c

42 $\frac{2}{5}$ rms. Triton Bond, 17×28, 25 $\frac{1}{2}$ lbs. per rm., at 15 $\frac{1}{4}$ c

NOTE: Writing paper is manufactured in rolls, cut in sheets of standard sizes and wrapped in ream packages. When a purchase involves less than a ream, it is expressed in fractions. The number of pounds is ascertained by multiplying the number of reams by the weight per ream. The value of each quantity of paper is ascertained by multiplying the number of pounds by the price per pound. The standard ream contains 480 sheets made up of twenty quires, 24 sheets to the quire. Some mills wrap their paper in ream packages containing 500 sheets to the ream. Use 480 sheets to the ream in all of your calculations unless otherwise instructed in the problem.

7. Verify the following inventory of automobile accessories in the stock of the Western Auto Supply Company, December 31:

Merchandise Inventory, December 31, 19

45 Seat Covers.....	\$ 7.57	\$340	65		
26 Seat Covers.....	6.97 $\frac{1}{2}$	181	35		
88 pairs Gauntlets.....	1.96 $\frac{1}{2}$	173	92		
158 Steel Ratchet Valve Lifters...	.16 $\frac{1}{4}$	25	67		
253 Grease Guns.....	.46 $\frac{1}{2}$	122	64		
74 Rubber Mats.....	.45 $\frac{3}{4}$	34	03		
56 pkgs. Top Patching Material..	.26 $\frac{1}{3}$	12	85		
153 bottles Top Dressing.....	.35 $\frac{2}{3}$	50	55		
584 Carburetor Adjusters.....	.76 $\frac{1}{2}$	446	78		
352 Piston Rings.....	.08 $\frac{1}{2}$	29	78		
226 Timers.....	.36 $\frac{1}{4}$	81	83		
158 Headlamp Reflectors.....	.56 $\frac{1}{2}$	89	27		
74 Headlamp Rims.....	.27 $\frac{1}{4}$	20	27		
148 Stoplites.....	.57 $\frac{1}{4}$	84	73		
96 Aluminum Step Plates.....	.87 $\frac{1}{4}$	83	76		
Total				\$1778	08

Show on tablet paper: (a) the calculations necessary to verify the extensions; and (b) the correct total of the inventory.

EXERCISE No. 92, ORAL PROBLEMS

1. At $8\frac{1}{3}$ c a pound, how many pounds of sugar can you buy for a dollar?

2. If a messenger boy earns $\$1\frac{4}{5}$ a day, six days a week, how much does he earn in a week?

3. You and your school chum form a partnership for selling papers after school hours. The two of you together sell 800 papers for the week at 3c each. A profit of $1\frac{1}{4}$ cents is made on each paper sold. On Saturday night, when the profits are divided equally, how much does each one receive?

4. You have kept an accurate account of your school expenses for the month, including lunch, school supplies, carfare, etc. The total for the month amounts to $\$12\frac{1}{4}$. Counting 20 days to the school month, what were your expenses for the day?

5. You buy tickets for your carfare at the rate of 6 for 51c. You use 2 tickets per day for a school month of 20 days, what is your carfare per month? If your school runs ten months, what will your carfare cost for the year? How much will you save by walking $\frac{1}{5}$ of the time?

6. By selling eggs at 45c a dozen you gain $\frac{2}{3}$ of a cent apiece. How much would you have gained apiece by selling at 48c per dozen?

7. At 36c a dozen what would be the cost of 224 eggs?

8. At $16\frac{2}{3}$ c a pound, what will a 25 pound box of prunes cost?

9. At $16\frac{1}{2}$ c a pound what will $12\frac{1}{2}$ pounds of evaporated peaches cost?

10. I gain $1\frac{1}{3}$ c each by selling apples at 5c apiece; what would I lose by selling them 3 for 10c?

11. From a piece of cloth containing $43\frac{3}{4}$ yards there were cut 9 dresses of $4\frac{1}{2}$ yards each. How many yards remained?

12. If $3\frac{1}{4}$ yards of cloth are required for one suit of clothes, how many suits can be cut from 39 yards?

13. You go to market with butter and eggs. The eggs bring you \$4.00, which is $\frac{2}{3}$ of the value of the butter. What did you get for the butter?

14. Mrs. Brown bought two chickens weighing $3\frac{1}{4}$ and $3\frac{1}{2}$ pounds respectively, at 28c per pound. How much change will she get from a \$5.00 bill?

15. A tract of land was sold at \$110 per acre which was $\frac{2}{3}$ above cost. What was the cost per acre?

DIVISION OF COMMON FRACTIONS

The division of common fractions involves: (a) division of one fraction by another; (b) division of an integer by a fraction, or a fraction by an integer; (c) division of one mixed number by another; and (d) division of a mixed number by an integer, or an integer by a mixed number. The purpose of division of fractions is to ascertain the number of times, or fractional number of times, one fraction is contained in another fraction.

EXERCISE No. 93, DIVISION OF FRACTIONS

The following problems provide oral practice in the division of fractions. The numerator of the quotient is obtained by dividing the numerator of the dividend by the numerator of the divisor; the denominator of the quotient is obtained by dividing the denominator of the dividend by the denominator of the divisor. When the quotient is an improper fraction, reduce it to a mixed number, with the fraction of the mixed number in its lowest terms. In connection with each answer, state whether the quotient is of greater or less value than the dividend.

1	2	3	4	5
a. $\frac{1}{2} \div \frac{1}{2} =$	a. $\frac{8}{21} \div \frac{4}{7} =$	a. $\frac{6}{15} \div \frac{2}{3} =$	a. $\frac{13}{36} \div \frac{6}{18} =$	a. $\frac{13}{60} \div \frac{13}{15} =$
b. $\frac{1}{4} \div \frac{1}{2} =$	b. $\frac{8}{25} \div \frac{2}{5} =$	b. $\frac{4}{10} \div \frac{4}{5} =$	b. $\frac{22}{34} \div \frac{11}{17} =$	b. $\frac{18}{75} \div \frac{9}{25} =$
c. $\frac{3}{4} \div \frac{3}{4} =$	c. $\frac{9}{26} \div \frac{3}{2} =$	c. $\frac{7}{22} \div \frac{7}{2} =$	c. $\frac{27}{45} \div \frac{9}{15} =$	c. $\frac{27}{90} \div \frac{9}{30} =$
d. $\frac{4}{9} \div \frac{2}{3} =$	d. $\frac{14}{39} \div \frac{7}{3} =$	d. $\frac{6}{32} \div \frac{1}{2} =$	d. $\frac{18}{34} \div \frac{9}{17} =$	d. $\frac{55}{66} \div \frac{5}{11} =$

EXERCISE No. 94, DIVISION OF FRACTIONS

When the numerator of the divisor is not exactly contained in the numerator of the dividend, and the denominator of the divisor is not exactly contained in the denominator of the dividend, it is more convenient to invert the divisor and multiply. As explained in Exercise No. 71, the effect on the value of a fraction is the same whether the denominator is divided, or the numerator multiplied by a given number. $\frac{7}{8} \div \frac{7}{16}$ is the same as $\frac{7}{8} \times \frac{16}{7} = 2$. The sevens cancel each other, and eight is contained in sixteen two times, hence the quotient is 2. In other words $\frac{7}{16}$ is con-

tained in $\frac{7}{8}$, two times. This may be proved by multiplying $\frac{7}{16}$ by 2. ($\frac{7}{16} \times \frac{2}{1} = \frac{14}{16} = \frac{7}{8}$.)

The following problems provide oral practice in division when it is necessary to invert the divisor.

$$\begin{array}{llll}
 1. \frac{1}{2} \div \frac{1}{4} = & 6. \frac{1}{5} \div \frac{1}{2} = & 11. \frac{5}{8} \div \frac{2}{3} = & 16. \frac{5}{12} \div \frac{5}{8} = \\
 2. \frac{1}{2} \div \frac{1}{3} = & 7. \frac{1}{5} \div \frac{1}{4} = & 12. \frac{1}{8} \div \frac{3}{5} = & 17. \frac{9}{22} \div \frac{2}{7} = \\
 3. \frac{1}{3} \div \frac{1}{2} = & 8. \frac{1}{5} \div \frac{1}{8} = & 13. \frac{1}{7} \div \frac{7}{8} = & 18. \frac{2}{3} \div \frac{4}{5} = \\
 4. \frac{2}{3} \div \frac{1}{2} = & 9. \frac{1}{2} \div \frac{1}{5} = & 14. \frac{4}{7} \div \frac{3}{7} = & 19. \frac{1}{2} \div \frac{5}{6} = \\
 5. \frac{1}{2} \div \frac{2}{3} = & 10. \frac{1}{3} \div \frac{1}{6} = & 15. \frac{5}{9} \div \frac{3}{5} = & 20. \frac{2}{35} \div \frac{7}{8} = \\
 21. \frac{2}{5} \div \frac{1}{9} = & 22. \frac{1}{6} \div \frac{5}{7} = & 23. \frac{2}{5} \div \frac{3}{8} = & 24. \frac{3}{4} \div \frac{3}{5} = \\
 25. \frac{2}{5} \div \frac{3}{8} = & & &
 \end{array}$$

EXERCISE No. 95, DIVISION OF MIXED NUMBERS

When a problem requires the division of an integer by a mixed number, or vice versa, change the mixed number to an improper fraction, invert the divisor, multiply, and reduce the quotient to a mixed number with the fraction in its lowest terms. If desired, the integer and mixed number may each be multiplied by the denominator of the fraction, thereby removing the fraction. The process of division is then completed as in whole numbers with the quotient expressed as a mixed number, with the fraction reduced to its lowest terms.

The solution at the left shows the process when a mixed number is changed to an improper fraction, and the divisor inverted.

The solution at the right shows the process when both the dividend and divisor are multiplied by the denominator of the fraction, and the division completed as in whole numbers.

The following problems provide written practice in dividing an integer by a mixed number, and a mixed number by an integer. You may solve the problems by either of the methods illustrated above, and prove the results by the other method. Show on tablet paper all of your calculations.

1	2	3
a. $176 \div 7\frac{2}{5} =$	a. $427\frac{1}{3} \div 16 =$	a. $729 \div 8\frac{2}{7} =$
b. $132 \div 5\frac{6}{7} =$	b. $141\frac{1}{2} \div 23 =$	b. $520 \div 8\frac{3}{8} =$
c. $237 \div 3\frac{3}{7} =$	c. $375\frac{2}{3} \div 41 =$	c. $360 \div 5\frac{2}{5} =$
d. $435 \div 8\frac{5}{8} =$	d. $245\frac{5}{6} \div 17 =$	d. $475 \div 8\frac{3}{4} =$
e. $682 \div 9\frac{2}{5} =$	e. $308\frac{2}{5} \div 21 =$	e. $178 \div 5\frac{2}{5} =$

EXERCISE No. 96, DIVISION OF MIXED NUMBERS

When a problem requires the division of one mixed number by another mixed number, change both mixed numbers to improper fractions, invert the divisor, multiply, and reduce the quotient to a mixed number with the fraction in its lowest terms. Cancellation should be used when possible.

The following problems provide written practice in the division of mixed numbers. Show all necessary calculations on tablet paper.

1	2	3
<i>a.</i> $3\frac{1}{2} \div 2\frac{1}{2} =$	<i>a.</i> $16\frac{2}{3} \div 6\frac{2}{3} =$	<i>a.</i> $18\frac{3}{5} \div 12\frac{2}{3} =$
<i>b.</i> $7\frac{1}{3} \div 2\frac{1}{2} =$	<i>b.</i> $18\frac{3}{4} \div 6\frac{1}{4} =$	<i>b.</i> $58\frac{1}{3} \div 12\frac{1}{2} =$
<i>c.</i> $9\frac{1}{4} \div 7\frac{1}{2} =$	<i>c.</i> $12\frac{1}{2} \div 4\frac{1}{2} =$	<i>c.</i> $93\frac{3}{4} \div 16\frac{2}{3} =$
<i>d.</i> $5\frac{1}{3} \div 4\frac{1}{3} =$	<i>d.</i> $22\frac{4}{9} \div 7\frac{2}{3} =$	<i>d.</i> $64\frac{3}{8} \div 15\frac{1}{3} =$
<i>e.</i> $6\frac{1}{6} \div 3\frac{1}{3} =$	<i>e.</i> $27\frac{3}{8} \div 3\frac{1}{2} =$	<i>e.</i> $52\frac{2}{9} \div 27\frac{4}{5} =$
<i>f.</i> $8\frac{2}{3} \div 2\frac{1}{3} =$	<i>f.</i> $32\frac{6}{7} \div 4\frac{3}{7} =$	<i>f.</i> $45\frac{3}{4} \div 22\frac{1}{2} =$

EXERCISE No. 97, FRACTIONAL RELATIONS

When it is desired to show the relation of one number to another, this is indicated by placing the numbers in the form of a fraction. Thus, if it is desired to know the relation of 5 to 20, the result expressed as a fraction would be $\frac{5}{20}$, which reduced to its lowest terms is $\frac{1}{4}$; hence, 5 is $\frac{1}{4}$ of 20.

The following problems provide oral practice in determining the relation of numbers.

The mental process of solution applied to the first problem would be: 2 is $\frac{1}{20}$ of 40, because the fraction, $\frac{2}{40} = \frac{1}{20}$ in its lowest terms; 4 is $\frac{1}{10}$ of 40, because the fraction, $\frac{4}{40}$, reduced to its lowest terms is $\frac{1}{10}$; 8 is $\frac{1}{5}$ of 40 because the fraction, $\frac{8}{40}$, reduced to its lowest terms is $\frac{1}{5}$, etc.

What fractional part of:

1. 40 is 2; 4; 8; 10; 20?
2. 24 is 2; 3; 4; 6; 8; 12?
3. 72 is 2; 3; 4; 6; 8; 9; 12; 18; 24; 36?
4. 48 is 2; 3; 4; 6; 8; 12; 24?
5. 50 is 2; 5; 10; 25?
6. 56 is 2; 4; 7; 8; 14; 28?
7. 80 is 2; 4; 5; 8; 10; 16; 20; 40?
8. 63 is 3; 7; 9; 21?

EXERCISE No. 98, FRACTIONAL RELATIONS

When the fractional part of a number is expressed as an integer, the value of the number may be determined by dividing the integer by the numerator and multiplying by the denominator of the fraction. If $\frac{2}{3}$ of a number is 20, and it is desired to ascertain the number, the following reasoning process will apply.

If $\frac{2}{3}$ of a certain number is 20, then $\frac{1}{3}$ of that number is $20 \div 2$ or 10; if $\frac{1}{3}$ of the desired number is 10, then $\frac{2}{3}$ of it would be three times 10, or 30.

The following problems provide oral practice in determining the number when a part of it is expressed as an integer.

1	2	3
<i>a.</i> 27 is $\frac{3}{4}$ of ?	<i>a.</i> 14 is $\frac{7}{8}$ of ?	<i>a.</i> 16 is $\frac{8}{9}$ of ?
<i>b.</i> 15 is $\frac{3}{5}$ of ?	<i>b.</i> 27 is $\frac{3}{5}$ of ?	<i>b.</i> 21 is $\frac{3}{5}$ of ?
<i>c.</i> 16 is $\frac{4}{5}$ of ?	<i>c.</i> 24 is $\frac{6}{7}$ of ?	<i>c.</i> 25 is $\frac{5}{6}$ of ?
<i>d.</i> 12 is $\frac{2}{3}$ of ?	<i>d.</i> 18 is $\frac{3}{4}$ of ?	<i>d.</i> 32 is $\frac{8}{9}$ of ?
<i>e.</i> 26 is $\frac{1}{4}$ of ?	<i>e.</i> 20 is $\frac{1}{11}$ of ?	<i>e.</i> 28 is $\frac{1}{5}$ of ?

EXERCISE No. 99, FRACTIONAL RELATIONS

The following problems provide written practice in determining the number when a part of it is expressed as a mixed number or a fraction.

1	2	3
<i>a.</i> $87\frac{1}{2}$ is $\frac{7}{8}$ of ?	<i>a.</i> $27\frac{1}{6}$ is $\frac{3}{5}$ of ?	<i>a.</i> $36\frac{3}{8}$ is $\frac{2}{5}$ of ?
<i>b.</i> $33\frac{1}{3}$ is $\frac{1}{3}$ of ?	<i>b.</i> $25\frac{3}{5}$ is $\frac{4}{7}$ of ?	<i>b.</i> $35\frac{3}{7}$ is $\frac{2}{7}$ of ?
<i>c.</i> $12\frac{5}{6}$ is $\frac{7}{8}$ of ?	<i>c.</i> $36\frac{1}{3}$ is $\frac{3}{5}$ of ?	<i>c.</i> $47\frac{1}{5}$ is $\frac{4}{5}$ of ?
<i>d.</i> $35\frac{2}{3}$ is $\frac{3}{4}$ of ?	<i>d.</i> $42\frac{1}{3}$ is $\frac{3}{7}$ of ?	<i>d.</i> $71\frac{1}{3}$ is $\frac{3}{4}$ of ?
<i>e.</i> $\frac{8}{9}$ is $\frac{3}{4}$ of ?	<i>e.</i> $\frac{3}{4}$ is $\frac{2}{3}$ of ?	<i>e.</i> $\frac{7}{9}$ is $\frac{1}{3}$ of ?

NOTE: The process is the same with a mixed number or a fraction as with an integer. When there is a mixed number in the problem, this should be changed to an improper fraction.

EXERCISE No. 100, COMPLEX FRACTIONS

The division of fractions may be expressed in the form of a complex fraction, that is, one fraction over another. If mixed numbers are involved, it is customary to change them to improper fractions when the expression is written in the form of a complex

fraction. The illustrations show the solution of the problem $7\frac{1}{8} \div 5\frac{1}{7}$.

$$7\frac{1}{8} \div 5\frac{1}{7} = \frac{7\frac{1}{8}}{5\frac{1}{7}} = \frac{\frac{57}{8}}{\frac{36}{7}} = \frac{57}{8} \times \frac{7}{36} = \frac{399}{288} = \frac{133}{96} = 1\frac{37}{96}$$

In this solution, the mixed numbers are changed to improper fractions expressed as a complex fraction. Since the numerator is the dividend and the denominator, the divisor, the process of division here can be expressed as a complex fraction either, in the form of mixed numbers or improper fractions. $\frac{57}{8} \div \frac{36}{7}$ is equivalent to multiplication with the terms of the divisor inverted. The product, reduced to the form of a mixed number, with the fraction in its lowest terms, gives the number of times $5\frac{1}{7}$ is contained in $7\frac{1}{8}$.

$$7\frac{1}{8} \div 5\frac{1}{7} = \frac{\frac{57}{8}}{\frac{36}{7}} = \frac{57}{8} \times \frac{7}{36} = \frac{399}{288} = 1\frac{133}{96} = 1\frac{37}{96}$$

Another method of performing the division indicated by a complex fraction is to multiply the two extreme numbers together for a new numerator and the two mean numbers for a new denominator. You will note by comparing this solution with the first illustration that the result is the same as inverting the divisor and multiplying.

$$7\frac{1}{8} \div 5\frac{1}{7} = \frac{\frac{19}{8}}{\frac{12}{7}} = \frac{7 \times 19}{8 \times 12} = \frac{133}{96} = 1\frac{37}{96}$$

In this solution, the cancellation is completed before multiplying the extremes together for a numerator, and the means together for a denominator. Referring to the second solution you will observe that cancellation could have been used before the multiplication process, but that the reduction was not made until after the improper fraction was reduced to a mixed number. The final results of the three processes are the same, hence one may be used as a means of proving the other.

The following problems provide written practice in the division of fractions when they are expressed in complex form. Show all of your calculations on tablet paper as in the illustrations.

1	2	3	4	5
a. $\frac{6\frac{3}{5}}{4\frac{1}{2}}$	a. $\frac{3\frac{3}{8}}{5\frac{6}{7}}$	a. $\frac{10\frac{2}{3}}{12\frac{3}{4}}$	a. $\frac{42\frac{7}{8}}{36\frac{1}{4}}$	a. $\frac{58\frac{1}{4}}{32\frac{1}{7}}$
b. $\frac{5\frac{2}{3}}{6\frac{1}{3}}$	b. $\frac{3\frac{2}{3}}{7\frac{8}{9}}$	b. $\frac{17\frac{1}{2}}{11\frac{1}{5}}$	b. $\frac{39\frac{1}{7}}{24\frac{1}{3}}$	b. $\frac{43\frac{2}{5}}{27\frac{1}{3}}$
c. $\frac{4\frac{2}{7}}{5\frac{3}{5}}$	c. $\frac{8\frac{3}{6}}{4\frac{1}{2}}$	c. $\frac{23\frac{2}{5}}{16\frac{3}{4}}$	c. $\frac{45\frac{1}{5}}{19\frac{5}{6}}$	c. $\frac{36\frac{1}{9}}{18\frac{1}{3}}$

EXERCISE No. 101, FRACTIONS APPLIED TO DENOMINATE NUMBERS

When a fraction is applicable to an abstract number, and is expressed in its lowest terms, there is no other means of expressing the fractional value. When a fraction is applicable to a concrete number, its value may be expressed in other concrete terms which bear a definite relation to the concrete number. Thus, the fraction " $\frac{3}{4}$ " indicates that an abstract unit is divided into 4 equal parts and 3 parts are expressed. The expression " $\frac{3}{4}$ of a dozen" is applicable to 12 because 12 units make a dozen, hence might be expressed as 9. The fraction " $\frac{1}{2}$ of a bushel" might be expressed as 2 pecks because there are 4 pecks in a bushel.

The following problems provide oral practice in expressing fractional values applicable to concrete numbers in another form. These problems serve two purposes: (a) as a review of measurements learned in the preceding study of arithmetic, and (b) the expression of a fraction in more than one form without a change in its value.

1	2	3
a. $\frac{3}{4}$ of a yard	a. $\frac{3}{4}$ of a pound	a. $\frac{1}{2}$ of a ton
b. $\frac{5}{8}$ of a bushel	b. $\frac{1}{2}$ of a quart	b. $\frac{3}{4}$ of a dollar
c. $\frac{3}{4}$ of a gallon	c. $\frac{2}{3}$ of a yard	c. $\frac{1}{5}$ of a quarter
d. $\frac{5}{6}$ of a foot	d. $\frac{1}{4}$ of a gallon	d. $\frac{1}{2}$ of a rod
e. $\frac{7}{12}$ of a dozen	e. $\frac{3}{8}$ of a pound	e. $\frac{1}{8}$ of a pound

EXERCISE No. 102, ORAL PROBLEMS

1. A merchant's debts amount to \$2,500, which is $\frac{1}{7}$ of his assets. What is the value of his assets and what is his net worth?

NOTE: Assets—debts=net worth.

2. The stock of a bankrupt merchant was sold for \$24,000, which was $\frac{2}{3}$ of the cost. What was the cost of the stock? What was the loss?

3. What will $2\frac{1}{4}$ yards of silk cost at \$5 $\frac{1}{2}$ per yard?

4. Bananas are priced at 40c per dozen. How many can you buy for 10c?

5. A merchant sells apples 3 for 10c. If a box contains 120 apples, what will he get for them? If he bought them at the rate of 4 for 10c, what is his gross profit on the box of apples?

6. A family saves \$960 per year, which is $\frac{3}{8}$ of its income. What is the income?

7. If you perform $\frac{1}{1\frac{1}{2}}$ of a certain piece of work in the first week, working 6 days per week, how many days will it take to complete the task?

8. If a boy can earn $3\frac{1}{2}$ dollars per day, how much will he earn in a month of 26 working days?

9. The athletic association of a school collected \$820 for the year. $\frac{1}{3}$ was for basket ball, $\frac{1}{4}$ for baseball, $\frac{1}{2}$ for football and the remainder for hockey. How much was collected for each?

10. A farmer sold $\frac{4}{9}$ of his apples for \$120.00. What is the value of the entire crop?

11. A girl bought 4 apples at $5\frac{1}{2}$ c each. How much change did she receive if she gave the clerk a quarter?

12. How much will be saved on 12 rides by buying street car tickets if the cash fare is 10 cents and tickets sell for $8\frac{1}{3}$ c each?

13. Mrs. Smith bought $2\frac{1}{2}$ yards crepe satin, \$2.50 a yard and 3 yards muslin at $18\frac{1}{2}$ c a yard. What was the amount of her purchase?

14. What is the cost of $3\frac{1}{3}$ dozens eggs at 60 cents per dozen?

15. A milliner sold one dozen hats for \$3.60 each, this was one third above cost. What was the cost price of each hat? What was the profit on all the hats?

CHAPTER X

PRACTICAL PROBLEMS IN COMMON FRACTIONS

The purpose of this chapter is to provide practical problems in addition, subtraction, multiplication, and division when common fractions are involved.

1. In a test an automobile ran 28 miles in 30 minutes. At this rate, how many miles would it travel in $6\frac{1}{2}$ hours?

2. A farm containing 640 acres of land has 200 acres planted in corn, $\frac{1}{4}$ of the remainder in oats, $\frac{1}{3}$ of the remainder in barley, $\frac{2}{3}$ of the remainder in orchard, and the balance in pasture. What is the acreage of each?

3. The distance between Chicago and Waukegan by automobile is 40 miles, which is $\frac{2}{5}$ of the distance between Chicago and Milwaukee. What is the distance between Chicago and Milwaukee?

4. An automobile, operated between Chicago and Milwaukee, consumed $6\frac{5}{8}$ gallons of gasoline at $22\frac{1}{2}$ c per gallon. (a) What did the gasoline cost for one trip? (b) What was the average number of miles per gallon?

5. Wheat was quoted at the opening of the market on Monday at \$1.57; on Tuesday, $\$1.57\frac{1}{2}$; on Wednesday, $\$1.56\frac{3}{4}$; on Thursday, $\$1.56\frac{1}{4}$; on Friday, $\$1.55\frac{1}{2}$; on Saturday, $\$1.55\frac{3}{4}$. What was the average quotation for the week?

6. If you pay 20c for three pounds of potatoes, how much do you pay (a) for a bushel of 60 pounds at the same price? (b) For a peck, which is $\frac{1}{4}$ of a bushel?

7. A retail merchant buys beans at \$4.50 per hundred pounds. He sells them at three pounds for 25c. What is his gain (a) per pound? (b) Per hundred pounds?

8. Divide \$90 between two boys so that the first boy will have (a) $\frac{2}{3}$ as much as the second boy; (b) $\frac{2}{3}$ less than the second boy; and (c) $\frac{2}{3}$ more than the second boy.

9. An office building was insured for \$60,000 and the policies were written by three companies as follows: Company A, policy for \$15,000; Company B, policy for \$20,000; Company C, policy for \$25,000. The building was damaged by fire to the extent of \$18,000. What part of the loss should each company pay?

10. You gave a clerk a five-dollar bill in payment for the following: 4 pounds of coffee at $35\frac{3}{4}c$; 6 pounds of onions at $6\frac{1}{2}c$; $12\frac{1}{4}$ pounds of flour at $4c$; $\frac{1}{2}$ pound of pepper at $48c$ a pound. (a) How much change should you receive? (b) In what denominations?

11. 36,630 pounds of wheat were put into sacks averaging $2\frac{1}{2}$ bushels (60 lbs. per bu.) to the sack. How many sacks were required?

12. By rearranging and installing new machinery, a factory increased production by $\frac{1}{3}$. At the same time the working hours were reduced from 10 to 8 hours. Was there an increase or decrease in production? How much?

13. Find the cost of $26\frac{1}{2}$ pounds of prunes at $27\frac{1}{2}c$ per pound; and $16\frac{1}{2}$ pounds of raisins at $18\frac{3}{4}c$ per pound.

14. I spent $\frac{1}{3}$ of my money and lent $\frac{5}{6}$ of the remainder. What part did I have left?

15. $\frac{1}{8}$ of a bushel of wheat (60 lbs.) is what part of $\frac{3}{8}$ of a bushel of wheat? Express your answer fractionally, and prove by the number of pounds.

16. What fraction of a ton of 2,000 pounds is 1,760 pounds?

17. A farmer sold to a merchant 14 dozen eggs at $42\frac{1}{4}c$, and $16\frac{1}{2}$ pounds of butter at $44c$. He purchased from the merchant 6 pounds of sugar at $7\frac{1}{2}c$ and 4 pounds of coffee at $38\frac{1}{2}c$. Who pays the difference, the farmer or the merchant? How much?

18. A dealer paid \$5.75 for a ton of coal (2000 lbs.), and retailed it at $42\frac{1}{2}c$ per bushel (80 lbs.). What was his profit?

19. $\frac{7}{8}$ added to a certain fraction gives $1\frac{5}{6}$. Find the fraction.

20. If the sum of two numbers is $95\frac{1}{2}$ and one of the numbers is $67\frac{1}{8}$, what is the other number?

21. What number divided by $2\frac{3}{8}$ will equal $12\frac{1}{4}$?

22. A. B. Jones and T. G. Smith contract to do a piece of work for \$625. Jones works 65 days and Smith 35 days. How should the money be divided?

23. If you travel 75 miles by automobile on 7 gallons of gasoline, how many gallons will be required to make a trip of 350 miles?

24. What is the cost of $14\frac{1}{2}$ gross note books at $4\frac{3}{4}$ c each?

NOTE: A gross is 12 dozen.

25. A farmer sold $\frac{3}{4}$ of a bushel of seed potatoes for \$2.50. At the same price, what would he receive for his crop of 75 bushels?

26. Two brothers paid \$2,400 for a farm, each paying half. Each sold $\frac{1}{4}$ of his interest to a third brother at cost (a) What fractional part of the total investment does each brother now own, and (b) what is the value of the investment of each?

27. If you paid $8\frac{1}{4}$ c per yard for $10\frac{1}{2}$ yards of ribbon, and $16\frac{2}{3}$ c per yard for 9 yards of gingham, (a) what change would you receive from a five-dollar bill? (b) In what denominations?

28. A box of apples containing 144 apples was purchased for $2\frac{1}{2}$ c per apple and sold at $\frac{6}{5}$ of the cost. What was the profit?

29. Three railroads carry goods a distance of 1,560 miles for \$7.50. If the distance carried by the first road is 780 miles, the second, 520 miles, and the remainder by the third road, how would the charge of \$7.50 be divided?

30. A box of 10 dozen oranges cost \$4. If one dozen froze and the remainder were sold at $4\frac{1}{6}$ c each, what was the profit?

31. Five dressed hogs weighed $150\frac{1}{4}$ lbs., $180\frac{2}{3}$ lbs., $165\frac{1}{2}$ lbs., $138\frac{3}{4}$ lbs., and $186\frac{1}{3}$ lbs. What was (a) the average weight, and (b) the total cost of the five hogs at $16\frac{2}{3}$ c per pound.

32. A lot is $50\frac{1}{4}$ feet wide by $80\frac{2}{3}$ feet deep. How many rods ($16\frac{1}{2}$ ft.) of woven wire will be required to fence the lot?

33. Find the cost of the butter in 5 tubs weighing $33\frac{1}{2}$ pounds, $34\frac{1}{4}$ pounds, $31\frac{1}{8}$ pounds, $30\frac{3}{4}$ pounds, and $31\frac{1}{3}$ pounds, if the tubs average $4\frac{1}{2}$ pounds each, and the butter is billed at $37\frac{3}{4}$ c per pound.

34. Multiply the sum of $\frac{7}{8}$ and $\frac{2}{3}$ by their difference.

35. A boy rides a bicycle 7 miles in $\frac{3}{4}$ of an hour. At the same rate, how long would it take him to ride $20\frac{3}{4}$ miles?

36. A merchant owned $\frac{3}{8}$ of an interest in a store and sold $\frac{1}{2}$ of his interest for \$6,000. What was (a) the total value of the store, and (b) the value of his remaining interest?

37. A merchant asked $\frac{1}{2}$ more for an article than it cost. During a dull season he reduced the market price $\frac{1}{5}$. If he gained \$2.40, what was (a) the cost, and (b) the selling price?

38. Jones and Smith, two merchants, each made a bid on a certain lot of merchandise. Jones' offer was $\frac{4}{5}$ of the cost value, and Smith's offer was $\frac{5}{7}$ of the cost value. If their offers differed to the amount of \$960, (a) what was the cost value of the merchandise; (b) how many dollars did Jones offer for it; and (c) how many dollars did Smith offer for it?

39. Find the cost of the following: $37\frac{1}{2}$ pounds of tea at $62\frac{1}{2}$ c per pound; $42\frac{1}{2}$ pounds of coffee at $41\frac{3}{4}$ c per pound; $5\frac{1}{2}$ pounds of pepper at $58\frac{1}{2}$ c per pound; $37\frac{1}{2}$ pounds of lard at $25\frac{1}{2}$ c per pound.

40. Find the sum of the results:

$$(16\frac{2}{3} + 27\frac{1}{3} - 12\frac{1}{6}) \times 3\frac{5}{7}$$

$$90\frac{1}{2} \div 10\frac{1}{2} + 125\frac{1}{7} - 81\frac{1}{3}$$

$$(12\frac{1}{6} + 50\frac{5}{6} + 40\frac{1}{3} - 76\frac{2}{3}) \times 3\frac{1}{4}$$

41. Find the sum of the sum, difference, product, and quotient of $16\frac{1}{2}$ and $3\frac{1}{4}$.

42. A merchant bought 12 pieces of denim as follows: $40\frac{1}{2}$, $43\frac{1}{4}$, $41\frac{1}{4}$, $41\frac{1}{2}$, $44\frac{1}{4}$, $43\frac{3}{4}$, $40\frac{1}{2}$, $40\frac{1}{4}$, $42\frac{1}{2}$, $41\frac{3}{4}$, $42\frac{1}{4}$, $40\frac{1}{2}$ yards at $16\frac{2}{3}$ c per yard. What was the cost?

43. A dealer bought grain at $\frac{9}{10}$ of the market price and sold it later for $\frac{1}{1}\frac{3}{2}$ of the former market price. If he gained \$66, how much did he pay for the grain?

44. Alfred Johnson works the following number of hours during the seven days beginning with Monday, May 9: Monday, $9\frac{3}{5}$; Tuesday, $9\frac{3}{4}$; Wednesday, $10\frac{3}{10}$; Thursday, $7\frac{7}{10}$; Friday, $9\frac{3}{4}$; Saturday, $9\frac{3}{5}$; Sunday, $8\frac{1}{2}$. His wages are $\$1.23\frac{1}{4}$ per hour for eight hours each day, time and a half for all time over eight hours each day, and double time for Sunday. How much was due him for the time worked during the seven days?

45. What is the number whose $\frac{4}{5}$ exceeds its $\frac{2}{3}$ by 60?

46. By what number must $\frac{5}{6}$ be multiplied to produce 1,704?

47. I bought 25 turkeys for \$145.50. (a) If the average weight is $10\frac{1}{2}$ pounds, how much did I pay per pound? (b) If I desire a profit of $\frac{1}{5}$ of cost, at what price per pound must I sell them?

48. Four men are partners in the general merchandise business and gain the first year, \$6,250. If they share the profits $\frac{2}{5}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{3}{20}$ respectively, how much will each get?

49. A man's liabilities are equal to $\frac{3}{4}$ of his assets. If his assets amount to \$48,000, what is his net worth?

50. A speculator gained $\frac{1}{10}$ on stock he sold at \$84. Had he sold 10 days later when it was quoted at \$90, (a) what would have been his gain? (b) What would he have lost, if he had sold at \$75?

51. A merchant bought a box of 120 apples at the rate of 3 for 5c. 18 were bruised and unfit for sale. He sold the others at the rate of 3 for 10c. What was his profit?

52. I bought 10 bags of wheat weighing $120\frac{1}{2}$, $122\frac{1}{4}$, $119\frac{3}{4}$, $121\frac{1}{2}$, $122\frac{3}{4}$, $119\frac{1}{2}$, $122\frac{1}{2}$, $120\frac{1}{4}$, 121, $123\frac{1}{4}$ pounds respectively. If I paid \$1.55 per bushel, allowing one pound each for the weight of the bags, what was the cost of the wheat?

53. I sold $\frac{3}{5}$ of my property for \$7,500. The part I have remaining at the same rate is worth $\frac{4}{5}$ of the original cost of the entire property. What was the cost?

54. A, B, and C are partners in a business, each owning $\frac{1}{3}$. A sold $\frac{1}{3}$ of his interest to B, and the remainder to C. What part does B and C each now own?

55. A man speculated in grain and lost $\frac{1}{5}$ of his investment. The next year he gained \$600, and then had \$2,600. What was his original investment?

56. The sum of two numbers is $48\frac{1}{4}$. Their difference is $6\frac{3}{4}$. What is the product of the numbers?

57. A, B, C, and D invest \$1,200, \$1,500, \$2,500, and \$1,000 respectively. They are to share gains or losses in proportion to capital invested. In the first year their profit was \$3,459.60. What will each receive?

58. A box of books weighed 384 pounds gross. If the box weighed 36 pounds, what is the fractional weight of both the box and its contents?

59. The distance between Harrisburg, Pennsylvania, and Erie, Pennsylvania, by one route, is $340\frac{1}{2}$ miles. Lock Haven, Pennsylvania, is on the same line of railroad as that between Harrisburg and Erie, and is $117\frac{7}{10}$ miles from Harrisburg. (a) What is the distance between Lock Haven and Erie? (b) How much farther is Lock Haven from Erie than from Harrisburg? (c) What fractional part of the entire distance between Harrisburg and Erie is the distance from Emporium which is $191\frac{4}{5}$ miles from Harrisburg?

60. Find the sum of the sum, difference, and product of $42\frac{7}{8}$ and $15\frac{1}{3}$.

61. A merchant bought 3,650 yards of silk at $\$1.12\frac{1}{2}$ per yard and sold $\frac{4}{5}$ of it at $\frac{3}{5}$ above cost. He had a special sale and reduced the selling price of the remainder $\frac{1}{5}$. It was all sold. What was his profit?

62. In a manufacturing plant $\frac{2}{5}$ of the space is given to machinery, $\frac{3}{10}$ to stores, $\frac{1}{5}$ to assembling and shipping, and the remainder, 1,200 square feet, is used for offices. How many square feet is occupied by each?

63. A grocer purchased a case of canned peaches containing 4 dozen cans for $\$6.36$. He retailed them at $17\frac{1}{2}$ c per can. (a) What was his profit, and (b) what fraction of the cost price was this profit?

64. Two fields are the same width, but one is $\frac{1}{5}$ longer than the other. If the smaller contains 35 acres, how many acres are there in both fields?

65. Multiply $145\frac{2}{3}$ by $12\frac{1}{4}$, and divide the product by $25\frac{1}{2}$.

66. A stock of merchandise is marked $\frac{1}{4}$ above cost, but on account of damage by water is sold at $\frac{1}{3}$ of the marked price. If the loss is $\$600$, what was (a) the cost and (b) the selling price?

67. What will $31\frac{1}{2}$ gallons of vinegar cost if $2\frac{1}{2}$ gallons cost 94c?

68. I gave a twenty-dollar bill in payment of 9 pounds of nuts at $16\frac{2}{3}$ c per pound, 12 pounds of coffee at $37\frac{1}{2}$ c per pound, and $12\frac{1}{2}$ pounds of butter at 48c per pound. What change should I receive?

69. A. L. Day, W. O. Marx, and E. F. Watts formed a partnership to operate a garage. It was agreed that each partner should share in the profits according to the amounts invested. Day invested \$1,500; Marx invested \$900; Watts invested \$1,200. After operating the garage for three months, the partners sold the business for \$5,400. You are to ascertain how the sale price was divided among the partners in accordance with their agreement.

70. Andrews, Arnold, and Anderson rented a pasture for \$48 per month. Andrews had 6 cows, Arnold, 5 cows, and Anderson 8 cows. It was agreed that each would pay a portion of the rent based on the number of cows pastured. How much was each required to pay based on this agreement?

71. Johnson and Smith operated a bus line between two towns. The profit was shared in proportion to the investment. Johnson invested \$2,750 and Smith, \$3,750. During the month of June the profits were \$128.50. What part of this profit did each partner receive?

72. A book contained 416 pages. It was printed on paper 36 by 48 inches, 32 pages on each side of a sheet. (a) How many pounds of paper would be needed to print 10,000 of these books allowing $\frac{1}{50}$ for waste, if each ream of paper weighs 120 pounds and contains 500 sheets? (b) What would be the cost of the paper at $11\frac{5}{8}$ c per pound?

73. C. E. Herzog sold a lot fronting $62\frac{1}{2}$ feet on Main Street for \$1,718.75. How much did I. L. Campbell receive for the adjoining lot which has a frontage of $72\frac{1}{2}$ feet, if he sold it at the same price per front foot.

74. Robert Douglas owed Brown Brothers, retail merchants, \$187.65. What was the balance of his indebtedness, if Brown Brothers accepted in part payment, 127 dozen eggs at $41\frac{1}{2}$ c per dozen and 62 pounds of butter at $39\frac{1}{2}$ c per pound?

75. How much profit is made by purchasing 1760 lbs. of wool at $17\frac{1}{2}$ c per pound, and selling it at $32\frac{3}{8}$ c per pound, after paying freight charges of $\$1.25\frac{1}{4}$ per hundred pounds?

76. The A. & B. railroad purchased a right of way through the farm of Russell Mann. It was agreed to pay him \$120 per

acre for the ground used by the railroad. How much did he receive if the right of way was 99 feet wide and 480 rods long?

NOTE: A rod is $16\frac{1}{2}$ feet, and an acre contains 160 square rods.

77. A lot, 150 feet by 300 feet, was sold for \$22,500. What was the price per acre?

78. What is the selling price of three bales of cotton weighing $498\frac{1}{2}$ pounds, $502\frac{3}{4}$ pounds, $501\frac{3}{8}$ pounds at $20\frac{7}{8}$ c per pound?

79. A merchant had a pecan grove of 125 trees that averaged 55 pounds to the tree. He sold one half of his crop at $10\frac{1}{2}$ c per pound, and the other half for $12\frac{1}{2}$ c per pound. How much did he receive for the entire crop?

80. H. B. Fall, a farmer, sold 22,581 pounds of seed cotton at $7\frac{3}{8}$ c per pound. How much did he receive for it?

81. Find the sum of the difference and product of $61\frac{1}{4}$ and $38\frac{2}{3}$.

82. Mrs. Smith bought the following dry goods: $4\frac{1}{2}$ yards pongee at \$1.12 per yard; 6 yards muslin at $18\frac{1}{3}$ c per yard; 16 yards filet lace edging at $8\frac{1}{2}$ c per yard. What is the total amount of her purchase?

83. How much will it cost to build 94 feet of wire fence at $\$1.56\frac{1}{2}$ per foot?

84. A boy saves one fifth of his income each week and gives one tenth of the remainder to charity. If his income is \$25 per week, (a) how much will he save each week; (b) how much will he give to charity in 12 weeks time?

85. A boy earns \$6 for $10\frac{1}{2}$ hours work each week; another boy earns 50 cents an hour for 2 hours and 15 minutes work on each of six days. Each boy deposits $\frac{1}{4}$ of his weekly earnings in a savings account in a bank. (a) What is the difference in the weekly earnings of each boy; (b) in the weekly savings?

CHAPTER XI

DECIMAL FRACTIONS

There are two methods of writing a fractional value, (a) by placing the numerator over the denominator, and (b) by writing the numerator and indicating the denominator by the decimal point; when both the numerator and denominator are written, it is known as a common fraction, and when the denominator is expressed by a decimal point, as a decimal fraction. A fractional value can be expressed decimally only when the denominator is ten or a power of ten. The denominator of a decimal fraction is determined by the position of the figures which express the value of the numerator and their relation to the decimal point.

The fractional value, one tenth, written as a common fraction is $\frac{1}{10}$, and as a decimal fraction, .1; seven tenths, as $\frac{7}{10}$ or .7; twenty-seven hundredths, as $\frac{27}{100}$ or .27; seven hundredths, as $\frac{7}{100}$ or .07. You will observe that when there are not a sufficient number of figures in the numerator to indicate the denominator that ciphers are placed between the decimal point and the figure or figures in the numerator. Thus, in writing the fractional value, seven hundredths, as a decimal, it is necessary to place a cipher between the 7 and the decimal point, otherwise the value of the fraction would be seven tenths. There must be as many figures at the right of the decimal point as there are ciphers in the denominator if the fraction were expressed as a common fraction.

EXERCISE No. 103, EXPRESSING FRACTIONS

The problems on page 141 provide oral practice in expressing fractional values both by common and decimal fractions. As the fractions are read you are to describe the method of writing both as a common fraction and as a decimal fraction.

The enumeration is from the right of the decimal point to the right beginning with tenths. Thus, a decimal fraction with a

denominator of hundredths will have two decimal places; thousandths, three decimal places; ten-thousandths, four decimal places; hundred-thousandths, five decimal places, etc.

NOTE: The word "and" in decimal fractions, indicates a mixed decimal, and denotes the decimal point; it connects the whole number and the decimal fraction. It is also used in reading a complex decimal.

1. Ninety-nine hundredths
2. One hundred seventy-five thousandths
3. One hundred and seventy-five thousandths
4. Sixteen ten-thousandths
5. Seven hundred fifty-five ten-thousandths
6. Seven and two thirds of a thousandth
7. Seven and three and two thirds thousandths
8. Thirty-one and thirteen hundredths
9. Seventy-seven hundred-thousandths
10. Two thousand and twenty-five ten-thousandths
11. Two hundred fifteen ten-thousandths
12. Three thousand seventy-five hundred-thousandths
13. Forty-four hundredths
14. Seventy-eight hundred-thousandths
15. Sixty-five ten-thousandths
16. Four hundred ten thousandths
17. Five hundred sixty-five hundred-thousandths
18. Five hundred and one ten-thousandths
19. Seventeen thousandths
20. Five thousand five hundred fifty-five hundred-thousandths
21. Thirty-one thousandths
22. One hundred thousandths
23. Five hundred one ten-thousandths
24. Eighteen thousand three hundred forty-one hundred-thousandths
25. Thirteen ten-thousandths
26. One hundred and three thousand one hundred thirty-six ten-thousandths
27. One and sixteen thousandths

EXERCISE No. 104, CHANGING COMMON FRACTIONS TO DECIMALS

When it is desired to express a fractional value shown in common fraction form as a decimal fraction, it is necessary to reduce the fraction to its equivalent value in tenths, hundredths, thousandths, etc. This reduction may be made (a) by multiplying the numerator and the denominator by a number which will express the denominator as tenths, hundredths, thousandths, etc., or (b) to divide the numerator by the denominator. The method of reducing common fractions to decimals by multiplying the denominator and the numerator by the required number is illustrated in this exercise; the method of reduction by division will be illustrated in a subsequent exercise.

When it is desired to express a common fraction in decimal form, and ten or a power of ten is divisible by the denominator without a remainder, divide by the denominator and multiply the numerator by the quotient. The illustration below shows the process of expressing the fractions, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{5}{8}$ in decimal form.

$$\frac{1}{2} \times \frac{5}{5} = \frac{5}{10} = .5$$

$$\frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = .75$$

$$\frac{2}{5} \times \frac{2}{2} = \frac{4}{10} = .4$$

$$\frac{5}{8} \times \frac{125}{125} = \frac{625}{1000} = .625$$

To express the fraction, $\frac{1}{2}$, as a decimal, it is necessary to secure a denominator of 10 or a power of 10. Since 2 is contained in 10 without a remainder, both the numerator and denominator are multiplied by the quotient resulting from dividing 10

by 2. The value of the fraction will be expressed in tenths. This plan is followed in the illustration at the left. The fraction $\frac{5}{10}$ can be changed to decimal form by omitting the denominator and placing the decimal point at the left of the numerator.

The fraction, $\frac{3}{4}$, cannot be expressed as tenths, because 4 is not contained in 10 an even number of times. However, it can be expressed as hundredths, since 4 is contained in 100, 25 times.

The fraction, $\frac{2}{5}$, can be expressed as tenths because 5 is contained in 10, 2 times.

The fraction, $\frac{5}{8}$, cannot be expressed as tenths or hundredths, because 8 is not contained an even number of times in 10 or 100, but it is contained in 1,000 an even number of times.

The following problems provide written practice in changing common fractions to decimal fractions by the process explained and illustrated above. Arrange your work on tablet paper in the same form as in the illustration.

1.	2.	3.	4.
a. $\frac{7}{25}$	a. $\frac{3}{4}$	a. $\frac{4}{25}$	a. $\frac{4}{5}$
b. $\frac{27}{125}$	b. $\frac{15}{80}$	b. $\frac{9}{40}$	b. $\frac{21}{400}$
c. $\frac{9}{50}$	c. $\frac{18}{25}$	c. $\frac{1}{5}$	c. $\frac{63}{125}$
d. $\frac{5}{8}$	d. $\frac{17}{20}$	d. $\frac{11}{40}$	d. $\frac{5}{18}$

EXERCISE No. 105, CHANGING DECIMALS TO COMMON FRACTIONS

When it is desired to express the value of a decimal fraction as a common fraction, place the denominator indicated by the value of the numerator under the numerator, and reduce the fraction to its lowest terms. Thus, .5 indicates the value, five tenths. This expressed as a common fraction would be $\frac{5}{10}$, which reduced to its lowest term is $\frac{1}{2}$.

The following problems provide written practice in expressing decimal fractions in the form of common fractions of equivalent value. Show all of your calculations on tablet paper.

1.	2.	3.	4.
a. .55	a. .375	a. .36	a. .46
b. .82	b. .1875	b. .750	b. .09
c. .06	c. .0025	c. .3125	c. .07
d. .625	d. .4755	d. $.33\frac{1}{3}$	d. $.08\frac{1}{3}$
e. .45	e. .0125	e. $.12\frac{1}{2}$	e. .31

ADDITION OF DECIMAL FRACTIONS

The addition of decimal fractions is for the same purpose as the addition of common fractions; that is, to ascertain the sum of two or more fractional values. The process of addition in decimal fractions is the same as in whole numbers after the numbers to be added are arranged so that units are under units, tens under tens, etc. As in the addition of whole numbers or common fractions, the sum will be greater than the value of any one of the numbers added. The sum should be written as in whole numbers keeping the decimal point in the same position in the sum as it is in the numbers added. If the sum of the decimals added is equal to or greater than one, the whole number is written at the left of the decimal point.

EXERCISE No. 106, ADDITION OF DECIMAL FRACTIONS

The following problems provide written practice in the addition of fractional values expressed as decimal fractions. Show on tablet paper, (a) the sum of each column added, (b) the sum of all the columns, and (c) the proof by reverse addition.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
.7	.3	.4975	.09	.9518	.723	.1824	.902
.16	.19	.5421	.7	.321	.12	.2071	.738
.395	.457	.9814	.625	.576	.519	.6925	.476
.9999	.3119	.7746	.8712	.743	.4897	.7298	.35
.2142	.5678	.2132	.3875	.9182	.7139	.5802	.4
.756	.245	.6539	.506	.8273	.4532	.9009	.712
.85	.37	.9001	.9	.123	.898	.3116	.29
.9	.5	.2078	.12	.4561	.424	.8195	.9916

EXERCISE No. 107, CALLING DECIMALS

As explained in a previous exercise, two parties are involved in calling figures, because the only purpose in calling is to write or check the amount called. The two parties involved should agree upon a code so that there may be no misunderstanding.

The following problems provide practice in calling and writing decimals, and in addition of decimals. You will show on tablet paper: (a) the sum of each column added; (b) the sum of all the columns; and (c) the proof by reverse addition.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
.769	.3	.3014	.509	.7568	.3225	.8	.2029
.131	.05	.3247	.1492	.3539	.5	.39	.0102
.8694	.225	.4639	.675	.209	.23	.009	.112
.37	.3579	.2001	.5559	.7	.4071	.881	.295
.4	.2468	.1918	.1263	.3215	.0003	.9532	.48
.0006	.875	.3575	.2198	.03	.913	.7581	.002
.3769	.19	.1827	.48	.9182	.429	.133	.21
.538	.37	.2112	.048	.6	.0005	.325	.301
.06	.7	.1444	.01	.3008	.192	.29	.221
.9	.9	.5053	.0001	.17	.02	.4	.848

EXERCISE No. 108, ADDITION OF MIXED DECIMALS

The process of addition of mixed decimals is the same as the addition of decimals. The whole number is written at the left of the decimal point and the fraction at the right. Place tenths under tenths, hundredths under hundredths, etc. The enumeration of the decimal is from left to right beginning with tenths, and the enumeration of the whole number is from right to left beginning with units.

The following problems provide written practice in addition of mixed decimals. Show on tablet paper: (a) the sum of each column; (b) the sum of all the columns; and (c) proof by reverse addition.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
2098.1801	1598.09	9.8	1234.5678
5756.1982	7.75	13.21	356.102
9521.0019	918.625	625.505	27.01
1532.1492	1098.1444	1492.7981	3.3
1444.0509	509.7	1001.4332	7.8
7582.6257	37.67	509.111	31.67
1337.2118	112.111	21.75	999.107
3022.6029	1.0007	1.1	1776.1924

EXERCISE No. 109, CALLING MIXED DECIMALS

The following problems provide practice in calling and writing mixed decimals when the fractional values are expressed decimally. Find the sum of each problem and prove your results by reverse addition.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
3578.1329	721.001	1925.1926	21.98
821.098	9.1	1898.1001	666.625
43.11	85.48	7501.6145	3.7
6.9	2000.0002	3198.7152	21.0001
2.1	5243.8756	2000.1444	155.671
75.16	2978.3519	5053.2198	2198.1444
221.133	1795.2901	2222.1886	10.1

SUBTRACTION OF DECIMAL FRACTIONS

The purpose of subtracting one decimal fraction from another is to ascertain the difference between the two decimals. The process of subtraction in decimal fractions is the same as in whole numbers. Since the position of the decimal always determines the value of the fraction, it is necessary to have the decimal points under one another before subtracting.

The illustrations below show the solution of .487 subtracted from .5, and .5 subtracted from .787.

The solution at the left shows the process of subtraction when three figures are required to express the fraction to be taken from one which can be expressed with one number. The subtraction of 7 is from 10, 8 from 9,

$\begin{array}{r} .5 \\ .487 \\ \hline .013 \end{array}$	and 4 from 4 by the process of borrowing, or adding to 7 the number which will make 10. Ciphers could be added to the right of 5, but this is not necessary, since they may be assumed. In the process at the right, the value of the fraction to be subtracted is expressed in one figure, and the	$\begin{array}{r} .787 \\ .5 \\ \hline .287 \end{array}$
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fraction from which it is to be subtracted, in three figures. Ciphers could be added to the right of the 5, but this is not necessary. The process is to bring down the figures in the hundredths and thousandths columns, and subtract the figures in the tenths column.

Keep in mind the fact that it is not possible to subtract a decimal fraction with a larger value from one with a smaller value, no more than it is possible to subtract a whole number with a larger value from a whole number with a smaller value.

EXERCISE No. 110, SUBTRACTION OF DECIMAL FRACTIONS

The following problems provide written practice in subtraction of decimal fractions. Show on tablet paper the solution of each problem in the same form as in the above illustrations.

1	2	3	4	5
a. $\begin{array}{r} .25 \\ .125 \\ \hline \end{array}$	a. $\begin{array}{r} .2 \\ .0001 \\ \hline \end{array}$	a. $\begin{array}{r} .7 \\ .699 \\ \hline \end{array}$	a. $\begin{array}{r} .2 \\ .0221 \\ \hline \end{array}$	a. $\begin{array}{r} .14449 \\ .12315 \\ \hline \end{array}$
b. $\begin{array}{r} .796 \\ .509 \\ \hline \end{array}$	b. $\begin{array}{r} .5019 \\ .0201 \\ \hline \end{array}$	b. $\begin{array}{r} .001 \\ .0001 \\ \hline \end{array}$	b. $\begin{array}{r} .6374 \\ .3528 \\ \hline \end{array}$	b. $\begin{array}{r} .76852 \\ .49736 \\ \hline \end{array}$
c. $\begin{array}{r} .7 \\ .235 \\ \hline \end{array}$	c. $\begin{array}{r} .325 \\ .0475 \\ \hline \end{array}$	c. $\begin{array}{r} .9801 \\ .5766 \\ \hline \end{array}$	c. $\begin{array}{r} .0901 \\ .0176 \\ \hline \end{array}$	c. $\begin{array}{r} .21359 \\ .09078 \\ \hline \end{array}$

EXERCISE No. 111, SUBTRACTION OF MIXED DECIMALS

The process of subtracting mixed decimals is the same as when subtracting decimal fractions. The decimal points must

be placed in a vertical line, with the whole number values on the left, and the fractional values on the right, of the decimal point. The difference must be arranged in the same manner.

The following problems provide practice in subtracting decimals. You will observe when mixed decimals are involved, the fraction in the subtrahend of the mixed decimal may be larger than the fraction in the minuend of the mixed decimal because it is possible to increase the value of the fraction by borrowing.

1	2	3	4
<i>a.</i> 1000.0001 <u>1.1</u>	<i>a.</i> 5138.7343 <u>4917.9872</u>	<i>a.</i> 256.303 <u>199.545</u>	<i>a.</i> 243.66 <u>128.35</u>
<i>b.</i> 2109.0098 <u>143.7786</u>	<i>b.</i> 1774.1896 <u>296.0041</u>	<i>b.</i> 405.001 <u>31.786</u>	<i>b.</i> 267.89 <u>165.01</u>
<i>c.</i> 8778.1561 <u>5489.3547</u>	<i>c.</i> 3149.9876 <u>2214.7784</u>	<i>c.</i> 443.687 <u>357.441</u>	<i>c.</i> 143.67 <u>1.99</u>

EXERCISE No. 112, SUBTRACTION OF MIXED NUMBERS

When the fractional value in one mixed number is expressed as a common fraction and in the other as a decimal fraction, it is necessary to express both values either as decimal fractions or as common fractions before subtracting. The nature of the problem will determine whether it is better to change the decimal to a common fraction or the common fraction to a decimal.

The following problems provide written practice in subtraction when both common and decimal fractions are involved. If both forms of the fraction are expressed, you may use your choice of changing the common fraction to a decimal, or the decimal fraction to a common fraction before subtracting. You will show on tablet paper all the calculations necessary to ascertain the desired answer and prove its correctness.

1	2	3
<i>a.</i> $\frac{3}{4} - .325$	<i>a.</i> $.9876 - \frac{9}{17}$	<i>a.</i> $24\frac{3}{8} - 18.244$
<i>b.</i> $\frac{9}{11} - .4465$	<i>b.</i> $.844 - \frac{9}{16}$	<i>b.</i> $63\frac{3}{8} - 52.4$
<i>c.</i> $\frac{7}{16} - .1716$	<i>c.</i> $.3444 - \frac{3}{41}$	<i>c.</i> $58.72\frac{1}{3} - 44.89\frac{4}{9}$
<i>d.</i> $\frac{12}{25} - .007$	<i>d.</i> $.9 - \frac{9}{13}$	<i>d.</i> $75.5\frac{3}{7} - 67.987\frac{1}{3}$
<i>e.</i> $\frac{2}{27} - .5$	<i>e.</i> $15\frac{3}{4} - 12.0055$	<i>e.</i> $4.567\frac{1}{2} - 2.999\frac{1}{9}$
<i>f.</i> $\frac{4}{5} - .125$	<i>f.</i> $36\frac{1}{3} - 24.97$	<i>f.</i> $28.093\frac{1}{3} - 25$

EXERCISE No. 113, REVIEW

The following problems provide a review of addition and subtraction of decimals. Show on tablet paper all the calculations necessary to arrive at the desired answer in each problem, and the proof of its correctness. If you can solve any of these problems without the use of the pencil, write the answer, and after it the letter "M" to indicate mental solution.

1. Add four and fifty-five ten-thousandths, seventy-five hundred-thousandths, one and sixteen hundredths, five thousand and one millionth.

2. From the sum obtained in problem 1 take the sum of the following: sixteen ten-thousandths, one and nine hundred fifteen ten-thousandths, and seven eighths of a hundred-thousandth.

3. Add 75 units, 75 tenths, 75 hundredths, 75 thousandths, and 75 ten-thousandths.

4. $\frac{3}{4} + \frac{7}{8} + \frac{2}{3} + 1.6\frac{2}{3} + 10\frac{1}{4} + 37\frac{3}{8} + 1.9 + \frac{5}{8} + 12\frac{3}{8} + 2\frac{1}{8} =$

5. Subtract $15.99\frac{1}{5}$ from 49.0016 .

6. From $77.4\frac{1}{2}$ take $65.87\frac{1}{8}$.

7. From $3.45\frac{2}{5}$ take $2.99\frac{3}{4}$.

8. How much greater is $\frac{1}{2}$ of a tenth than $\frac{2}{3}$ of a hundredth?

9. How much more is nine and three fifths of a hundredth than six and two fifths of a tenth?

10. From 300 take 3 hundredths.

11. What is the sum of $9.45 + .06\frac{2}{5} + 5.6 + 4.16\frac{3}{4} + 7.8\frac{1}{5}$?

12. Add $.24\frac{1}{5}$, $.1745\frac{3}{4}$, 46.5 , and 38.0007 .

13. $.3708 + \frac{1}{2} + 2.06\frac{2}{5} + \frac{3}{4} + .17\frac{3}{5} + .0\frac{1}{4} = ?$

14. $81.26\frac{1}{2} + .046\frac{1}{2} + 9.6\frac{2}{5} + 76.7895\frac{3}{4} = ?$

15. From $\frac{2}{3}$ of a tenth take $\frac{1}{2}$ of a hundredth.

16. To 19 hundredths add $\frac{2}{3}$ of a tenth.

17. What is the sum in decimals, of $3\frac{7}{8}$, $1\frac{2}{5}$, $5\frac{1}{2}$, $2\frac{3}{4}$?

18. Subtract $16\frac{1}{2}$ tenths from 4609 thousandths.

19. From $\frac{1}{18}$ of a hundredth take $\frac{5}{9}$ of a thousandth.

20. From $\frac{1}{5}$ of $\frac{1}{2}$ take $.0925$.

CHAPTER XII

DECIMALS (Continued)

MULTIPLICATION OF DECIMALS

The purpose of multiplication of decimal fractions is to ascertain the product of two decimals. The process of multiplication in decimal fractions is the same as in whole numbers, except it is necessary to take into consideration the fractional value of the product, and to indicate this by having as many figures at the right of the decimal point as there are figures at the right of the decimal points in both the multiplier and multiplicand.

$\begin{array}{r} .671 \\ .015 \\ \hline 3355 \\ 671 \\ \hline .010065 \end{array}$	The solution at the left shows the process of multiplication of two decimals which have the same denominator, and the one at the right the multiplication of two decimals that have different denominators. You will observe the process of multiplication is the same as that in integers, and there must be as many figures at the right of the decimal point as there are in both the multiplier and multi-	$\begin{array}{r} .047 \\ .35 \\ \hline 235 \\ 141 \\ \hline .01645 \end{array}$
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plicand. When there are not enough figures in the product, sufficient ciphers must be prefixed to make the decimal places the same.

EXERCISE No. 114, MULTIPLICATION OF DECIMALS

The following problems provide written practice in the multiplication of decimal fractions. Show on tablet paper the solution of each problem in the same form as in the illustration above, and prove by reverse multiplication. Each problem requires two solutions: one showing the multiplier beneath the multiplicand; and the other, the multiplier above the multiplicand.

1	2	3
<i>a.</i> $.0725 \times .072$	<i>a.</i> $.3765 \times .012$	<i>a.</i> $.009965 \times .9$
<i>b.</i> $.0075 \times .005$	<i>b.</i> $.7564 \times .87$	<i>b.</i> $.009 \times .008$
<i>c.</i> $.2653 \times .325$	<i>c.</i> $.6562 \times .598$	<i>c.</i> $.2009 \times .675$
<i>d.</i> $.8642 \times .967$	<i>d.</i> $.0083 \times .7$	<i>d.</i> $.315 \times .18$

EXERCISE No. 115, MULTIPLICATION OF DECIMALS

The following problems provide practice in: (a) writing decimals; (b) multiplication of decimals; and (c) expressing the product in figures, and (d) in words.

1. Multiply six hundred seventy-five thousandths by seven hundredths.
2. Multiply seven thousand nine ten-thousandths by two thousandths.
3. Multiply three hundred twelve thousandths by fifty-seven hundredths.
4. Multiply sixty-seven hundredths by five thousandths.
5. Multiply four hundred fifty-four ten-thousandths by three hundred sixty-seven thousandths.
6. Multiply nine thousand nine hundred nine ten-thousandths by six ten-thousandths.
7. Multiply four hundred seventeen thousandths by twenty-nine hundredths.
8. Multiply one thousand four hundred forty-four ten-thousandths by five hundred nine thousandths.

EXERCISE No. 116, MULTIPLICATION OF INTEGERS AND DECIMALS

When it is desired to multiply a decimal fraction by an integer the process is the same as in multiplying two decimal fractions. The important element is to ascertain the number of decimal places in the product.

The following problems provide written practice in the multiplication of integers and decimals. Show on tablet paper all the calculations necessary: (a) to ascertain the product; and (b) to prove the product by reverse multiplication.

1	2	3
a. $4981 \times .9009$	a. $3374 \times .1445$	a. $1742 \times .1492$
b. $762 \times .635$	b. $225 \times .13$	b. $1883 \times .005$
c. $3774 \times .1045$	c. $6725 \times .7631$	c. $4752 \times .367$
d. $7 \times .1673$	d. $3456 \times .33$	d. $298 \times .7$

EXERCISE No. 117, MULTIPLICATION OF MIXED DECIMALS

When an integer and a decimal are involved in both the multiplier and multiplicand, the process is the same as when multiplying decimals or a decimal and a whole number.

The following problems provide written practice in multiplying mixed decimals. Show on tablet paper all the calculations necessary (a) to ascertain the product; and (b) to prove the product by reverse multiplication.

1	2	3
<i>a.</i> 9.887×100.6	<i>a.</i> 763.25×2214.6	<i>a.</i> 321.175×174.258
<i>b.</i> 35.42×625.8	<i>b.</i> 4356.2×329.43	<i>b.</i> 75.3908×214.432
<i>c.</i> 321.9×1.653	<i>c.</i> 55.424×1.2345	<i>c.</i> 625.107×4.00807
<i>d.</i> 277.3×14.25	<i>d.</i> 892.15×1007.1	<i>d.</i> 3675.42×275.314
<i>e.</i> 6.517×216.4	<i>e.</i> 56.432×1492.4	<i>e.</i> 1.00006×56.1472
<i>f.</i> 1.111×111.11	<i>f.</i> 1314.8×3.6	<i>f.</i> 36314.1×96.21
<i>g.</i> 313.1×8.136	<i>g.</i> 1912.6×11.83	<i>g.</i> 10009.2×83.161
<i>h.</i> 919.5×63.18	<i>h.</i> 1001.3×318.1	<i>h.</i> 1921.6×328.92

EXERCISE No. 118, BILLING

The multiplication of integers and fractions expressed decimally occur frequently in the extension of bills and inventories. The reason for this is because it is not always possible to express by integers the quantity of merchandise sold and the price.

The following problems provide practice in making extensions when the fractional values are expressed as decimals. You are to ascertain the correct amount of each sale which is the sum of the values of each item sold.

1. Brown Brothers, provision merchants, Buffalo, New York, sold J. C. Warren the following provisions:

20 pieces, 421 lbs., Bacon Sides	@	\$.1375
25 Hams, $395\frac{1}{2}$ lbs.,	@	.225
50 cans, 2500 lbs., Lard	@	.2175
5 sacks, 385 lbs., Dried Apples	@	.145

CAUTION: When decimal fractions are involved in the extensions of invoices, be sure to place the decimal point in either the multiplier, multiplicand, or both, before beginning the multiplication. Unless this plan is followed, there may be an error in placing the decimal point in the product. Thus, in extending the first item of the above invoice, there is no decimal point in the number of pounds, but there are four figures at the right of the decimal point in the price, hence there must be four figures at the right of the decimal point in the product.

2. W. E. Sutton & Company, Stock Brokers, sold the following stocks to W. O. Crosswhite, January 5:

25 shares, American Express	@	136 $\frac{3}{4}$
35 shares, American Locomotive	@	116 $\frac{1}{2}$
52 shares, American Radiator	@	114 $\frac{1}{8}$
36 shares, American Smelting & Refining Co.,	@	133 $\frac{1}{2}$
41 shares, Delaware-Lackawanna	@	145 $\frac{3}{4}$
45 shares, Gulf States Steel	@	90 $\frac{5}{8}$
15 shares, International Harvester	@	128 $\frac{3}{4}$
32 shares, Union Pacific Ry.	@	149 $\frac{7}{8}$
24 shares, U. S. Steel	@	133 $\frac{7}{8}$

NOTE: The quotation on American Express means \$136.75 per share, hence the value of 25 shares would be ascertained by multiplying \$136.75 by 25. This will give a product of \$3,418.75. The extensions of the other items are made in the same manner. Where the price is expressed in eighths, each eighth equals 12 $\frac{1}{2}$ c.

3. The American Sugar Company sold the following items to J. B. Lipscomb, a retail grocer, 156 Walnut St., November 17:

5 bbls., 1,642 lbs., Granulated Sugar	@	\$6.35 per cwt.
3 bbls., 917 lbs., Brown Sugar	@	5.95 per cwt.
25, 100 lb. bags, Granulated Sugar	@	7.05 per cwt.
25, 50 lb. bags, Granulated Sugar	@	7.07 per cwt.
10, 25 lb. sacks, Granulated Sugar	@	7.09 per cwt.
200 boxes Domino Sugar	@	.23 per box
50 boxes Powdered Sugar	@	.27 per box

4. The Hale Justis Wholesale Drug Co. sold the following items to The Corner Drug Store, June 6:

6 doz. bxs. Aspirin	@	\$1.41
5 doz. bottles Witch Hazel	@	2.30
36 bxs. Talcum Powder	@	.18
20 jars Vaseline	@	.11 $\frac{3}{4}$
8 doz. bxs. Zinc Stearate	@	1.80
24 bottles Rubbing Alcohol	@	.52 $\frac{1}{2}$

EXERCISE No. 119, AUDITING INVENTORY

On December 31, the Peoples Grocery Company had in stock the merchandise given on the list below. You are to audit this inventory and show on tablet paper: (a) the calculations necessary to verify each extension; (b) errors in the extensions; (c) the corrected total

MERCHANDISE INVENTORY, DECEMBER 31, 19—

8	cases of Tag Soap.....	\$4.75	\$ 38		
7 $\frac{1}{2}$	doz. LaCreme Mustard....	1.75	13 13		
225	lbs. Corn Meal.....	.03 $\frac{1}{2}$	7 85		
30	sacks Gold Medal Flour...	1.45	43 50		
103	jars Preserves.....	.12 $\frac{1}{2}$	12 88		
87 $\frac{1}{2}$	doz. No. 2 Canned Peas...	2.28 $\frac{3}{4}$	200 36		
275	pkgs. Brillo.....	.09 $\frac{1}{4}$	25 44		
8	kegs Herring.....	1.23 $\frac{1}{2}$	19 92		
35 $\frac{3}{4}$	doz. cans of Peaches.....	2.75	99 31		
45 $\frac{1}{2}$	lbs. Mixed Candy.....	.22 $\frac{1}{2}$	10 24		
286 $\frac{1}{2}$	lbs. Chocolate Creams.....	.15	43 98		
78	pkgs. Mello Wheat.....	.15	11 70		
325	lbs. Bokar Coffee.....	.55	178 75		
146	lbs. Santos Coffee.....	.33 $\frac{1}{3}$	47 95		
248	lbs. 8 o'clock Coffee.....	.39	98 62		
986	lbs. Potatoes.....	.049	49 31		
216	lbs. Rome Beauty Apples..	.055	12 85		
427	lbs. Leaf Lard, unrendered.	.145	61 91		
46	lbs. Pork Sausage.....	.33	15 18		
36	doz. Tangerines.....	.35	12 75		
58	doz. Oranges.....	.40	24 20		
24	doz. Grapefruit.....	.98 $\frac{1}{3}$	23 61		
18	lbs. Grapes.....	.15	3 70		
26 $\frac{1}{2}$	doz. Strictly Fresh Eggs...	.485	12 85		
24	lbs. Figs.....	.23	6 52		
10	lbs. Dates.....	.30	3 00		
50	bxs. Superior Salt.....	.04	2 00		
36	lbs. Nut Filled Dates.....	.225	8 10		
Total.....				\$1087 61	

EXERCISE No. 120, COMPENSATION

Very often in business the compensation of a salesman is a definite number of cents on each dollar of the sales. When this is the case, it is necessary to multiply the sales by the number of cents, expressed decimally in hundredths, to find the compensation. The important element in such problems is the correct expression of the product by the decimal point.

The following problems provide practice in ascertaining the compensation of salesmen based on an agreed number of cents on each dollar of sales.

1. The Atlas Manufacturing Company employs twelve salesmen. Their compensation is a commission of $9\frac{1}{2}\text{c}$ on each dollar of their sales. The sales made by each of the salesmen during the month of May were as follows:

a. Davis.....	\$2,604.65	g. Older....	\$3,152.40
b. Malcolm....	2,732.55	h. Curry...	2,318.85
c. Jones.....	2,346.50	i. Marx....	3,155.50
d. Miller.....	2,541.95	j. Mason...	2,242.75
e. Roberts....	2,887.42	k. Hardin..	2,714.40
f. Kaufmann..	2,247.45	l. Wilder...	2,127.58

You are to ascertain: (a) the amount of commission earned by each salesman; (b) the total sales; and (c) the total commission.

2. At the beginning of June, the sales manager agreed to allow those salesmen whose sales were larger than the average established in May, an extra cent commission on the sales in June above the May average. The sales in June were as follows:

a. Davis.....	\$2,648.25	g. Older....	\$2,968.42
b. Malcolm....	2,832.47	h. Curry...	2,472.28
c. Jones.....	2,462.94	i. Marx....	3,147.75
d. Miller.....	2,645.31	j. Mason...	2,537.68
e. Roberts....	2,784.35	k. Hardin..	2,674.32
f. Kaufmann..	2,137.56	l. Wilder...	2,243.49

You are to ascertain: (a) the amount of compensation received by each salesman; (b) the total compensation of the

twelve salesmen; and (c) the total sales made by all of the salesmen.

3. At the beginning of July, the sales manager agreed to give the salesmen the same rate of compensation as in June, but the bonus of one cent is to apply on the sales above the June average. The sales in July were as follows:

<i>a.</i> Davis.....	\$2,742.50	<i>g.</i> Older.....	\$2,687.48
<i>b.</i> Malcolm....	2,784.85	<i>h.</i> Curry.....	2,458.46
<i>c.</i> Jones.....	2,543.27	<i>i.</i> Marx.....	2,894.37
<i>d.</i> Miller.....	2,523.58	<i>j.</i> Mason.....	2,604.45
<i>e.</i> Roberts....	2,957.29	<i>k.</i> Hardin.....	2,684.37
<i>f.</i> Kaufmann..	2,427.60	<i>l.</i> Wilder.....	2,420.75

You are to ascertain: (a) the amount of compensation received by each salesman; (b) the total compensation of the twelve salesmen; and (c) the total sales made by all of the salesmen.

DIVISION OF DECIMALS

The division of decimal fractions is for the same purpose as the division of common fraction and integers; that is, to ascertain the number of times one decimal fraction is contained in another decimal fraction. The process of division in decimal fractions is the same as in whole numbers, and common fractions, except it is necessary to take into consideration the fractional value of the quotient and to indicate this by having as many figures at the right of the decimal point as the number of decimal places in the dividend exceed those in the divisor. Thus, if the dividend has four figures at the right of the decimal point, and the divisor has one figure at the right of the decimal point, the quotient will have three figures at the right of the decimal point. When the decimal places in the divisor exceed those in the dividend, a sufficient number of ciphers should be annexed at the right of the decimal point to make those in the dividend and divisor equal. When all of the figures or ciphers in the dividend have been used through the process of division, additional ciphers may be annexed at the right of the dividend, and the process of division continued; the figures in the quotient after the ciphers are annexed will be at the right of the decimal point.

The illustration below shows the process of dividing .123 by .315 and .414 by .0375.

$ \begin{array}{r} .3904 + \\ \hline .315 \overline{) .1230000} \\ \underline{945} \\ 2850 \\ \underline{2835} \\ 1500 \\ \underline{1260} \\ 240 \end{array} $	The solution at the left shows the process of division with decimals which have the same denominator; the solution at the right, the division of two decimals which have a different denominator. The quotient contains as many decimal places as the number of decimal places in the dividend exceed the number in the divisor.	$ \begin{array}{r} 11.04 \\ \hline .0375 \overline{) .414000} \\ \underline{375} \\ 390 \\ \underline{375} \\ 1500 \\ \underline{1500} \end{array} $
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EXERCISE No. 121, DIVISION OF DECIMALS

The following problems provide written practice in the division of decimal fractions. Show on tablet paper: (a) the quotient obtained in the same form as illustrated above; and (b) proof of the quotient by multiplication. If the division is not exact, annex ciphers to the dividend until three decimal places are obtained in the quotient.

1	2	3
a. $.3544 \div .22$	a. $.21 \div .0007$	a. $.26 \div .0076$
b. $.2096 \div .04$	b. $.45 \div .1312$	b. $.2245 \div .17$
c. $.7853 \div .37$	c. $.63 \div .0021$	c. $.42 \div .0006$
d. $.4598 \div .55$	d. $.75 \div .0025$	d. $.75 \div .0015$
e. $.2778 \div .03$	e. $.98 \div .7743$	e. $.2216 \div .14$
f. $.4935 \div .21$	f. $.64 \div .3124$	f. $.7308 \div .63$

EXERCISE No. 122, DIVISION OF DECIMALS

The following problems provide written practice in: (a) expressing, with figures, fractional values in decimal form; (b) division of one decimal by another; and (c) proof of division by multiplication. You are to show on tablet paper all calculations necessary to secure the desired quotient.

1. Twenty-six hundredths divided by forty-six thousandths.
2. Two thousand forty-four ten-thousandths divided by six hundred eight thousandths.
3. Five hundred eighty-six thousandths divided by forty-one hundredths.

4. Eight thousand nine hundred nine ten-thousandths divided by eight ten-thousandths.
5. Twenty-one hundredths divided by four hundred sixty-five thousandths.
6. Four hundred fifty-four ten-thousandths divided by sixty-five hundred-thousandths.
7. One hundred four millionths divided by three hundred thousandths.
8. Six hundred thirty-eight hundred-thousandths divided by sixty-five hundredths.
9. Six hundred twenty thousandths divided by forty-five ten-thousandths.
10. Three thousandths divided by twenty-seven ten-thousandths.

EXERCISE No. 123, DIVISION OF DECIMALS AND INTEGERS

When a problem requires the division of an integer by a decimal, or a decimal by an integer, the process is the same as in the division of decimals. The quotient must contain as many decimal places as those in the dividend exceed those in the divisor.

The following problems provide written practice in ascertaining the quotient resulting from the division of an integer by a decimal, or a decimal by an integer. Show on tablet paper (a) all of your calculations and (b) prove the quotient by multiplication. When the divisor is not contained in the dividend an equal number of times, the process of division should be continued until three decimal places are obtained in the quotient.

1	2	3
a. $7642 \div .21$	a. $.6875 \div 43$	a. $.4756 \div 1137$
b. $5375 \div .55$	b. $.3413 \div 13$	b. $.5837 \div 21137$
c. $9847 \div .87$	c. $.8009 \div 11$	c. $.6847 \div 458$
d. $6845 \div .34$	d. $.2754 \div 21$	d. $.9283 \div 302$
e. $2098 \div .64$	e. $.9834 \div 67$	e. $.5738 \div 4573$
f. $4753 \div .49$	f. $.9672 \div 56$	f. $.7843 \div 6009$

EXERCISE No. 124, DIVISION OF MIXED DECIMALS

When the problem requires the division of one mixed decimal by another the process of division is the same as when dividing a decimal fraction by a decimal fraction, or by an integer.

The following problems provide written practice in dividing mixed decimals. Arrange your work on tablet paper in the same form as illustrated in Exercise No. 120, and prove by multiplication.

1	2	3
<i>a.</i> $77.54 \div 21.42$	<i>a.</i> $2781.4 \div 76.398$	<i>a.</i> $276.259 \div 139.117$
<i>b.</i> $2.980 \div 1.206$	<i>b.</i> $879.35 \div 396.67$	<i>b.</i> $9876.54 \div 25.7643$
<i>c.</i> $355.6 \div 121.9$	<i>c.</i> $43.006 \div 2.5467$	<i>c.</i> $42.7108 \div 7.64371$
<i>d.</i> $98.75 \div 567.3$	<i>d.</i> $778.24 \div 221.16$	<i>d.</i> $554.872 \div 23.1581$
<i>e.</i> $443.7 \div 5.432$	<i>e.</i> $9876.5 \div 33.489$	<i>e.</i> $3724.05 \div 221.786$
<i>f.</i> $0.009 \div 3.508$	<i>f.</i> $67.309 \div 9.3274$	<i>f.</i> $629.405 \div 321.456$

EXERCISE No. 125, DIVISION OF MIXED DECIMALS

When the problem requires the division of a mixed decimal by an integer, or an integer by a mixed decimal, the process of division is the same as when dividing a decimal fraction by an integer, or by a decimal fraction. The important element is the proper location of the decimal in the quotient.

The following problems provide written practice in dividing an integer by a mixed decimal, and a mixed decimal by an integer. Arrange your work on tablet paper in the same form as illustrated in Exercise No. 120, and prove by multiplication.

1	2	3
<i>a.</i> $13.487 \div 367$	<i>a.</i> $4877 \div 25.574$	<i>a.</i> $643.14 \div 48374$
<i>b.</i> $134.57 \div 32$	<i>b.</i> $5869 \div 234.67$	<i>b.</i> $374656 \div 23.46$
<i>c.</i> $1283.6 \div 845$	<i>c.</i> $5948 \div 4351.6$	<i>c.</i> $23.473 \div 47362$
<i>d.</i> $1.3785 \div 1389$	<i>d.</i> $3945 \div 2.3875$	<i>d.</i> $548.23 \div 47321$
<i>e.</i> $14.675 \div 147$	<i>e.</i> $1374 \div 23.463$	<i>e.</i> $485712 \div 2.374$
<i>f.</i> $123.75 \div 168$	<i>f.</i> $2137 \div 574.46$	<i>f.</i> $584731 \div 34.54$

CHAPTER XIII

PRACTICAL PROBLEMS IN DECIMAL FRACTIONS

The purpose of these problems is to provide additional practice in the application of the four fundamental operations to problems involving decimal fractions, and in applying fractions to the solution of business problems. You are to show on tablet paper: (a) all your calculations necessary to ascertain the desired facts; and (b) proof of your calculations.

1. At the beginning of a ten day vacation trip the speedometer on J. F. Sherwood's car registered 9,647.8 miles. The mileage registered at the close of each of the ten days was as follows: 9,935.4; 10,191.8; 10,487.1; 10,792.8; 11,105.4; 11,401.9; 11,714.4; 11,989.7; 12,242.6; and 12,512.1. It is desired to know: (a) the distance traveled each day; (b) the total distance traveled; and (c) the average mileage each day.

2. During the ten days vacation trip mentioned in problem No. 1, Mr. Sherwood purchased gasoline as follows: first day, 15 gals. at $22\frac{1}{2}c$ and 15 gals. at $23c$; second day, 15 gals. at $24\frac{1}{2}c$; third day, 15 gals. at $22c$ and 15 gals. at $22\frac{1}{2}c$; fourth day, 15 gals. at $23c$ and 15 gals. at $23\frac{1}{2}c$; fifth day, 15 gals. at $23\frac{1}{2}c$ and 15 gals. at $22\frac{1}{2}c$; sixth day, 15 gals. at $22\frac{1}{2}c$ and 15 gals. at $21\frac{1}{2}c$; seventh day, 15 gals. at $24c$; eighth day, 15 gals. at $23c$ and 15 gals. at $22\frac{1}{2}c$; ninth day, 15 gals. at $22\frac{1}{2}c$ and 15 gals. at $22c$; tenth day, 15 gals. at $22\frac{1}{2}c$ and 15 gals. at $22c$. It is desired to know: (a) the cost of the gasoline used on the trip; and (b) the number of miles per gallon, expressing the answer decimally in thousandths.

3. Reduce $\frac{7\frac{1}{2}}{30}$ to decimal form and divide by seventeen thousandths.

4. A ton (2,000 lbs.) of ground feed contains 200 lbs. of flax seed, 250 lbs. corn, 750 lbs. barley, and the remainder is oats. Express the fractional part of each in decimal form.

5. The cost of manufacturing an order for 5,240 knives was \$642.58. What was the cost in cents and thousandths of a cent per knife?

6. Multiply $408.06\frac{2}{3}$ by 75.125, and divide the product by 4.

7. The cost of manufacturing 6,500 spools was \$4.29. What was the cost per spool?

8. On July 19, the Pittsburgh "Pirates" had won 55 games and lost 29. The New York "Giants" had won 52 games and lost 30. What were the League standings of the two Clubs? (Express in thousandths.)

9. If the "Giants" defeat the "Pirates" in the next two games, (a) which will be ahead; (b) how many thousandths?

10. From $7.03\frac{1}{3}$ subtract $4\frac{3}{8}$.

11. The Mitchell Publishing Company of Chicago wishes to ship a quantity of books to San Francisco. Upon inquiry, the following rates are quoted: by rail direct from Chicago in a car load, minimum weight 30,000 lbs., \$2.775 per cwt.; from Chicago to New York by rail, \$0.945 per cwt.; and from New York to San Francisco by water, \$0.90 per cwt. It is desired to know: (a) the freight cost on a car weighing 36,487 lbs. direct by rail; (b) by water and rail; and (c) the amount saved if the shipment is made by rail and water, and the marine insurance is \$42.

12. The passenger fare from Cincinnati to Chicago is \$14.01. The rate per mile is \$0.036. It is desired to know the distance between these two cities expressed as a whole number, or as a mixed number with the fraction expressed decimally in hundredths.

13. Express decimally the difference between two hundred fifteen thousandths, and two hundred and fifteen thousandths.

14. U. B. Joy bought six United States bonds at $102\frac{5}{8}$ and sold them for $103\frac{1}{4}$. (a) How much did he make on the transaction? (b) What fractional part of the cost is the profit? Express your answer decimally in hundred-thousandths.

NOTE: $102\frac{5}{8}$ means \$102.62 $\frac{1}{2}$.

15. From three hundred forty-six and two hundred five ten-thousandths, take three hundred forty-six ten-thousandths.

16. To $7.0852\frac{1}{3}$, add $\frac{1}{8}$ of it.
17. The distance between two cities is 477.68 miles. What is the cost of a railroad ticket at \$3.60 per hundred miles?
18. Twenty-four hundredths of a farm was planted in corn. The yield was 48.4 bushels per acre, and was sold at 75c per bushel. If the corn crop was sold for \$1,742.40, how many acres were there in the farm?
19. The freight rate on sugar between two cities is 79.27 cents per hundred pounds. What will be the freight cost on five barrels weighing as follows: 327 pounds, 339 pounds, 325 pounds, 342 pounds, and 336 pounds?
20. Find the cost of $27\frac{1}{2}$ lbs. of coal at \$8.50 per ton of 2,000 lbs.
21. A restaurant had two cakes of equal size. No. 1 was cut into 12 pieces and sold at 20c a piece, and No. 2 was cut into 18 pieces and sold at 15c a piece. It is desired to know: (a) the fractional difference, expressed decimally, between the size of the pieces in the two cakes; (b) the difference between the selling price of the two cakes; (c) the fractional relation, expressed decimally, of this difference as compared with the selling price of cake No. 1; and (d) as compared with the selling price of cake No. 2.
22. Find (a) the value of the following items and (b) the total value.
- a. 125 lbs. at \$4 per hundred pounds
 - b. 20 lbs. at \$0.035 per pound
 - c. 450 lbs. at \$2.50 per hundred pounds
 - d. 14 yds. at \$0.075 per yard
 - e. 40 bu. at \$1.005 per bushel
 - f. 4.5 tons at \$20 per ton
 - g. 3000 lbs. at \$8.25 per thousand pounds
 - h. 15 gal. at \$0.425 per gallon
 - i. 25 lbs. at \$0.025 per pound
 - j. 2.5 bu. at \$2.50 per bushel

23. The following problems provide written practice in addition. Speed can be acquired only through combinations as explained in Chapter II. Find the sum of each problem and prove your answers.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
7,864,265	\$4,540.75	2.3	763.445	78,641
2,217,682	7,168.21	7.9	1.008	2,344,098
3,496,572	59.33	99.67	27.9	4,776
4,210,710	2,914.45	71.43	898.26	54,304
5,796,423	349.66	473.561	43.756	9,103,725
5,021,349	1,009.75	592.298	9.22	6,251,428
1,127,534	5.25	312.001	75.36	375
8,210,396	2,210.16	575.761	417.683	291,743
1,404,507	327.41	67.09	7.06	7,824,325
9,817,654	9,201.37	37.51	28.34	209,804
2,256,341	2,176.51	8.1	567.202	8,341,829
3,914,673	86.10	5.4	4.017	5,950,763

24. The space occupied by each department of a factory was as follows:

Department A.....	4,700 square feet
Department B.....	6,100 square feet
Department C.....	8,300 square feet
Department D.....	9,500 square feet
Department E.....	4,600 square feet
Department F.....	5,800 square feet

The annual rent was \$7,200. You are to ascertain: (a) the amount of annual rent allocated to each department; and (b) the amount charged to each department when the rent was paid on the first of each month.

25. C. M. Orr paid \$60,000 for an eight-apartment building. His net income from the rent of this building at the end of the first year was .092 of the cost of the property. (a) How much was the net income? (b) What was the net monthly rent for each of the eight apartments?

26. James Dobson, a traveling salesman received $9\frac{3}{4}$ c on each dollar and fraction thereof of sales which he made. In addition to that, he was compensated for his expenses in making the sales. During one year his sales were \$89,462.75 and his expenses, \$2,689.37. During that year his employers paid him \$5,600. How much was due him at the end of the year?

27. A certain automobile company advertised that one of its models was driven from Washington, D. C. to Oakland, California, 3,127 miles, in 96 hours and 18 minutes. What was the average number of miles traveled per hour?

NOTE: Your answer should be shown as a mixed decimal with the fraction expressed in thousandths.

28. The present standing of the Champion Baseball Club is .875; of the Challengers .625. At the present time each club has played 48 games. If the Champions win only four out of the next twelve games, and the Challengers win eight out of the next twelve games, how will the clubs stand?

29. Divide forty-five hundredths by fifteen ten-thousandths.

30. The tariff rate on a certain commodity was .00225 of the import value. If the tariff (tax) amounted to \$570, what was the total value of the product imported?

31. The following prices were quoted in a recent newspaper advertisement. You are to show (a) the saving in dollars and cents, (b) the fractional value of this saving based on the original sale price expressed as a decimal fraction, and (c) the total saving, in dollars and cents, to a customer who should buy one each of these articles.

<i>a.</i> \$ 15.00 dresses	\$12.00	<i>k.</i> \$1.00 men's shirts	\$0.80
<i>b.</i> 20.00 dresses	15.00	<i>l.</i> 1.25 men's shirts	1.00
<i>c.</i> 25.00 dresses	20.00	<i>m.</i> 1.50 men's shirts	1.20
<i>d.</i> 30.00 dresses	24.00	<i>n.</i> 2.00 men's shirts	1.50
<i>e.</i> 35.00 dresses	28.00	<i>o.</i> 2.50 men's shirts	1.80
<i>f.</i> 40.00 dresses	32.00	<i>p.</i> 3.00 men's shirts	2.25
<i>g.</i> 48.00 dresses	36.00	<i>q.</i> .40 towels	\$0.35
<i>h.</i> 60.00 dresses	40.00	<i>r.</i> .48 towels	.40
<i>i.</i> 100.00 dresses	70.00	<i>s.</i> .56 towels	.45
<i>j.</i> 125.00 dresses	90.00	<i>t.</i> .75 towels	.60

32. The "Simplex" auto makes 255.5 miles on five gallons of oil, and the "Complex" makes 175.25 miles on three and a half gallons. What is the difference in distance per gallon, expressed in thousandths of a mile?

33. The Losey Novelty Company bought one gross of steel pens at five tenths of a cent each and sold them at the rate of eight for five cents. State the gain or loss decimally on each.

34. January 1, James Whitcomb received a statement from the bank showing a balance of \$762.55 with check No. 44 for \$12.65 (the last one written) unpaid. During the month of January, he wrote checks and made deposits as follows:

January 2	Check No. 45	\$ 58.50
January 4	Check No. 46	17.44
January 5	Check No. 47	89.90
January 7	Check No. 48	48.16
January 8	Check No. 49	21.45
January 10	Check No. 50	37.50
January 10	Deposited	300.00
January 11	Check No. 51	75.00
January 12	Check No. 52	95.98
January 13	Check No. 53	100.00
January 14	Deposited	75.00
January 15	Check No. 54	35.61
January 17	Check No. 55	25.75
January 19	Check No. 56	36.67
January 20	Deposited	20.00
January 23	Check No. 57	8.95
January 26	Check No. 58	13.25
January 27	Deposited	35.00
January 28	Check No. 59	28.95
January 29	Check No. 60	13.55

February 1, the statement from the bank shows all checks have been paid with the exception of Check No. 60.

Show on tablet paper: (a) the bank balance after each check has been subtracted or deposit added; and (b) the balance shown by the monthly statement received from the bank.

35. If sugar sells at .075 per pound, how many pounds can you buy for \$75.75?

36. Corn meal is advertised at a special sale, 12.25 lbs. for 62c. What is the cost per pound.

37. Three freight cars loaded with coal weighed respectively, 39.75 tons, 42.285 tons, and 40.72 tons. What was (a) the total weight and (b) the value at \$6.75 per ton?

38. The information given below shows the value of the building permits issued in certain Ohio cities during October and November of the same year, and November of the previous year. It is desired to know, (a) the increase or decrease in each city of November over October, and (b) of November this year over November last year, and (c) the increase or decrease ascertained in (b) expressed decimally in thousandths.

	Nov. this Year	Nov. last Year	Oct. this Year
Akron	\$ 707,848	\$1,005,798	\$1,547,273
Cleveland	6,575,075	4,330,800	6,045,350
Columbus	1,233,800	1,368,500	3,240,700
Dayton	462,296	576,359	580,100
Hamilton	148,897	106,465	154,459
Lima	101,795	167,305	64,320
Mansfield	124,480	122,775	153,050
Springfield	78,560	354,350	78,055
Toledo	1,462,282	902,597	1,486,644
Youngstown	330,000	1,221,050	3,464,630

39. Reduce to decimals and add: $\frac{5}{12} + \frac{5}{9} + \frac{3}{4} + \frac{1}{64} + \frac{2}{3} + \frac{7}{50} + \frac{1}{6} + 1\frac{5}{8}$. Carry to four decimal places, expressing the remainders as common fractions.

40. A publishing company sold the following books during the year: January, 926; February, 842; March, 530; April, 285; May, 248; June, 196; July, 495; August, 1,548; September, 5,490; October, 4,560; November, 1,249; December, 539. Ascertain: (a) the total number of books sold during the year; (b) the average monthly sales; and (c) the number each month is above or below the average, indicating above by "+", and below by "-".

41. If the books mentioned in problem 40 were sold at \$1.125 per copy, what was (a) the total sales for each month? (b) The total sales for the year?

42. The following problems provide practice in addition, subtraction, multiplication, and division. Complete the process in the order indicated by the sign. (a) Show the results of each horizontal column; and (b) add these totals and prove by reverse addition.

$$a. 4 \times 4 - 3 \times 2 + 6 - 4 \times 2 - 6 \times 2 - 4 \div 8 - 5 \times 2 =$$

$$b. 3 \times 6 + 9 - 7 \div 2 + 8 \times 2 - 6 \times 3 \div 2 - 5 \div 8 + 6 =$$

$$c. 7 \times 7 - 9 \div 4 + 6 \times 2 - 5 \div 9 + 7 \times 2 - 6 \times 2 \div 7 =$$

$$d. 6 \times 3 \div 9 + 6 - 2 \times 3 + 6 \div 2 \times 3 - 6 \div 3 + 4 \div 7 =$$

$$e. 5 \times 5 - 4 \div 7 \times 2 + 4 \times 2 - 6 \div 7 + 8 \times 3 \div 5 \times 4 =$$

$$f. 2 \times 9 \div 3 + 5 \times 2 - 6 + 4 \div 5 + 6 \times 3 \div 6 + 7 + 4 =$$

$$g. 5 \times 6 \div 3 - 4 \times 3 \div 9 \times 7 + 6 - 9 \times 2 + 6 \div 7 \times 4 =$$

$$h. 6 \times 2 \div 4 \times 6 + 9 \div 3 - 6 \times 5 + 6 \div 7 \times 4 - 4 \times 3 =$$

$$i. 5 \times 9 \div 3 - 5 + 9 \times 2 - 8 \div 3 \times 2 - 7 - 3 \times 2 \div 8 =$$

$$j. 2 \times 9 \div 6 + 4 \times 2 - 4 \div 5 \times 9 + 6 \div 8 \times 6 - 8 + 6 =$$

43. The limited telephone service in a certain office is charged at the rate of \$3.50 per month with an additional charge of five cents per telephone call on all calls exceeding fifty calls per month. During the month of February, ninety-seven calls were made. What was the amount of the February telephone bill?

44. The following facts were published by Farm Paper No. 1 for the information of its advertisers and as evidence of its value as an advertising medium:

	Paper No. 1	Paper No. 2	Paper No. 3	Paper No. 4	Paper No. 5	Paper No. 6
	Lines	Lines	Lines	Lines	Lines	Lines
Farm implements and machinery.....	9,194	3,277	2,475	1,551	806	783
Nursery stock, seeds, etc....	5,427	2,969	2,716	380	1,969	1,251
Poultry feeds, remedies and supplies.....	4,496	2,312	2,128	1,239	1,012	860
Light plants, heating and water systems and equipment	2,643	1,555	656	446	347	227

You are to ascertain (a) the total number of lines of advertising in each of the six papers, and (b) the excess of the number of lines in Farm Paper No. 1 over each of the other five Farm Papers.

45. The table below shows the cost of operating the various departments of the United States Government for two consecutive years. You are to ascertain (a) the total expenses for each year, (b) the increase of one year over the other year, and (c) the amount of this increase expressed decimally in ten-thousandths.

	1st Year	2d Year
Legislative establishment.....	\$ 14,915,001	\$ 16,498,384
Executive office.....	489,960	441,960
Independent Offices—		
Civil Service Commission.....	1,008,092	1,001,592
Employes' Compensation Commission.....	2,301,500	2,742,040
Federal Board of Vocational Education.....	8,227,000	8,210,620
Federal Trade Commission.....	1,008,000	997,000
General Accounting Office.....	3,701,960	3,714,400
Housing Corporation.....	743,915	674,398
Interstate Commerce Commission.....	6,853,962	6,033,309
Public Buildings and Public Parks.....	2,282,505	2,293,850
Tariff Commission.....	721,500	699,000
Shipping Board and Emergency Fleet.....	24,330,000	14,198,574
Smithsonian Institution and National Museum,	874,020	858,240
United States Veterans' Bureau.....	405,700,000	458,965,000
Other independent offices.....	1,578,045	4,879,876
Department of Agriculture.....	133,075,191	140,717,758
Department of Commerce.....	28,542,129	30,402,847
Department of the Interior.....	262,255,603	250,967,602
Department of Justice.....	24,205,822	24,367,627
Department of Labor.....	8,627,625	8,567,305
Navy Department.....	302,862,378	322,869,430
State Department.....	16,277,652	16,614,932
Treasury Department.....	153,708,404	157,563,713
War Department, including Panama Canal ...	339,765,931	335,641,525
District of Columbia.....	36,032,853	35,626,579

46. L. B. Reid purchased five \$1,000 bonds of the Inland Steel Company for $97\frac{3}{4}$ and sold them for $99\frac{3}{8}$. You are to ascertain: (a) the total amount paid for the bonds; (b) the amount received for them; (c) the profit; and (d) the profit expressed decimally in thousandths.

NOTE: " $97\frac{3}{4}$ " means $\$97\frac{3}{4}$ on each \$100 of the bond or \$977.50 per bond.

47. James Brown purchased an automobile on the deferred payment plan. The cost price of the automobile was \$1,680; the insurance, \$44; and the extras, \$120. He is to pay cash for the insurance, the extras, and four tenths of the cost price of the machine. The balance of the cost, plus the carrying charge of \$180 is to be paid in twelve equal installments payable monthly. You are to ascertain: (a) the amount of cash paid; (b) the amount of each monthly payment; and (c) the total cost of the automobile when the twelve installments have been paid.

48. After using the automobile mentioned in the above problem for two years, Mr. Brown exchanged it for a new one, and was allowed an exchange value of \$750. You are to ascertain: (a) the total loss resulting from this exchange; and (b) the average monthly loss of operating the automobile.

49. On a certain day the market price of the English pound expressed in dollars was \$4.7865. Ascertain (a) the cost of a draft for 235 pounds and 5 shillings; and (b) the difference between this and the par value which is \$4.8665.

NOTE: 20 shillings make a pound.

50. The distance between Los Angeles and San Francisco is 475 miles. What is the cost of a ticket at \$.036 per mile?

CHAPTER XIV

ALICUOT PARTS

An integer or mixed number which is contained in another number without a remainder is known as an *aliquot part* of that number. Three is an aliquot part of six, because it is contained in six without a remainder. Two and one half is an aliquot part of five, because it is contained in five without a remainder.

The aliquot parts of one hundred may be used to advantage in the multiplication of numbers when dollars and cents are involved. Thus, if the price of an article is $16\frac{2}{3}$ c and the number of articles involved is 63, the value of the 63 articles can be ascertained by multiplying 63 by $16\frac{2}{3}$ c or dividing 63 by 6. The solutions below show the application of both methods.

$\begin{array}{r} 63 \\ .16\frac{2}{3} \\ \hline 42 \\ 378 \\ 63 \\ \hline \$10.50 \end{array}$	The solution at the left shows the process when multiplication is used. The first amount, 42, is the product of $\frac{2}{3} \times 63$; the second amount is 6×63; and the third amount is 1×63. Two places are pointed off in the product, because of the two decimal places in the multiplier. The solution at the right shows the process when 63 is divided by 6. Six is contained in 63 ten times with a remainder	$\begin{array}{r} 6)63.00 \\ \hline \$10.50 \end{array}$
---	---	--

of 3; into 30 five times with no remainder, hence the result of dividing 63 by 6 is ten and fifty hundredths, or \$10.50.

The process of reasoning when applying aliquot parts to the solution of this problem would be as follows: if the price is $16\frac{2}{3}$ c for one article, six articles could be purchased for \$1 because $16\frac{2}{3}$ is $\frac{1}{6}$ of a dollar; if the price of each article were a dollar, the total cost of 63 articles would be \$63, but since the price is only $\frac{1}{6}$ of a dollar, the total cost would be $\frac{1}{6}$ of \$63 or \$10.50.

EXERCISE No. 126, ALICUOT PARTS

The problems on page 170 provide oral practice in ascertaining the value of a given number of articles when the price is an aliquot part of a dollar. The aliquot parts of a dollar most frequently used are shown in the second column of the table on page 171.

1

- a. 9 bu. oats at $33\frac{1}{3}c$
- b. 24#* sugar at $6\frac{1}{4}c$
- c. 120# candy at $16\frac{2}{3}c$
- d. 32# lard at $12\frac{1}{2}c$
- e. 22# tea at 50c
- f. 24 doz. eggs at 50c
- g. 32 yds. ribbon at $33\frac{1}{3}c$
- h. 45# bacon at $12\frac{1}{2}c$
- i. 84# butter at $33\frac{1}{3}c$
- j. 75 gals. oil at $16\frac{2}{3}c$
- k. 720# bacon at 25c
- l. 96# sugar at 10c

2

- a. 48# apples at $8\frac{1}{3}c$
- b. 56# beans at $12\frac{1}{2}c$
- c. 60 doz. eggs at $33\frac{1}{3}c$
- d. 96 doz. eggs at $33\frac{1}{3}c$
- e. 48 yds. cloth at $12\frac{1}{2}c$
- f. 84# sugar at $8\frac{1}{3}c$
- g. 720 yds. ribbon at 25c
- h. 75 gals. oil at 20c
- i. 114# soap at $16\frac{2}{3}c$
- j. 15 gals. gas at $33\frac{1}{3}c$
- k. 55# lard at 20c
- l. 60 gals. cider at 50c

*NOTE: When the symbol “#” is placed after figures, it is interpreted as lbs.; when placed before figures, it is interpreted as No.

EXERCISE No. 127, ALIUOT PARTS

If the quantity is an aliquot part of one hundred, or the price is an aliquot part of a dollar, the process of division may be applied just the same. Thus, the cost of 50 pounds at 27c per pound would be 27 divided by 2 or \$13.50, just the same as 27 pounds at 50c would be \$13.50.

The following problems provide additional oral practice in aliquot parts, when the price is an aliquot part of a dollar, or the quantity is an aliquot part of 100.

1

- a. 25 gals. at 28c
- b. 28 gals. at 25c
- c. $33\frac{1}{3}\#$ at 18c
- d. 96# at $8\frac{1}{3}c$
- e. 75 hrs. at 50c
- f. 39 yds. at $33\frac{1}{3}c$
- g. 66 yds. at $12\frac{1}{2}c$
- h. 25 gals. at 42c

2

- a. 42 gals. at 25c
- b. 56# at $12\frac{1}{2}c$
- c. $12\frac{1}{2}$ hrs. at \$1.20
- d. 50 hrs. at \$1.20
- e. 36 gals. at $33\frac{1}{3}c$
- f. 99# at $16\frac{2}{3}c$
- g. 59 bu. at 50c
- h. 50 gals. at 44c

3

- a. 28 doz. at $33\frac{1}{3}c$
- b. $12\frac{1}{2}$ doz. at 28c
- c. 42 yds. at 20c
- d. 20 yds. at 45c
- e. 51 bu. at $33\frac{1}{3}c$
- f. 25 gals. at 80c
- g. 81# at $16\frac{2}{3}c$
- h. 64# at $33\frac{1}{3}c$

TABLE OF ALIQOT PARTS

\$1.00	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	.50														
3	$.33\frac{1}{3}$	$.66\frac{2}{3}$													
4	.25	.50	.75												
5	.20	.40	.60	.80											
6	$.16\frac{2}{3}$	$.33\frac{1}{3}$	.50	$.66\frac{2}{3}$	$.83\frac{1}{3}$										
8	$.12\frac{1}{2}$	.25	$.37\frac{1}{2}$	.50	$.62\frac{1}{2}$	.75	$.87\frac{1}{2}$								
10	.10	.20	.30	.40	.50	.60	.70	.80	.90						
12	$.08\frac{1}{3}$	$.16\frac{2}{3}$	.25	$.33\frac{1}{3}$	$.41\frac{2}{3}$	.50	$.58\frac{1}{3}$	$.66\frac{2}{3}$	.75	$.83\frac{1}{3}$	$.91\frac{2}{3}$				
16	$.06\frac{1}{4}$	$.12\frac{1}{2}$	$.18\frac{3}{4}$	.25	$.31\frac{1}{4}$	$.37\frac{1}{2}$	$.43\frac{3}{4}$	.50	$.56\frac{1}{4}$	$.62\frac{1}{2}$	$.68\frac{3}{4}$	.75	$.81\frac{1}{4}$	$.87\frac{1}{2}$	$.93\frac{3}{4}$

The table above shows the aliquot parts of a dollar and of a hundred, and the multiples of these aliquot parts. The numbers in the first vertical column indicate the denominators of the fractions representing the aliquot parts of a dollar, and the numbers at the top in the horizontal column, the numerators of the fractions representing the multiples of these aliquot parts. The aliquot parts of a dollar or a hundred are shown in the square where the two columns meet. 50c is $\frac{1}{2}$ of a dollar; $33\frac{1}{3}$ c is $\frac{1}{3}$ of a dollar; $66\frac{2}{3}$ c is twice $33\frac{1}{3}$ c, hence is a multiple of the aliquot part, $33\frac{1}{3}$ c. 25c is $\frac{1}{4}$ of a dollar, and 50c and 75c are both multiples of 25c. 20c is an aliquot part of a dollar, and 40c, 60c, and 80c are multiples of this aliquot part.

EXERCISE No. 128, MULTIPLES OF ALIQOT PARTS

When the price of an article is a multiple of an aliquot part, the value of a given number of articles is ascertained by dividing the quantity by the denominator of the fraction representing the aliquot part, and multiplying the quotient by the numerator of the fraction representing the multiple of the aliquot part. If the quantity is an aliquot part of 100, the same principle will apply to the price.

If it is desired to ascertain the value of 48 articles at $37\frac{1}{2}$ c each, divide 48 by 8 and multiply by 3: $48 \div 8 = 6$; $6 \times 3 = 18$; when the aliquot part is of \$1.00, this product, 18, is \$18.00.

The problems at the top of page 172 provide oral practice in applying the information given in the table. The results are the same whether you divide and multiply, or multiply and divide.

1	2	3
a. 48 bu. at 75c	a. 48 gals. at $41\frac{2}{3}$ c	a. 50 gals. at 75c
b. 32 lbs. at $62\frac{1}{2}$ c	b. 54 bu. at $83\frac{1}{3}$ c	b. 98 doz. at $37\frac{1}{2}$ c
c. 96 bu. at $91\frac{2}{3}$ c	c. 85 yds. at 40c	c. 96 gals. at $18\frac{3}{4}$ c
d. 64 gals. at $87\frac{1}{2}$ c	d. 50 bu. at 65c	d. 66 gals. at $41\frac{2}{3}$ c
e. 45 yds. at 80c	e. 75 gals. at 24c	e. 88 gals. at $87\frac{1}{2}$ c
f. 128 lbs. at $81\frac{1}{4}$ c	f. $83\frac{1}{3}$ yds. at 48c	f. 105 lbs. at 60c
g. 55 gals. at 40c	g. 96 bu. at $91\frac{2}{3}$ c	g. 75 lbs. at 25c
h. 72 bu. at 75c	h. 45 yds. at 40c	h. 25 gals. at 50c

EXERCISE No. 129, MULTIPLES OF ALIUOT PARTS

The following problems provide written practice in applying multiples of aliquot parts. Show on tablet paper: (a) the value of the one aliquot part by division; and (b) the value of the required multiple by multiplication. Refer to the table and find the multiple as instructed in the oral exercise; when the divisor is not contained an equal number of times, multiply first and divide the product. Think twice before placing the decimal point.

1	2	3
a. 376 yds. at 75c	a. 516.8# at $62\frac{1}{2}$ c	a. 16.25 bu. at $87\frac{1}{2}$ c
b. 529 gals. at $18\frac{3}{4}$ c	b. 6,875# at 25c	b. 481 yds. at $31\frac{1}{4}$ c
c. 756 gals. at $83\frac{1}{3}$ c	c. 91.6 bu. at $68\frac{3}{4}$ c	c. 2,265.75gals. at $83\frac{1}{3}$ c
d. 1,236# at 30c	d. 9,672.75# at $18\frac{3}{4}$ c	d. 15.75 bu. at $93\frac{3}{4}$ c
e. 3,264 bu. at $43\frac{3}{4}$ c	e. 762.5 doz. at $83\frac{1}{3}$ c	e. 742.5 doz. at $37\frac{1}{2}$ c
f. 9,782.5# at $31\frac{1}{4}$ c	f. 275.5 doz. at $31\frac{1}{4}$ c	f. $37\frac{1}{2}$ doz. at 42c
g. 421.75# at 40c	g. 762 bu. at 75c	g. $62\frac{1}{2}$ bu. at 97c
h. 99.6 bu. at $56\frac{1}{4}$ c	h. 9,964.75# at $91\frac{2}{3}$ c	h. $81\frac{1}{4}$ gal. at 23c

EXERCISE No. 130, MULTIPLES OF ALIUOT PARTS

When the price is more than \$1.00, or the quantity more than 100, and the excess of \$1.00 or 100 is an aliquot part or a multiple of an aliquot part, the product may be found by dividing and multiplying in the same manner as when they are less than \$1.00 or 100. Thus, if the price is $\$1.37\frac{1}{2}$, divide the quantity by 8, and multiply by 3 plus 8, or 11. In other words, the price is $\frac{8}{8}$ of a dollar plus $\frac{3}{8}$ of a dollar.

The following problems provide written practice in applying aliquot parts, and multiples of aliquot parts.

1

- a. 7,261 lbs. at $\$1.33\frac{1}{3}$
- b. $127\frac{1}{2}$ bu. at $\$1.12\frac{1}{2}$
- c. 81.25 lbs. at 32c
- d. 45 doz. at $\$1.87\frac{1}{2}$
- e. $227\frac{1}{2}$ bu. at $\$1.62\frac{1}{2}$
- f. $387\frac{1}{2}$ lbs. at $18\frac{3}{4}$ c
- g. $75\frac{3}{4}$ doz. at $\$1.75$
- h. 427.05 bu. at $\$1.30$
- i. 55.25 gals. at $58\frac{1}{3}$ c
- j. 216.5 bu. at $\$1.41\frac{2}{3}$

2

- a. 768 bu. at $\$1.12\frac{1}{2}$
- b. 22.05 gals. at $\$1.37\frac{1}{2}$
- c. $214\frac{1}{2}$ lbs. at $18\frac{3}{4}$ c
- d. 516.75 bu. at $\$1.16\frac{2}{3}$
- e. 271.5 bu. at $87\frac{1}{2}$ c
- f. 621.25 lbs. at $\$1.08\frac{1}{3}$
- g. 217.5 bu. at $\$1.06\frac{1}{4}$
- h. 79 gals. at $\$1.83\frac{1}{3}$
- i. $177\frac{1}{2}$ doz. at $43\frac{3}{4}$ c
- j. 991 bu. at $\$1.06\frac{1}{4}$

EXERCISE No. 131, BILLING

Aliquot parts are used to advantage in billing and extending the value of merchandise on inventories. The application is the same as in the preceding exercises.

The following problems provide written practice in making extensions of bills rendered.

Show all the calculations necessary to arrive at: (a) extension of each item; and (b) the total amount of the bill.

1. The Johnson Paper Company sold the Krehbiel Printing Company the following merchandise:

- 112,500 envelopes at $\$1.20$ per thousand
- 14,765# of Regal Bond paper at $12\frac{1}{2}$ c per pound
- 2,500# of ledger paper at $41\frac{2}{3}$ c per pound
- 6,580# of news paper at $6\frac{1}{4}$ c per pound
- 5,407# Crown book paper at $8\frac{1}{3}$ c per pound
- 2,586# of Champion Coated paper at $16\frac{2}{3}$ c per pound
- 3,175# of Old Hampshire paper at $41\frac{2}{3}$ c per pound
- 1,250 Columbian Clasp envelopes at $\$2.12\frac{1}{2}$ per thousand
- 5,500 XXX white envelopes at $\$1.68\frac{3}{4}$ per thousand

2. February 1, the Kroger Grocery Company, dealers in wholesale groceries, sold H. M. Jameson, a retail grocer, the following merchandise:

3 sacks (300 lbs.) granulated sugar at $8\frac{1}{3}c$ per pound

25 doz. Red Cross soap at $55c$ per dozen

32 doz. Canned peaches at $\$1.12\frac{1}{2}$ per dozen

75 doz. Canned apples at $\$1.06\frac{1}{4}$ per dozen

135 lbs. of rice at $8\frac{1}{3}c$ per pound

118 $\frac{3}{4}$ lbs. of coffee at $56c$ per pound

3. May 6, the Shillito Dry Goods Company sold Jenkins Bros., retail dry goods merchants, the following bill of goods:

18 yds. of flannel at $\$2.91\frac{2}{3}$ per yard

66 $\frac{2}{3}$ yds. of black satin at $\$3.90$ per yard

24 yds. of ribbon at $6\frac{1}{4}c$ per yard

12 yds. of muslin at $16\frac{2}{3}c$ per yard

16 yds. of gingham at $18\frac{3}{4}c$ per yard

9 pairs of white hose at $\$1.33\frac{1}{3}$ per pair

12 cards of buttons at $12\frac{1}{2}c$ per card

4. The Dow Drug Company had the following merchandise in stock on December 31. You are to show all the calculations necessary to verify (a) the extensions, and (b) the total value of the merchandise in stock.

488 tubes of Ipana tooth paste @ $37\frac{1}{2}c$	$\$183.00$
360 tubes Pebecco tooth paste @ $30c$	10.80
356 tubes Forhan's tooth paste @ $25c$	89.00
246 tubes of Squibb's tooth paste @ $33\frac{1}{3}c$	8.20
366 tubes of Kolynos tooth paste @ $16\frac{2}{3}c$	68.00
288 bottles of toilet water @ $41\frac{2}{3}c$	120.00
112 boxes of Woodbury's soap @ $43\frac{3}{4}c$	490.00
128 boxes of Packer's tar soap @ $43\frac{3}{4}c$	56.00
128 boxes of Castile soap @ $18\frac{3}{4}c$	24.00
363 bottles of Honey and Almond Cream @ $33\frac{1}{3}c$	123.00
328 boxes of talcum @ $12\frac{1}{2}c$	41.00
128 bottles of Palmolive shampoo @ $25c$	46.00
320 bars of Armour's bath soap @ $6\frac{1}{4}c$	40.00

APPLICATION OF ALIQUOT PARTS

Corn, wheat and other grain are listed on the grain exchanges at a price per bushel, but when delivered, the quantity is usually expressed in pounds, because it is not possible to provide containers holding an even number of bushels. When the price is per bushel, and the quantity expressed in pounds, there are three methods of ascertaining the value of a given quantity: (a) by dividing the quantity by the number of pounds in a bushel, and multiplying by the price per bushel; (b) by multiplying the number of pounds by the price per bushel and dividing by the number of pounds in a bushel; and (c) by multiplying the number of pounds by a fraction in which the numerator is the price per bushel, and the denominator the number of pounds in a bushel. The illustrations below show the three methods of ascertaining the value of 4,704 pounds of corn at 72c per bushel of 56 pounds.

a. $4704 \div 56 = 84$	b. $4704 \times .72 = 3386.88$	c. $\frac{72}{56} = \frac{9}{7}$
$84 \times .72 = 60.48$	$3386.88 \div 56 = 60.48$	672
$60.48 = \$60.48$	$60.48 = \$60.48$	$4704 \times \frac{9}{7} = 6048$
		$6048 = \$60.48$

In the problem given, the quantity can be expressed as an even number of bushels, hence solution (a) would be the most satisfactory. If the number of pounds had not been an even number of bushels, the fraction would have made the calculations more complicated. In solution (b) it is assumed that the price is per pound, hence the total number of pounds is multiplied by the price, then the product is divided by 56, the number of pounds in a bushel. In solution (c) the price per bushel is used as the numerator of a fraction, the denominator of which is the number of pounds in a bushel. This fraction is reduced to its lowest terms. Solution (c) is the most satisfactory when the price and the number of pounds have factors in common, so the fraction may be reduced to lower terms.

EXERCISE No. 132, APPLICATION OF ALIQUOT PARTS

The problems at the top of page 176 provide written practice in ascertaining the value of corn, using the method explained above. Show on tablet paper, using one of the three methods illustrated above, all the calculations necessary to ascertain the value of the given quantity at the price stated in each problem. The figures in the first column indicate the number of pounds of corn purchased; the amount in the second column show the price per bushel of 56 pounds.

1			2		
a. 5,159#	at	\$0.64	a. 84,000#	at	\$0.84
b. 2,144#	"	.63	b. 91,562#	"	1.05
c. 5,004#	"	.84	c. 17,642#	"	1.12
d. 1,008#	"	.49	d. 11,200#	"	.88
e. 9,884#	"	.48	e. 98,000#	"	.91
f. 8,010#	"	.76	f. 28,000#	"	.84
g. 4,800#	"	.77	g. 72,568#	"	.56
h. 5,901#	"	.88	h. 66,480#	"	.77
i. 5,146#	"	.56	i. 56,700#	"	.70
j. 3,988#	"	.98	j. 98,000#	"	1.05

EXERCISE No. 133, APPLICATION OF ALIUOT PARTS

The following problems provide written practice in ascertaining the value of wheat when the price is per bushel of sixty pounds.

1			2		
a. 6,475#	at	\$1.32	a. 98,576#	at	\$0.90
b. 3,781#	"	1.20	b. 17,645#	"	1.08
c. 5,650#	"	1.26	c. 86,664#	"	.75
d. 9,864#	"	1.30	d. 56,878#	"	1.50
e. 7,500#	"	1.08	e. 24,000#	"	1.10
f. 2,400#	"	.96	f. 99,960#	"	1.26
g. 9,996#	"	1.25	g. 20,000#	"	1.05
h. 7,884#	"	1.10	h. 72,000#	"	1.32
i. 9,964#	"	1.05	i. 11,164#	"	1.10
j. 7,765#	"	.96	j. 67,566#	"	1.35

EXERCISE No. 134, APPLICATION OF ALIUOT PARTS

As explained in Exercise No. 128 multiples of aliquot parts may be used to an advantage when extending an invoice where these multiples are involved. These occur quite frequently in ascertaining the value of grain. When the quantity is stated in pounds and the price per bushel is a multiple of an aliquot part

of the number of pounds in a bushel, the value can be ascertained by multiplying the number of pounds by the fraction representing this multiple.

Thus if it is desired to ascertain the value of 862 pounds of oats at 48c per bushel the process would be as follows: $862 \times \frac{48}{32} = 862 \times \frac{3}{2} = \frac{2586}{2} = \12.93 . A bushel of oats weighs 32 pounds hence, one half of a bushel is 16 pounds. 16 is an aliquot part of 32 also of 48.

The following problems provide written practice in the use of multiples of aliquot parts in ascertaining the value of oats with 32 pounds to a bushel. The quantity is given in the first column, and the price per bushel in the second column. Show on tablet paper the solution of each problem in the same form as in the illustration.

1			2		
a. 7,846#	at	\$0.80	a. 74,000#	at	\$1.12
b. 2,568#	"	.36	b. 96,468#	"	1.04
c. 3,894#	"	.48	c. 20,468#	"	.64
d. 8,760#	"	.64	d. 11,676#	"	.88
e. 7,400#	"	.40	e. 29,684#	"	1.32
f. 2,696#	"	.56	f. 67,856#	"	.68
g. 9,896#	"	.68	g. 39,568#	"	1.00
h. 5,672#	"	1.04	h. 87,652#	"	.72
i. 3,784#	"	.48	i. 54,800#	"	.36
j. 6,392#	"	.72	j. 20,000#	"	.56

NOTE: The aliquot parts of a bushel used to the best advantage are: 4 lbs. = $\frac{1}{8}$; 8 lbs. = $\frac{1}{4}$; 16 lbs. = $\frac{1}{2}$.

EXERCISE No. 135, APPLICATION OF ALIQUOT PARTS

Logs are sawed into lumber in pieces with the desired length, width, and thickness, and sold at a price per thousand board feet. When the quantity or the price is an aliquot part, or a multiple of an aliquot part of a thousand board feet, the price may be ascertained by multiplying by a fraction, the same as in grain. Thus, if it is desired to ascertain the value of 750 feet of lumber at \$36 per thousand, the value may be ascertained by using 750 as the numerator of a fraction and 1,000 as the denom-

inator. This reduced to its lowest terms is $\frac{3}{4}$. The price per thousand, \$36, is then multiplied by $\frac{3}{4}$, giving a product of \$27, which is the price of 750 feet.

The following problems provide written practice in ascertaining the value of a given quantity of lumber expressed in board feet with the price per thousand board feet. You are to show on tablet paper all the calculations necessary to ascertain the desired information using the aliquot parts of one thousand.

1				2			
a.	1,500 ft.	at \$45.00	per 1000	a.	625 ft.	at \$55.50	per 1000
b.	3,000 "	" "	48.00 "	b.	875 "	" "	52.25 "
c.	4,500 "	" "	52.00 "	c.	1,125 "	" "	44.00 "
d.	4,250 "	" "	38.75 "	d.	625 "	" "	72.50 "
e.	7,500 "	" "	42.50 "	e.	1,625 "	" "	54.50 "
f.	2,500 "	" "	65.00 "	f.	2,125 "	" "	21.50 "
g.	12,500 "	" "	45.50 "	g.	4,250 "	" "	42.50 "
h.	750 "	" "	41.50 "	h.	1,750 "	" "	75.00 "
i.	1,2655 "	" "	100.00 "	i.	1,250 "	" "	55.25 "
j.	125 "	" "	38.50 "	j.	6,288 "	" "	50.00 "

NOTE: The aliquot parts of a thousand used to the best advantage are: 125 ft. = $\frac{1}{8}$; 250 ft. = $\frac{1}{4}$; 500 ft. = $\frac{1}{2}$.

EXERCISE No. 136, APPLICATION OF ALIUOT PARTS

When the price is per ton of 2,000 pounds, and the quantity given in pounds, the number of pounds may be divided by 2,000 to ascertain the number of tons, and the number of tons multiplied by the price. If the number of pounds given is not an even number of tons, consider the remainder as the numerator of a fraction with 2,000 as the denominator. Multiply the price by this fraction to ascertain the value of the fractional part of a ton. If the entire quantity is more than a ton, this may be expressed as an improper fraction with the total number of pounds as the numerator, and 2,000 as the denominator. Multiply the price by the improper fraction, reduced to its lowest terms. The result will be the total cost. When the price per ton is expressed in dollars and cents, it is necessary to point off five places in the product; if the price is stated in dollars only, three places are pointed off in the product.

The following problems provide written practice in ascertaining the value of merchandise when the quantity is expressed in pounds and the price is per ton of 2,000 pounds. You are to show on tablet paper all the calculations necessary to arrive at the desired information using the aliquot parts of 2,000.

1	2
a. 1,500 lbs. of hay at \$22.50	a. 8,750 lbs. of coal at \$ 5.25
b. 2,500 " " coal " 7.25	b. 9,750 " " pig iron 13.95
c. 800 " " hay " 24.50	c. 8,020 " " hay at 22.00
d. 12,250 " " steel " 19.50	d. 4,200 " " coal " 6.00
e. 2,500 " " coal " 5.50	e. 8,964 " " pig iron 12.50
f. 5,000 " " pig iron 14.50	f. 75 " " coal at 6.00
g. 1,750 " " hay at 24.00	g. 8,562 " " hay " 25.00
h. 7,500 " " steel " 22.50	h. 150 " " hay " 20.00
i. 87 " " hay " 20.00	i. 8,764 " " steel " 25.00
j. 2,250 " " hay " 25.50	j. 17,500 " " pig iron 13.50

NOTE: The aliquot parts of a ton used to best advantage are 125 lbs. = $\frac{1}{16}$; 200 lbs. = $\frac{1}{10}$; 250 lbs. = $\frac{1}{8}$; 500 lbs. = $\frac{1}{4}$.

EXERCISE No. 137, ALIQUOT PARTS

Aliquot parts or multiples of aliquot parts can be used to an advantage whether they are involved in the quantity or price. Thus the value of 60 pounds of wheat at 85c per bushel of 60 pounds, may be ascertained by the same process as the value of 85 pounds of wheat at 60c per bushel of 60 pounds. You should learn to utilize aliquot parts and multiples of aliquot parts in all possible calculations, because their use will enable you to solve a problem more quickly.

The following problems provide oral practice in aliquot parts and multiples of aliquot parts as discussed in the preceding exercises.

1. 90 lbs. of wheat at \$1.10 per bu.
2. 90 bushels of oats at $33\frac{1}{3}$ c per bushel.
3. 250 ft. of lumber at \$44 per M.
4. 168 ft. of lumber at \$25 per M.
5. 88 lbs. of corn at 63c per bu. of 56 lbs.
6. 120 yds. of ribbon at $8\frac{1}{3}$ c per yd.

7. 30 yds. of tape at $3\frac{1}{3}c$ per yd.
8. 12 quarts of plums at $25c$ per quart.
9. 96 lbs. of hay at \$25 per ton.
10. 125 lbs. hay at \$24 per ton.
11. 128 lbs. coal at \$5 per ton.
12. 24 lbs. of butter at $37\frac{1}{2}c$ per lb.
13. 80 bu. of barley at $75c$ per bu.
14. 820 pounds of tobacco at $12\frac{1}{2}c$ per lb.
15. 40 bushels of corn at $87\frac{1}{2}c$ per bushel.
16. What will be the cost of 86 lbs. of green apples at \$2.50 per bu. of 50 lbs.?
17. What will 96 lbs. of barley cost at $80c$ per bu. of 48 lbs.?
18. How much will 63 lbs. of buckwheat cost at $90c$ per bu. of 42 lbs.?
19. What will 96 lbs. of cotton seed cost at $60c$ per bu. of 32 lbs.?
20. What will be the cost of 165 lbs. of flaxseed at \$1.20 per bu. of 55 lbs.?
21. How much will 88 lbs. of hempseed cost at \$1.10 per bu. of 44 lbs.?
22. How many yards of silk at $75c$ can be bought for \$8.50?
23. How many yards of cheese cloth at $16\frac{2}{3}c$ can be bought for \$7.20?
24. How many yards of linen at $\$1.16\frac{2}{3}$ can be bought for \$22.40?
25. How many yards of silk at $\$1.66\frac{2}{3}$ can be bought for \$20?
26. How many pounds of sugar at $8\frac{1}{3}c$ can be purchased for \$2.75?
27. How many bushels of wheat at $\$1.12\frac{1}{2}$ can be purchased for \$9.00?
28. What will 250 lbs. of millet cost at \$2.50 per bu. of 50 lbs.?
29. How much will 44 lbs. of onion seed cost at \$2.80 per bu. of 28 lbs.?
30. What will be the cost of 42 lbs. of orchard grass seed at \$3 per bu. of 14 lbs.?

CHAPTER XV

PERCENTAGE

Per cent, usually indicated by the symbol “%”, means by the hundred. It is used to describe fractional values expressing hundredths or a power of hundredths. The term “6%” is equivalent to “six hundredths”; “ $\frac{1}{2}\%$ ”, to “five thousandths”; “ $\frac{3}{4}\%$ ”, to “seventy-five ten-thousandths”; “27%”, to “twenty-seven hundredths.” The per cents mentioned, when expressed in the form of common and decimal fractions would be: $6\% = \frac{6}{100} = .06$; $\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = .005$; $\frac{3}{4}\% = \frac{\frac{3}{4}}{100} = .0075$; $27\% = \frac{27}{100} = .27$.

The subject of percentage relates to the solution of problems in which the term “per cent” is involved. The process of solving problems in percentage is the same as that applied in the solution of problems in common and decimal fractions; the denominator of the fraction representing the per cent in percentage problems is always 100. The usual process is to indicate the number preceding the words “per cent”, or the symbol “%”, as a decimal fraction, when making calculations in percentage.

24% of 25 is equivalent to $25 \times .24$ or a product of 6; 8% of 62 is equivalent to $62 \times .08$ or a product of 4.96; 15% of \$12.50 is equivalent to $12.50 \times .15$ or a product of 1.875 or \$1.88. When the value of the final product in the calculations of business problems is dollars and cents, and the fractional value of the cents is one half or more, it is customary to regard it as a full cent, and when less than one half, to discard it.

PERCENTAGE TERMS AND THEIR MEANING

The term *per cent* is used in business for indicating comparisons and is the basis for compensation. The *base* is the number with which the comparison is being made, or upon which the compensation is to be calculated. The *rate* is the fractional comparison. The *percentage* is the product of the base multiplied by the rate expressed as a decimal fraction.

Wilbur York, a salesman, is to receive 8% of the sales he makes as his compensation. During the month of January, his

sales were \$1,200. To ascertain the amount of his compensation (the percentage) it is necessary to multiply his sales (the base) by the rate per cent expressed as a decimal fraction; $\$1,200 \times .08 = 96.00$ or \$96 since the base is expressed in dollars.

During the month of February, Mr. York's sales were \$1,500 hence his compensation would be \$120; $\$1,500 \times .08 = 120.00$ or \$120. If it is desired to compare his February sales with the January sales and express the increase, \$300, as a rate per cent, the January sales will be the base, the \$300 the percentage, and the rate to be determined. Since the base multiplied by the rate equals the percentage, it is evident that the percentage divided by the base will equal the rate; $\$300 \div \$1,200 = .25$ or 25%, the rate of increase. Mr. York sold \$300 more merchandise in February than in January, and by so doing increased his sales 25%, because the \$300 increase was $\frac{1}{4}$ of the January sales.

Mr. York wishes to increase his income in March to \$160. To do this it is necessary for him to increase his sales because his income is 8% of his sales. Knowing his rate per cent and the income desired, he will naturally be interested in ascertaining the amount of goods he must sell in March in order to secure \$160. His reasoning would doubtless be something as follows: "If \$160, the income I desire, is 8% of some number, then 1% of that number would be $\frac{1}{8}$ of \$160 or \$20; if \$20 is 1% of the number, then 100% of the number must be 100 times \$20 or \$2,000. This means that I must sell \$2,000 worth of merchandise in March."

EXERCISE No. 138, PERCENTAGE REQUIRED

When the base and rate are given, the percentage is the product of the base multiplied by the rate expressed as a decimal fraction. The following problems provide oral practice in ascertaining the percentage when the rate (first column) and base (second column) are given.

1		2	
a. 50%	of 150	a. 8%	of \$250
b. 30%	" 60	b. 15%	" 300
c. 90%	" 90	c. 18%	" 300
d. 80%	" 27	d. 19%	" 400
e. 60%	" 60	e. 16%	" 100
f. 40%	" 40	f. 50%	" 500

EXERCISE No. 139, PERCENTAGE REQUIRED

The following problems provide written practice in ascertaining the percentage when the base and rate are given. Arrange your work on tablet paper in the same form as instructed in the multiplication of decimal fractions. When the base is expressed as dollars and cents, the product (percentage) will be dollars and cents, hence fractions of a cent are treated as a whole cent if one half cent or more, and discarded if less than one half cent.

1	2
a. 15% of 3,745	a. 78% of \$4,756.35
b. 37% " 5,846	b. 36% " 625.31
c. 23% " 9,283	c. 58% " 473.57
d. 21% " 1,473	d. 92% " 3,127.68
e. 34% " 8,463	e. 36% " 45.32
f. 16% " 8,136	f. 48% " 314.68
g. 22% " 8,415	g. 21% " 3,147.82
h. 14% " 9,236	h. 18% " 37.23
i. 37% " 1,947	i. 81% " 1.45
j. 45% " 8,273	j. 78% " .94
k. 16% " 4,836	k. 49% " 6.73
l. 34% " 9,281	l. 26% " 12.78

NOTE: The base is the *multiplicand*, the rate, expressed as a decimal fraction, is the *multiplier*, and the percentage is the *product*. If the base is an abstract number, the percentage is an abstract number; if the base is a concrete number, the percentage is of the same denomination as the base. 10% of 100 is 10; 10% of \$100 is \$10.

EXERCISE No. 140, PERCENTAGE REQUIRED

When the rate is an aliquot part of 100, the percentage may be ascertained (a) by multiplying the base by the rate expressed as a decimal fraction, or (b) by multiplying the base by the rate expressed as a common fraction reduced to its lowest terms.

If the rate is $12\frac{1}{2}\%$ and the base is 72, the percentage is (a) $72 \times .12\frac{1}{2} = 9.00 = 9$, or (b) $72 \times \frac{12\frac{1}{2}}{100} = 72 \times \frac{1}{8} = \frac{72}{8} = 72 \div 8 = 9$. By

this illustration you will note that multiplying the rate by the percentage expressed as a common fraction in its lowest terms,

is equivalent to dividing the base by the denominator of the fraction when the numerator is one.

The following problems provide oral practice in determining the percentage when the rate is an aliquot part of a dollar.

1	2
a. 25% of \$ 150.00	a. 4% of \$ 625.50
b. $12\frac{1}{2}\%$ " 125.84	b. 50% " 8,376.48
c. $16\frac{2}{3}\%$ " 209.64	c. $8\frac{1}{3}\%$ " 4,785.28
d. $8\frac{1}{3}\%$ " 1,067.40	d. $6\frac{1}{4}\%$ " 253.60
e. 10% " 4,856.00	e. 25% " 7,486.48
f. 5% " 3,064.80	f. $33\frac{1}{3}\%$ " 3,743.25
g. 20% " 584.75	g. $16\frac{2}{3}\%$ " 8,753.28
h. $33\frac{1}{3}\%$ " 437.64	h. $12\frac{1}{2}\%$ " 436.24
i. $12\frac{1}{2}\%$ " 31.80	i. 50% " 318.42
j. $66\frac{2}{3}\%$ " 291.30	j. 20% " 1,921.35

EXERCISE No. 141, RATE REQUIRED

When the *base* and *percentage* are given, the *rate* is ascertained by dividing the percentage by the base. The principle involved is the same as in the proof of multiplication by division. The base is the *multiplicand*, the rate the *multiplier*, and the percentage the *product*.

The following problems provide oral practice in ascertaining the rate when the percentage and base are given.

1	2
a. 25 is what per cent of 100?	a. 28# is what per cent of 32#?
b. 14 is what per cent of 56?	b. $17\frac{1}{2}$ is what per cent of 35?
c. 17 is what per cent of 51?	c. 18 is what per cent of 54?
d. \$10 is what per cent of \$100?	d. 15# is what per cent of 60#?
e. \$125 is what per cent of \$1,000?	e. 25c is what per cent of \$1.00?
f. 21# is what per cent of 56#?	f. 44# is what per cent of 220#?
g. 10 is what per cent of 100?	g. 25 is what per cent of 625?
h. 9 is what per cent of 81?	h. 18 is what per cent of 90?
i. 13 is what per cent of 91?	i. 125 is what per cent of 625?
j. $66\frac{2}{3}$ is what per cent of 200?	j. 61 is what per cent of 183?

EXERCISE No. 142, RATE REQUIRED

Since the percentage is the result of the base multiplied by the rate, it is evident that the percentage divided by the base will give the rate.

$$\begin{array}{r}
 .181+ \\
 287.64 \overline{) 52.15000} \\
 \underline{28 \ 764} \\
 23 \ 3860 \\
 \underline{23 \ 0112} \\
 37480 \\
 \underline{28764} \\
 8716
 \end{array}$$

The illustration at the left shows the method of ascertaining the rate when the base is \$287.64 and the percentage, \$52.15. The percentage is the dividend and the base is the divisor. Since the quotient is in thousandths, the rate is $.181+ = 18.1+ \%$.

NOTE: It is necessary to keep in mind the fact that the rate is the numerator of a fraction, the denominator of which is hundredths; hence, the quotient in the process of division, in which the base is the divisor and the percentage is the dividend, will be a decimal fraction. When the base is not contained in the percentage an exact number of hundredths of times, the division should be continued, in decimal fractions, to at least three decimal places.

The following problems provide written practice in ascertaining the rate when the base and percentage are given. Show your solutions on tablet paper as in the illustration.

1. 445 is what per cent of 1,262?
2. 318 is what per cent of 900?
3. 756 is what per cent of 252?
4. 1,644 is what per cent of 32,508?
5. \$375.55 is what per cent of \$2,408.40?
6. \$252.50 is what per cent of \$2,500?
7. 1,018 is what per cent of 16,485?
8. 327 is what per cent of 585?
9. 25,562 ft. is what per cent of 100,570 ft.?
10. 750 is what per cent of 150?
11. 3,265 is what per cent of 12,440?
12. \$90.90 is what per cent of \$9,090?

EXERCISE No. 143, BASE REQUIRED

When the rate and percentage are given, the base is ascertained by dividing the percentage by the rate expressed as a decimal fraction. The principle involved is the same as in the proof of multiplication by division. The base is the multiplicand, the rate, the multiplier, and the percentage, the product.

The formula given in the third illustration of Mr. York's sales on page 182 may be of assistance in helping you to understand the solution of these problems. Applied to problem "a" in No. 1, the statement would be as follows: if 144 is 12% of some number, 1% of the number would be $\frac{1}{12}$ of 144 or 12; if 1% of the number is 12, 100% of the number would be 100 times 12, or 1,200.

The following problems provide oral practice in ascertaining the base when the rate (first column) and percentage (second column) are given.

1	2	3
a. 12%...144	a. 7%...81.00	a. $12\frac{1}{2}\%$... 24.00
b. 10%...280	b. 11%... 2.20	b. 20%... 5.00
c. 15%...450	c. 15%... 4.50	c. $16\frac{2}{3}\%$... 6.00
d. 3%...225	d. 15%...45.00	d. 75%... 2.25
e. 6%...120	e. 14%... 2.80	e. 40%... 1.60
f. 7%...490	f. 7%... .72	f. 25%... 5.00
g. 15%...600	g. 7%... 8.10	g. 25%... 25.00
h. 20%...320	h. 9%... 1.80	h. 10%... .10

EXERCISE No. 144, BASE REQUIRED

When the rate and percentage are given, and it is necessary to write the solution, it is advisable to express the rate in the form of a decimal fraction before dividing. Thus if the rate is $18\frac{1}{4}\%$, and the percentage is \$66.66, the solution requires that \$66.66 be divided by .1825, as illustrated below.

$$\begin{array}{r}
 365.26 \\
 .1825 \overline{) 66.660000} \\
 \underline{54 \ 75} \\
 11 \ 910 \\
 \underline{10 \ 950} \\
 9600 \\
 \underline{9125} \\
 4750 \\
 \underline{3650} \\
 11000 \\
 \underline{10950} \\
 50
 \end{array}$$

In the problem at the left, there are four decimal places in the divisor, and only two in the dividend, hence it is necessary to annex two ciphers. The result of dividing the dividend by the divisor with these ciphers annexed will be dollars. Since there is a remainder after finding the dollars, it is necessary to annex additional ciphers in order to ascertain the cents. Since the remainder is less than one half, it is discarded. If it had been more than one half, another cent would have been added to the quotient.

The following problems provide written practice in ascertaining the base when the rate and percentage are given. Show on tablet paper, the base expressed as dollars and cents, if the percentage is in dollars and cents, or as a mixed number with the fraction expressed decimally in ten-thousandths.

1	2	3
a. 27% .. \$ 22.52	a. 7.4% ... 361	a. $12\frac{1}{2}\%$... \$287.62
b. 7% .. 181.65	b. 29% ... 127	b. 33% 55.42
c. 9% ... 54.55	c. $3\frac{1}{4}\%$... 525	c. 16% 34.14
d. 28% ... 43.15	d. $21\frac{1}{2}\%$... 87.5	d. 75% 241.04
e. 13% ... 38.65	e. 3.68% ... 72.60	e. 66% 205.87
f. 15% ... 44.44	f. 15.7% ... 261.3	f. 8% 51.65
g. 44% ... 98.89	g. 8.8% ... 541.52	g. 20% 201.11
h. 31% ... 36.42	h. $16\frac{3}{4}\%$... 72.5	h. $37\frac{1}{2}\%$... 55.56

EXERCISE No. 145, APPLICATION OF THE TERMS

The following problems provide written practice in the solution of percentage problems. Copy on tablet paper in the same form as the problems are stated, inserting the correct figures where question marks are given; mark the figures you have supplied by drawing a line under them. Submit with the problems all your calculations, expressing the rate decimally in ten-thousandths.

When the base and percentage are given in one amount and the percentage is a separate amount, subtract to ascertain the base; when the base, less the percentage, is given in one amount, and the percentage is a separate amount, the base is the sum of the two. With any two of the three essential elements given, the third can be ascertained by multiplication or division.

	Base	Rate	Percentage	B. + P.	B. - P.
1.	\$1,678.40	25%	?	?	?
2.	?	?	\$37.08	\$301.93	?
3.	\$1,264.75	?	\$189.71	?	?
4.	?	?	77.27	?	\$566.62
5.	\$7,843.20	18%	?	?	?
6.	\$1,657.45	8%	?	?	?
7.	\$9,864.37	?	\$3,452.53	?	?
8.	?	?	192.34	\$1,671.89	?

	Base	Rate	Percentage	B. + P.	B. - P.
9.	?	?	122.06	?	\$1,621.59
10.	?	?	1,865.51	?	6,614.10
11.	\$6,543.90	$16\frac{2}{3}\%$	?	?	?
12.	\$7,864.85	?	?	\$8,651.34	?
13.	\$3,487.70	?	?	?	\$3,175.41
14.	\$4,874.44	?	536.19	?	?
15.	?	?	563.22	\$9,950.22	?
16.	?	?	361.85	9,408.05	?
17.	\$5,580.10	$2\frac{1}{2}\%$	?	?	?
18.	\$4,589.40	?	917.88	?	?
19.	246.00	?	?	258.30	?

EXERCISE No. 146, SALES PROBLEMS

Whether a salesman receives a salary as compensation for his services, a combination of salary and commission, or commission only, the value of his services to the business concern which employs him is always measured by his sales. If he receives a salary, and his sales do not compare favorably with those of the other salesmen working under the same conditions, he will eventually be replaced by a more efficient salesman.

It is necessary for a business concern to keep a record of all sales in order to measure the efficiency of its sales organization. For the reasons just explained, it is necessary to record the sales made by each salesman, and use the facts resulting from this record as a means of measuring the efficiency of each salesman. The comparison of sales, and the compensation based on sales, are best expressed as a rate per cent.

The following problems provide written practice in the application of percentage to the problems of the owner of a business or the one responsible for the sales.

1. The Brown Grocery Company employed four salesmen. During the month of January their sales were as follows:

Brown.....	\$2,750.45	Smith.....	\$2,309.16
Clark.....	2,396.48	Snyder.....	2,845.87

Each salesman was allowed a commission of $8\frac{1}{2}\%$ of his sales. You are to ascertain: (a) the total sales of the four salesmen; (b) the average sales for the month; (c) the rate per cent each salesman was above or below the average; (d) the commission due each salesman for the month.

2. During February, the sales of the four salesmen mentioned in problem 1 were as follows:

Brown.....	\$2,954.55	Smith.....	\$2,145.72
Clark.....	2,463.68	Snyder.....	2,800.85

Show all your calculations necessary to ascertain: (a) the average sales for February; (b) the per cent of increase or decrease of the average as compared with the January sales; (c) the increase or decrease of each salesman's sales over the preceding month; (d) the per cent of this increase or decrease; (e) the commission due each salesman on the February sales.

3. The L. J. Mann Hardware Company has four salesmen each of whom receives a salary for his services. During the month of July the sales of each salesman were as follows:

Bates.....	\$829.63	Day.....	\$1,017.30
Davis.....	641.92	Jones.....	913.55

You are to ascertain: (a) the total sales made by the four salesmen; (b) the average sales of the four salesmen; (c) what per cent each salesman's sales were above or below the average.

4. Warren Brothers, retail dry goods merchants, have four salesmen, each of whom receives a definite salary, plus a commission of 2% on the sales he makes above the average sales. During the month of August the sales of these four salesmen were as follows:

Hays.....	\$1,142.96	Knight.....	\$1,429.94
Joyce.....	1,233.96	Ross.....	1,646.42

You are to show on tablet paper all of your calculations necessary to ascertain (a) the average sales, and (b) the commission due each salesman who is above the average.

5. The twelve salesmen listed below comprise the sales force of The Hunt Manufacturing Company. At the close of the current business year the sales of each salesman were as follows:

Bates...	\$13,167.44	Davis...	\$15,095.33	King...	\$11,412.96
Bauer..	13,498.72	Daly....	15,809.88	Rowe...	12,331.97
Craig...	13,782.22	Hale....	14,166.18	Saylor...	13,209.76
Culp...	10,773.50	Joyce...	13,224.65	Smith...	12,709.43

Each salesman is to receive as compensation for his services a

salary of \$200 per month and a commission of $3\frac{1}{2}\%$, if his sales are the average or more than the average, and only $2\frac{1}{2}\%$, if his sales are less than the average. You are to ascertain: (a) the total sales for the year; (b) the average sales; (c) the commission due each salesman for his sales during the year as per agreement; (d) the total income of each salesman for the year; and (e) the rate per cent of cost of each salesman's sales on \$100 unit basis.

NOTE: \$100 unit basis means that the salesmen's sales are expressed in even hundred dollars. Thus, the sales made by Mr. Bates will be expressed as \$13,200; Mr. Daly, \$15,800; Mr. Hale, \$14,200; Mr. Saylor, \$13,200, etc.

EXERCISE No. 147, RATE PER CENT OF PROFIT

When property is purchased and sold by an individual who is not engaged in a business enterprise, it is customary to use the cost price as the base to ascertain the per cent of profit or loss.

The following problems provide oral practice in ascertaining the per cent of profit or loss when the cost is used as the base.

When the percentage (profit or loss) is an aliquot part of the base (cost price) the rate will be the same aliquot part of 100%. Thus, in the first problem the profit, \$100, is one third of the base; hence the rate is $33\frac{1}{3}\%$, because this is one third of one hundred per cent.

1.		2.	
Cost	Selling Price	Cost	Selling Price
a. \$300.00	\$400.00	a. \$ 2.50	\$ 3.50
b. 200.00	300.00	b. 125.00	150.00
c. 250.00	300.00	c. 16.00	20.00
d. 800.00	900.00	d. 17.50	20.00
e. 500.00	750.00	e. 37.50	50.00
f. 25.00	30.00	f. 15.00	12.50
g. 2.50	5.00	g. 17.50	15.00
h. 12.50	15.00	h. 125.00	100.00

EXERCISE No. 148, PROFIT AND LOSS

The problems on page 191 provide written practice in ascertaining the profit or loss and per cent of profit or loss based on *cost*. You are to show on tablet paper all the calculations necessary to ascertain (a) the profit or loss, and (b) the per cent of

profit or loss expressed as an integer, or a mixed number with the fraction expressed decimally in hundredths.

1.		2.	
Cost Price	Selling Price	Cost Price	Selling Price
a. \$ 1,650.45	\$ 1,995.50	a. \$335.00	\$330.00
b. 550.00	725.00	b. 825.50	820.25
c. 225.45	200.50	c. 99.95	97.65
d. 775.25	815.65	d. 104.14	115.66
e. 12,500.00	14,650.00	e. 4.75	6.25
f. 200.50	192.25	f. 44.50	41.20

EXERCISE No. 149, PROFIT AND LOSS

It is customary for the merchant or manufacturer to use the selling price as the base in all calculations. Thus, if a merchant sells an article for \$1.25 and the cost price is \$1, the per cent of profit is calculated on \$1.25, the selling price, instead of \$1, the cost price. The same mathematical process is applied, whether the selling price or cost price is used as a base; but the rate per cent will be different because the cost price is different from the selling price. Thus, the per cent of profit on the sale referred to above, if based on the selling price, would be 20%, and on the cost price, 25%.

The following problems provide oral practice in ascertaining the per cent of profit or loss resulting from a sale, when the selling price is used as a base.

1.		2.	
Cost Price	Selling Price	Cost Price	Selling Price
a. \$1,250.00	\$1,500.00	a. \$25,000.00	\$30,000.00
b. 1,000.00	1,200.00	b. 21,000.00	28,000.00
c. 1,500.00	1,800.00	c. 1,500.00	1,200.00
d. 1,400.00	1,600.00	d. 37,500.00	50,000.00
e. 7,500.00	9,000.00	e. 32,000.00	40,000.00
f. 8,400.00	9,600.00	f. 1,250.00	1,000.00
g. 13.50	15.00	g. 7,500.00	8,000.00
h. 5,280.00	6,000.00	h. 9,900.00	9,000.00
i. 7,200.00	8,000.00	i. 8,500.00	8,000.00

EXERCISE No. 150, RATE PER CENT OF PROFIT OR LOSS

The following problems provide written practice in ascertaining the per cent of profit or loss when the cost and selling price are given, and the selling price is to be used as the base.

1.		2.	
Cost Price	Selling Price	Cost Price	Selling Price
a. \$22,451.75	\$24,526.60	a. \$17,716.65	\$17,005.04
b. 1,237.55	1,152.40	b. 7,622.55	8,004.05
c. 12,862.65	16,911.40	c. 2,262.65	2,141.50
d. 7,827.65	8,001.01	d. 8,887.18	9,919.90
e. 32,786.65	38,242.55	e. 4,465.86	4,392.99
f. 9,900.50	10,400.25	f. 1,255.15	1,400.75

NOTE: Express the rate per cent as an integer, or mixed number with the fraction expressed decimally in thousandths.

EXERCISE No. 151, DEPRECIATION

When a business man purchases an automobile, a motor truck, a typewriter, or other property of similar nature for use in the operations of his business, it is necessary to take into consideration the decrease in the value of this property because of its use, otherwise his records will not show the correct amount of profit or loss resulting from the operation of the business. This decrease in value is taken care of through recording as depreciation a certain per cent of the cost value of the property, usually at the end of each business year during its ownership. The per cent decided upon as the amount of depreciation is based on cost price.

If a motor truck costs \$2,500 and it is decided that it should be depreciated 20% each year, \$500 or one fifth of the cost value is recorded each year as the depreciation. The present, or "book", value of the truck is ascertained by subtracting the depreciation from the cost value of the truck.

The following problems provide written practice in ascertaining the present value of property which will depreciate because of its use. Copy the problems as given, inserting the correct amounts where the question marks appear. When completed, the first column will show the cost of the article; the second, the rate of depreciation each year; the third, fourth, fifth, and sixth the depreciation for each of the four years; and the seventh, the value of the property after deducting the depreciation for the four years.

Cost	Rate	1st Year	2d Year	3d Year	4th Year	Present Value
1. \$2,200	20%	?	?	?	?	?
2. 2,500	15%	?	?	?	?	?
3. 100	18%	?	?	?	?	?
4. 375	12½%	?	?	?	?	?
5. 5,000	17%	?	?	?	?	?
6. 3,500	20%	?	?	?	?	?
7. 750	15%	?	?	?	?	?
8. 1,000	16%	?	?	?	?	?
9. 125	20%	?	?	?	?	?
10. 140	14%	?	?	?	?	?
11. 2,850	17%	?	?	?	?	?
12. 1,675	15%	?	?	?	?	?

NOTE: When the property is sold or exchanged for new property, a record of the cost and depreciation will be of assistance in arriving at an equitable price. In case of a fire, it is necessary to know the present value in order to collect the loss from the insurance company.

EXERCISE No. 152, COST OF MERCHANDISE SOLD

The merchant does not always know the exact cost of each article sold, but he does know, at the end of the year, the cost of all merchandise sold during the year. This information is ascertained from (a) the cost of merchandise on hand at the beginning of the year, (b) the cost of merchandise bought during the year, and (c) the merchandise on hand at the end of the year. The cost of merchandise bought includes (a) the invoice cost of the merchandise bought, less the value of any merchandise returned, plus (b) the transportation cost.

Percentage may be used to an advantage in comparing the various items which enter into the cost of merchandise purchased. The merchant is interested in having a comparison of these facts in order to better control the cost of future purchases.

If the value of the merchandise on hand at the beginning of the year was \$2,000 and the purchases cost of merchandise during the year, \$3,000, it is quite evident that there should be in stock at the end of the year merchandise to the value of \$5,000 if none has been sold. If the inventory shows that the value of merchandise on hand is only \$1,500, it is quite evident that the merchandise sold during the year cost \$3,500. This is the difference between the amount that should have been on hand if none had been sold and the amount that is on hand.

The following problems provide written practice in applying percentage to purchase costs.

1. A. L. Young had \$12,416.84 worth of merchandise on hand at the beginning of the year and \$11,642.50 on hand at the end of the year. It is desired to know (a) the difference between the two inventories, and (b) the per cent of decrease in the value of the inventory at the end of the year when compared with the beginning inventory.

2. J. C. Smith purchased merchandise amounting to \$19,871.62 during a certain year and paid transportation cost on this amounting to \$1,141.36. It is desired to know the per cent of transportation cost on purchases.

3. During the same year applicable to problem 2, the returned purchases, that is the merchandise returned to creditors, was \$1,004.65. What per cent of the merchandise bought was returned?

4. The following facts are recorded in the ledger of George Busse, a retail merchant: total purchases, \$61,748.55; transportation cost, \$2,748.65. During the previous year the total purchases were \$75,527.86 and the transportation cost, \$3,165.54. Mr. Busse wishes to know (a) the comparison in per cent of the purchases for the two years, (b) the comparison in per cent of the transportation cost for the two years, (c) the per cent of transportation cost on purchases for this year, and (d) the per cent of transportation cost on purchases for the previous year.

5. The bookkeeping records show the following facts affecting the goods sold this year: beginning inventory, \$14,651.72; purchases during the year, \$28,765.92; returned purchases, \$562.90; freight, \$864.95; ending inventory, \$9,756.82. The same facts for the preceding year were: beginning inventory, \$15,562.40; purchases, \$25,987.62; returns, \$659.80; freight cost, \$1,242.41; ending inventory, \$14,651.72. You are required to show the calculations necessary to ascertain: (a) cost of the goods sold this year; (b) cost of the goods sold last year; (c) the per cent of increase or decrease; (d) the per cent of comparison of the beginning inventory this year with last year; and (e) the per cent of comparison of the ending inventory this year with last year.

EXERCISE No. 153, SALES

The profit on a sale of merchandise is ascertained by subtracting from the net sales the net cost of the merchandise sold. The net sales are the total sales less any returns made by customers. The method of ascertaining the cost of merchandise sold is explained in Exercise No. 152. It is customary to use the net sales as a base when ascertaining the rate per cent of income on sales or operating expense. It is only when a comparison of items which enter into the cost is desired that the cost is used.

The following problems provide written practice in applying percentage to business problems involving buying and selling of merchandise and expenses in connection therewith.

1. Cost of merchandise sold, \$14,786.95; selling price, \$17,562.40. You are required to show on tablet paper the calculations required to find the per cent of profit.

2. Total sales during the year were \$32,561.40; returns of merchandise by customers, \$846.65. What per cent of the merchandise sold was returned?

3. Total sales during the year were \$85,962.40; total selling expenses including total expense of salesmen, \$9,762.40. What per cent of the sales are the selling expenses?

4. Total sales, \$587,642.95; merchandise returned by customers for credit, \$1,262.40; allowances granted customers because goods were not up to standard, \$462.75; total selling cost, \$32,562.54; total administrative cost, \$31,562. You are required to show the calculations necessary to ascertain (a) the per cent of merchandise returned by customers, (b) the per cent of allowances to customers, (c) the rate per cent of selling expense, and (d) the rate per cent of administrative expense.

5. Cost of merchandise sold, \$458,962.95; sales, \$654,527.55. What was the per cent of profit?

CHAPTER XVI

PERCENTAGE PROBLEMS

The following problems provide written practice in applying the principles of percentage as explained in the preceding chapter. Show on tablet paper all the calculations necessary to arrive at the facts desired, and prove all of your answers.

1. The following items are quoted from a bargain sale advertisement. The first amount is the former sale price, and the second is the present sale price. You are to ascertain (a) the reduction in price, and (b) the per cent of this reduction based on the former price.

- | | |
|----------------------------------|--------------------------------|
| <i>a.</i> \$15 coats at \$12.50 | <i>g.</i> \$30 dresses at \$18 |
| <i>b.</i> \$4 dresses at \$3 | <i>h.</i> \$50 rugs at \$40 |
| <i>c.</i> \$5 dresses at \$4.50 | <i>i.</i> \$120 rugs at \$90 |
| <i>d.</i> \$3 galoshes at \$2.50 | <i>j.</i> \$6 shoes at \$3.50 |
| <i>e.</i> \$35 coats at \$20 | <i>k.</i> \$8 shoes at \$5 |
| <i>f.</i> \$20 coats at \$12 | <i>l.</i> \$10.50 shoes at \$7 |

2. January 2, James Bigelow purchased a used Buick automobile for \$1,250. January 4, he sold this car for \$1,400. Ascertain (a) the profit and (b) the per cent of profit.

3. January 2, C. H. Reed paid \$2,150 for a new automobile. December 31 of the same year he purchased another automobile, and received \$950 credit on the purchase price for the used car. (a) How much was the depreciation; that is, the difference between the cost of the used car and its exchange value? (b) What was the rate per cent of monthly depreciation?

4. A. L. Downing paid \$5,500 for a farm and sold it for \$6,250. How much was his per cent of profit after paying \$55.80 for taxes?

5. The Brown Manufacturing Company paid \$2,700 for a machine on January 2. The bookkeeper is instructed to depreciate

the machine at the rate of $12\frac{1}{2}\%$ each year. After keeping the machine for four years, it was exchanged for a new machine at an exchange allowance of \$972. (a) What was the book value of the machine when exchanged? (b) What should have been the rate of yearly depreciation to have absorbed all of the depreciation?

6. Walter Brown paid \$1,500 for a lot, and \$6,275.80 for the construction of a house. In addition to the cost of the house and lot, he paid \$250 for paving assessments, and \$82.50 for taxes. He sold the lot, and house upon completion, for \$9,000. What was (a) the net profit, and (b) the per cent of profit?

7. At the beginning of the business year, July 1, Martin Brothers, retail grocers, had in stock merchandise valued at \$9,654.55. June 30 of the next year, the record of purchases showed, \$12,621.65; transportation cost, \$927.75; total sales, \$20,467.75; returned sales of customers, \$527.75. The cost value of merchandise in stock June 30, as shown by the inventory was \$5,522.20. They desire to know (a) the profit on merchandise sold during the year, and (b) the per cent of profit.

When the rate per cent is a mixed number, show the value of the fraction decimally in hundredths.

8. From the following facts ascertain: (a) the cost of merchandise sold; (b) net sales; (c) the profit on sales; and (d) the per cent of profit on sales, based on net sales.

Beginning inventory.....	\$ 67,526.98
Purchases.....	127,642.75
Returned purchases.....	1,051.62
Transportation cost.....	4,827.38
Ending inventory.....	9,468.46
Total sales.....	199,461.58
Returns by customers.....	984.55

9. Beginning inventory, \$37,462.91; purchases during the year, \$52,827.36; ending inventory, \$30,562.44. (a) What per cent of the goods purchased was sold? (b) If the selling price was 25% in excess of the cost, what were the total sales for the year?

10. May 1, Adolph Howe paid \$8,500 for a house and lot which was occupied at a rental of \$75 per month. May 10, he paid for repairs and painting, \$285.65; June 1, collected rent of \$75; July 1, paid taxes, \$191.87; July 1, collected rent, \$75;

August 1, collected rent, \$75; August 15, paid insurance, \$24; September 1, collected rent, \$75; September 5, sold property to the occupant for \$11,250. Mr. Howe desires to know (a) the amount of profit made on this sale, and (b) the per cent of profit based on the purchase price of the property.

11. June 5, J. B. Hall paid \$775 for a used Studebaker automobile. June 10, he sold this car for \$825. He desires to know (a) the profit and (b) the per cent of profit based on the cost. June 15, he paid taxes which were assessed against the car while in his possession. (c) What effect will this have on the profit and the per cent of profit?

12. C. H. Dobson, a farmer, paid \$1,500 for a truck to be used in connection with the operations of his farm. The rate of depreciation set up by the tax commission was 30% the first year, 25% additional the second year, and 15% additional each year thereafter. This means that the taxable value of the truck at the end of the first year was cost less 30%, or \$1,050, and the second year, cost less 55%, the third year, cost less 70%, etc. At the end of the third year the truck was destroyed by fire. Mr. Dobson carried fire insurance of \$700. What amount should he collect from the insurance company based on the taxable depreciation?

13. I. L. Pierson, a farmer, collected in cash from the sales of farm products in a given year, \$3,562.48. He paid as expenses in operating the farm for the same year, \$722.65. You are to ascertain (a) his net profit, and (b) what per cent of his sales are his expenses.

14. The Model Laundry purchased a delivery truck for \$1,800 and kept it for four years. During the first year, it was depreciated 35%, the second year, 20%, the third year, 15%, and the fourth year, 10%, in accordance with the tax schedule. At the end of the fourth year it was exchanged for another truck at an exchange allowance of \$750. You are to show on tablet paper the calculations necessary to determine: (a) the book value of the truck at the time it was exchanged; (b) the difference between the exchange value and the book value; and (c) the rate per cent this difference is above or below the book value.

15. The sales manager for the Van Horn Manufacturing Company wishes to establish an average rate per cent of cost of sell-

ing. The records for the current year show total sales, \$862,981.50, and total cost of sales, \$90,613.06. (a) What was the rate per cent of cost of selling, and (b) how much above or below this average was J. H. Hurdig whose sales were \$29,684.72 and whose cost of selling was \$4,155.86?

16. Your father paid \$1,200 for a lot on which to build a home. Before building operations were begun, he sold the lot at a profit of $66\frac{2}{3}\%$, and bought another lot at 20% less than the selling price of the first one. (a) What did he receive for the first lot? (b) What did he pay for the second lot? (c) What was the net profit? (d) What was the per cent of net profit?

17. The number of high school graduates increased from 21,880 at the end of the nineteenth century to 224,367 thirty years later. What was the per cent of increase?

18. In a recent year there were 81,034 high school teachers as compared to 9,120 thirty years before. What was the per cent of increase?

19. The population of a certain city was 407,621 in one census year, and 502,682 in the next census year. (a) What was the increase in population in this city; and (b) the per cent of increase?

20. A certain city shows an increase in birth rate of 2,475 in one year. This is $1\frac{7}{8}\%$ increase. Find the birth rate for the previous year.

21. A star ballplayer batted for the week as follows: Monday, at bat 5 times and made 3 hits; Tuesday, 4 times, 1 hit; Wednesday, 4 times, 4 hits; Thursday, 3 times, 2 hits; Saturday, up 7 times in a "double header" and made 5 hits. What was his batting average for the week?

22. The population of a certain city was 363,891. During one year the number of deaths was 6,330. What was the per cent of death rate per thousand population?

23. Robert B. Hase operates The Electric Shop and carries a stock of electrical fixtures; he also provides mechanics for installing these. At the end of the business year, his net cost of merchandise purchased and labor cost was \$12,487.39; inventory of electrical fixtures in stock, \$1,855.42; his sales during the year, which includes the sale of both services and fixtures, were \$21,749.82; his expenses, which include rent, heat, light, in-

insurance and office help, were \$5,827.36. He desires to know: (a) the cost of merchandise and services sold; (b) the profit on these sales; (c) the net profit after deducting the expenses from the profit on sales; and (d) the per cent of net profit.

24. S. L. Robinson buys job lots of paper from the paper mills and sells to the printers. April 5, he purchased from the Beckett Paper Mill 215,500 lbs. of paper at 8c per lb.; April 7, he sold 55,200 lbs. at $9\frac{1}{2}$ c; April 8, he sold 27,500 lbs. at $9\frac{3}{4}$ c; April 9, 62,850 lbs. at $9\frac{1}{4}$ c; April 10, sold the balance at \$8.95 per 100 lbs. The expense in connection with the sale of this paper was \$175.50. He desires to know (a) the profit made on the sale of this job lot of paper and (b) the per cent of profit based on the sales.

25. During a certain year R. L. Dougherty, a retail clothing merchant paid \$8,142.55 selling expenses, and his sales for the year amounted to \$65,287.82. What was the rate per cent of selling cost?

26. The purchases of merchandise by the Fair Store during the month of January were \$18,642.91. The returns of merchandise to creditors were \$762.91. The allowances by creditors for damaged goods were \$82.65. (a) What per cent of merchandise purchased was returned, and (b) what per cent of the total purchases was the allowances?

27. The gross sales of the Gidding Company during the month of December were \$28,672.91. Merchandise returned by customers for credit during this month was \$587.63. Credit was allowed to customers because the merchandise sold was not satisfactory. You are required to ascertain: (a) the net sales; (b) the per cent of returns based on gross sales. (c) What per cent of the net sales were the allowances?

28. A corporation increased its capital stock 50%. At the end of the first business year thereafter a stock dividend of \$9,000 was paid to stockholders, which was 6% of the present capital. What was the capital stock before the increase?

29. A dealer sold 315 hogs which was $37\frac{1}{2}\%$ of the number he had bought during the week. (a) How many did he have left, and what per cent was unsold?

30. The Brown Manufacturing Company, manufacturers of clothing, wish to know the rate per cent of the selling cost of merchandise manufactured and sold. The following facts affecting these operations are recorded in the ledger:

Traveling salesmen's salaries.....	\$ 35,500.00
Traveling salesmen's expenses...	28,642.50
Advertising.....	9,427.50
House salesmen.....	14,279.90
Total sales.....	786,962.87
Returns by customers.....	2,412.50

NOTE: Use as a base the total sales less the returns by customers.

31. The sales manager of the Day Manufacturing Company wishes to know (a) the cost per square mile, and (b) the cost of business per 1,000 inhabitants for each of the 36 salesmen employed by the company. The following are the facts necessary to ascertain the information for A. B. Short and C. H. Williams, two of these salesmen. The territory assigned to Mr. Short contains 70,500 square miles and 7,800,000 population, and the territory traveled by Mr. Williams contains 90,600 square miles and 6,750,000 population. Mr. Short's salary and expenses were \$6,250, and Mr. Williams' salary and expenses, \$7,150. Ascertain the desired information for the sales manager as applied to each of these two salesmen.

32. The sales manager of the Day Manufacturing Company mentioned in problem 31, wishes to know the sales in cents, or in cents and dollars, per 1,000 population for each of the 36 salesmen. The sales of A. B. Short were \$52,561 and Mr. Williams, \$61,587.40. You are to ascertain the desired information for the sales manager.

33. During the year A. L. Patton, a farmer, received cash as follows:

Farm products.....	\$2,761.55
Chickens and eggs.....	527.40
Milk and butter.....	722.52
Cattle and hogs.....	1,299.87

He desires to know: (a) the total sales; and (b) what per cent of the total sales is each of the products mentioned.

34. The Hayes Manufacturing Company, manufacturers of radio parts, finds it necessary to secure a new location and has a very favorable proposition from another city. However, the acceptance of this proposition would increase the freight cost on the merchandise and raw material which it must purchase, because it would be located farther from the source of supplies. The ledger shows the following facts:

Total purchases during the year . . .	\$22,875.36
Freight on this	962.88

It is desired to know what per cent of the purchases is the freight, that a comparison may be made with the estimated rate which would be applicable to the new location.

35. November 19, J. B. Daily paid \$14,500 for a house and lot. November 10, he paid \$140 for insurance on a \$7,500 fire insurance policy; December 1, collected rent, \$125; December 15, paid taxes, \$495.50; January 1, collected rent, \$125; February 1, collected rent, \$125. February 2, his house was damaged by fire. February 10, collected \$4,500 from the insurance company; March 1, collected rent, \$125; March 13, paid contractor for repairing building, \$5,200; April 1, collected rent, \$125; May 1, collected rent, \$125; and on May 2, sold the house for \$16,000. He wishes to know (a) whether he has made or lost on the purchase and sale of this property, and (b) the per cent of profit or loss based on the first cost of the property.

36. The sales of C. H. Hyatt, a clerk in the Peoples Department Store, for a certain week were \$544.65. If his salary that week was \$40 plus $1\frac{1}{2}\%$ commission on his sales, what was the rate per cent of cost of his sales?

37. A merchant realizes a profit of $12\frac{1}{2}\%$ above all cost in operating his business. If the cost of operation was \$2,750, and the cost of goods sold, \$10,000, (a) what were the total sales and (b) what was the net gain?

38. The gross profit of Roberts Bros., retail clothiers, during a certain year was \$65,000. During the same year the cost of operations was \$25,000. If the net profit was $12\frac{1}{2}\%$ of the total sales, what was the amount of the total sales?

39. C. C. McDaniels, a retail clothier, wishes to know whether he has made or lost on the operations of his business during the past year and submits the following facts as a basis for obtaining this information:

Merchandise on hand at the beginning of the year .	\$ 4,561.87
Cost of merchandise purchased during the year . .	21,962.70
Transportation cost.....	671.80
Returned purchases.....	317.65
Sales for the year.....	28,509.87
Returned sales.....	362.55
Operating expenses for the year.....	7,241.55

Show (a) the profit or loss on sales, and rate per cent based on net sales; (b) rate per cent of cost of merchandise sold based on sales; and (c) rate per cent of expenses based on sales.

40. January 3, the Prompt Service Electric Company paid \$1,450 for an automobile truck for delivery purpose. The rate of depreciation recorded on the books as fixed by the tax laws was 30% the first year, 20% additional the second year, 15% additional the third year, and 10% additional for each year thereafter. They kept this truck for 4 years and traded it in for a new one being allowed \$652.24 exchange value. (a) What was the book value of the truck according to the tax schedule, and (b) what is the difference in per cent between the exchange value and the book value compared to the cost value?

41. The Jones Printing Company paid \$2,965.50 for advertising during a certain year. The net sales during this same year were \$30,564.80. What per cent of the net sales was the amount paid for advertising?

42. The average daily circulation of a certain newspaper during one year was 16,587. The daily average circulation for the succeeding year was 17,962. What was the per cent of increase in the circulation?

43. The total valuation for tax purpose in a certain city was \$44,475,465. The estimated expenses for operating the city during a year was \$1,041,065. What tax rate per thousand dollars will be necessary in order to secure the estimated expenses?

44. The freight rate between two cities on a given commodity was \$1.415 per hundred lbs. The Interstate Commerce Commission reduced the rate to \$1.3575 per hundred lbs. What was the per cent of this decrease?

45. The tariff rate for rooms in a certain hotel in accordance with location was \$5, \$4, \$3, and \$2 per day. Competition forced the management to readjust the tariff rate, and the new rates were fixed at \$4.50, \$3.50, \$2.50, and \$1.75. What was the per cent of the reduction on each of the four classes of rooms, which you may indicate in your answer as a, b, c, and d.

46. The freight rate by rail transportation between two cities is \$0.445 per hundred lbs. The freight rate by motor bus transportation is \$3.75 per thousand lbs. (a) What per cent less is the motor rate than the rail rate; (b) what per cent greater is the rail rate than the motor rate; and (c) what would be the saving on a shipment weighing 15,575 pounds?

47. The cost of shipping a package by parcel post is \$1.45 and by express \$1.65. What per cent is saved by shipping by parcel post, assuming that it costs 10c to send the package to the post office, and the express company calls for the package without charge?

48. C. W. Curtis sold 50 hogs at the market price on date of delivery. He started them to market by truck and expected their delivery on January 18 at which time the market price was \$7.85 per hundred lbs. Owing to a break-down and a delay incident to repairs the shipment was not delivered until January 19, on which day the market price was \$7.77 per hundred lbs. The total weight of the hogs was 11,581 lbs. (a) How much did he lose because of this break-down, and (b) what was the per cent of loss?

49. C. H. Dewey paid \$11,500 for a farm containing 165 acres. He sold 5 acres at \$100 per acre, 25 acres at \$90 per acre, 40 acres at \$85 per acre and the remainder at \$60 per acre. (a) What profit did he make on the sale of this farm; and (b) what per cent of profit on the amount invested?

50. The freight rate on a certain classification of merchandise between Pittsburgh and Buffalo is \$0.4575 per 100 lbs., and between New York and Pittsburgh on the same classification,

\$0.4925. C. H. Longenecker in Pittsburgh has a price of 78c per unit on this classification of merchandise from a manufacturer in Buffalo, and 76c per unit from a manufacturer in New York. If he wishes to buy 8 gross of these units and each weighs $22\frac{1}{2}$ lbs. packed, (a) would it be cheaper to buy from the manufacturer in Buffalo or New York, the Buffalo manufacturer offering 3%, 10 days, and the New York manufacturer, 2%, 10 days; (b) how much would be saved; and (c) what would be the per cent of saving using the lowest cost price as the base?

51. Find the standings of the following baseball teams:

- A. Won 66, lost 47.
- B. Won 69, lost 39.
- C. Won 57, lost 53.
- D. Won 66, lost 41.
- E. Won 49, lost 57.
- F. Won 63, lost 60.
- G. Won 65, lost 40.
- H. Won 70, lost 31.

52. R. H. Kincaid received a tax bill from the county treasurer for \$241.75. What is the rate per thousand dollars if the assessed value of his property is \$21,500.

53. P. M. Davis paid a broker \$265.50 for five \$50 bonds. What was; (a) the cost per bond; and (b) the per cent above the par value of \$50?

54. John Smith, a shoe salesman, made the following sales during each of the months of a certain year:

January.....	\$3,671.45	July.....	\$3,556.75
February.....	4,857.55	August.....	3,889.55
March.....	3,985.35	September.....	4,076.50
April.....	4,240.75	October.....	4,175.25
May.....	4,769.25	November.....	4,555.75
June.....	3,750.80	December.....	4,965.25

It is desired to ascertain: (a) his total sales for the year; (b) his average sales per month; (c) a comparison of the sales each month with the average; and (d) the per cent of the increase or decrease.

55. The following facts appeared in a certain city daily where four newspapers are published. The figures given are the display lines, which is the unit of measurement for advertising used by local and national advertisers. The newspapers are described by number. The figures in the first column apply to advertising space by local businesses and the second column by businesses outside of the city. You are required to show on tablet paper the calculations necessary to ascertain: (a) the total number of display lines by each newspaper; (b) the total number of display lines carried by the four newspapers; (c) what per cent of the total is carried by each newspaper; and (d) the per cent of newspaper No. 1 over No. 2, No. 3, and No. 4. You may assume that you are preparing your statistics for newspaper No. 1 which desires to show the public how much it is in the lead.

Newspaper	Local	National	Total
No. 1	9,764,153	2,386,433	?
No. 2	5,125,330	1,322,447	?
No. 3	3,660,636	817,390	?
No. 4	1,684,536	143,976	?

56. The number of pieces of first-class mail handled by the postoffice in a certain city during the month of January was 1,461,582. This was an increase of 181,652 over January of the preceding year. What was the per cent of increase?

57. A factory has five departments with floor space as follows: Department A, 11,500 square feet; Department B, 12,800 square feet; Department C, 1,500 square feet; Department D, 11,200 square feet; Department E, 21,000 square feet. What per cent of the total space does each department occupy?

58. Robert C. Cloud was touring over a state highway having a speed limit of 25 miles per hour. Tuesday morning at ten o'clock his speedometer registered 9,512.5 miles; at eleven o'clock it registered 9,549.3 miles; and at twelve o'clock it registered 9,589.3 miles. You are to show (a) the per cent of mileage over the legal speed limit during the first hour, (b) the per cent over the legal speed limit during the second hour, and (c) what was the per cent of increase for the second hour compared with the speed traveled during the first hour?

59. C. H. Penrod, salesman for the Acme Manufacturing Company, received \$9,426.87 as his salary and commission for a certain year. His salary was paid to him in monthly installments of \$500, and his commission was paid at the end of the year. He received a commission of $7\frac{1}{2}\%$ on his sales. (a) What was the amount of his sales, and (b) what was the amount of commission collected at the end of the year?

60. The distance from New York to Buffalo is 438.73 miles; from Buffalo to Cleveland 183.2; from Cleveland to Toledo, 113.5 miles; and from Toledo to Chicago, 254 miles: (a) what is the distance between New York and Chicago; (b) what per cent of the total distance is the distance from Chicago to Toledo; (c) what per cent of the total distance is the distance from Cleveland to Toledo?

61. The following bargain advertisement appeared in one of the newspapers of a certain city, "Fur coats at a saving of 25% to 40%":

- | | |
|--------------------------|--------------------------|
| a. \$100 coats for \$65 | e. \$195 coats for \$125 |
| b. \$135 coats for \$115 | f. \$225 coats for \$165 |
| c. \$175 coats for \$125 | g. \$235 coats for \$175 |
| d. \$295 coats for \$195 | h. \$150 coats for \$120 |

You are to determine (a) which of these reductions are in accordance with the advertisement, and (b) ascertain the percentage of reduction based on the original marked price. If the merchant sells one each of these coats at the bargain price advertised, (c) what would be the per cent of reduction on the whole sale based on the original marked price.

62. The following appeared in a bargain rug advertisement, "\$16.75 rugs for \$13.40". If these rugs cost the merchant \$11.64 per rug, what was the per cent of profit: (a) on the original marked price; (b) on the bargain price?

63. The following statement appeared in a bargain advertisement, "Inlaid linoleum, \$1.49 per yard, a reduction of 15%". (a) What was the original selling price? (b) If the linoleum cost \$1.18 a yard what per cent of profit was made on the original selling price? (c) What was the per cent of profit on the bargain price?

64. A merchant received a check from one of his customers for \$1,860.24, marked "In full of account". According to the terms of the sale, the customer was to have $2\frac{1}{2}\%$ discount if he paid the bill within ten days, and the check was received within this period. What was the amount of the sale for which this check was in payment?

65. The four salesmen for the Cramer Manufacturing Company made the following number of calls during the month of January: A. L. Coats, 101; D. C. Mayo, 126; J. J. Marshall, 112; and R. T. Downing, 118. (a) What was the total number of calls made by the four salesmen? (b) What per cent of the total calls was made by each salesman?

66. Mary Browne, a clerk in the Busy Bee Department store received a bonus of 2% on all of her sales, and 5% of the sales she made each day in excess of \$50. Her sales for the week were as follows: \$51.28, \$45.68, \$49.23, \$59.04, \$61.02, \$71.03. What amount of bonus did she receive for the week?

CHAPTER XVII

INTEREST

The term *interest* is used to describe the income derived from money loaned or the cost incurred by the use of money which has been borrowed. The one who uses the money pays for its use, hence the interest is a cost or an expense to him, and an income to the lender.

The amount borrowed or the amount on which the interest is calculated, is known as the *principal*. The per cent of the principal to be paid each year for the use of money is known as the *rate*. The *time* is the number of days, months, or years during which the money is used. The rate is based on one year, but if the borrower uses the money for more or less than a year, the amount of interest paid will be proportionally more or less.

The rate is controlled by the state law. By referring to the table on the next page, you will see the effect of the laws in the various states on interest rates. When a definite rate is fixed by state law, this is referred to as the *legal rate*, and is the rate to be applied when no other rate has been contracted. Some states allow a contract rate greater than the legal rate, but other states permit the legal rate only. Collecting more than the legal rate as interest is referred to as *usury*.

Interest calculations require the percentage application of decimal fractions with the rate based on one year. If Robert Jones, a salesman, is to receive 6% of the sales he makes as a compensation, this means that his income is .06 of his sales, or 6c on each dollar. If his income is 6% on money loaned, he will receive .06 of the amount loaned, or 6c on each dollar for each year it is used by one to whom it is loaned. The calculations necessary to arrive at the compensation are the same in each case, provided the money loaned is used by the borrower for a full year. If the borrower uses the money for less than a year, the compensation for its use will be less.

Interest Rates in the Various States

States and Territories	Legal rate Per cent	Contract rate Per cent	States and Territories	Legal rate Per cent	Contract rate Per cent
Alabama.....	8	8	Montana.....	8	10
Alaska.....	8	12	Nebraska.....	7	10
Arkansas.....	6	10	Nevada.....	7	12
Arizona.....	6	10	N. Hampshire....	6	Any rate
California.....	7	12	New Jersey.....	6	6
Colorado.....	8	12	New Mexico.....	6	10
Connecticut.....	6	12	New York.....	6	6
Delaware.....	6	6	North Carolina ..	6	6
Dist. of Col.....	6	8	North Dakota ...	6	9
Florida.....	8	10	Ohio.....	6	8
Georgia.....	7	8	Oklahoma.....	6	10
Hawaii.....	8	12	Oregon.....	6	10
Idaho.....	7	10	Pennsylvania....	6	6
Illinois.....	5	7	Porto Rico.....	6	12
Indiana.....	6	8	Rhode Island....	6	Varies
Iowa.....	6	8	South Carolina...	7	8
Kansas.....	6	10	South Dakota....	7	10
Kentucky.....	6	6	Tennessee.....	6	8
Louisiana.....	5	8	Texas.....	6	10
Maine.....	6	Any rate	Utah.....	8	12
Maryland.....	6	6	Vermont.....	6	6
Massachusetts...	6	Any rate	Virginia.....	6	6
Michigan.....	5	7	Washington.....	6	12
Minnesota.....	6	8	West Virginia...	6	6
Mississippi.....	6	8	Wisconsin.....	6	10
Missouri.....	6	8	Wyoming.....	7	10

When interest is calculated on the principal for the time the principal has been used, it is known as *simple interest*. When the contract provides that the interest at the end of a definite period may be added to the principal and the interest during the next period calculated on this new principal, it is known as *compound interest*. The usual plan in long time loans is for the borrower to pay simple interest on the loan at definite dates, rather than to compound the interest and increase the principal.

The method of calculating both simple and compound interest is the same in that the principal is multiplied by the rate expressed as a decimal fraction, the product being the interest for one year. In simple interest calculations the principal remains the same, but in compound interest calculations the principal increases at the beginning of stated periods due to the addition of interest.

EXERCISE No. 154, INTEREST AT 6% FOR ONE OR MORE MONTHS

A year is 12 months, hence interest for 1 month is for $\frac{1}{12}$ of a year; for 14 months, $\frac{1}{6}$ more than a year; 2 years and 9 months, $2\frac{3}{4}$ times that for one year. Contract or custom may affect the interpretation of "month" in interest calculations but unless governed by contract or special conditions, a month is usually regarded as one twelfth of a year.

The following problems provide oral practice in interest calculations based on the rate of 6% for one year of 12 months, each month being considered as equivalent to one twelfth of a year.

1	2
a. \$400.00 for 3 months	a. \$2,500.00 for 15 months
b. 250.00 for 6 months	b. 2,200.00 for 12 months
c. 100.00 for 3 months	c. 2,000.00 for 3 months
d. 600.00 for 2 months	d. 3,000.00 for 3 months
e. 500.00 for 4 months	e. 4,000.00 for 4 months
f. 750.00 for 12 months	f. 5,000.00 for 5 months
g. 60.00 for 6 months	g. 6,000.00 for 3 months
h. 150.00 for 4 months	h. 1,500.00 for 12 months
i. 350.00 for 12 months	i. 4,000.00 for 6 months
j. 90.00 for 6 months	j. 5,000.00 for 7 months
k. 80.00 for 3 months	k. 2,000.00 for 2 months
l. 75.00 for 4 months	l. 8,000.00 for 2 months

EXERCISE No. 155, INTEREST AT 6% FOR ONE OR MORE MONTHS

The following problems provide written practice in interest calculations with the rate at 6% for a year, each year consisting of 12 months, each month regarded as one twelfth of a year. The illustration below shows the method of calculating the interest on \$581.65 for 3 months at 6%.

$$\begin{array}{r}
 \$581.65 \\
 \quad .06 \\
 \hline
 4)34.8990 \\
 \hline
 8.7247
 \end{array}$$

The principal is multiplied by .06 giving a product of 34.8990 which is the interest for one year; since three months is $\frac{1}{4}$ of a year, dividing the interest for one year by 4 gives a quotient of 8.7247 or \$8.72, the interest for three months; had the two figures at the right been 50 or more, the interest would have been \$8.73.

1	2
<i>a.</i> \$315.45 for 3 months	<i>a.</i> \$5,123.45 for 14 months
<i>b.</i> 591.15 for 6 months	<i>b.</i> 2,145.65 for 9 months
<i>c.</i> 675.35 for 3 months	<i>c.</i> 8,125.55 for 6 months
<i>d.</i> 723.85 for 4 months	<i>d.</i> 3,456.75 for 8 months
<i>e.</i> 225.85 for 2 months	<i>e.</i> 4,567.25 for 3 months
<i>f.</i> 645.50 for 4 months	<i>f.</i> 6,024.35 for 7 months
<i>g.</i> 765.80 for 2 months	<i>g.</i> 9,210.10 for 1 month
<i>h.</i> 217.85 for 1 month	<i>h.</i> 1,245.90 for 2 months
<i>i.</i> 355.65 for 2 months	<i>i.</i> 5,216.75 for 5 months
<i>j.</i> 129.75 for 12 months	<i>j.</i> 3,150.85 for 6 months
<i>k.</i> 875.65 for 10 months	<i>k.</i> 1,750.25 for 9 months
<i>l.</i> 432.25 for 11 months	<i>l.</i> 5,500.15 for 12 months

EXERCISE No. 156, INTEREST FOR ONE OR MORE MONTHS

When the rate is more or less than 6%, the amount of interest is ascertained in the same manner as when it is 6%; that is, the principal is multiplied by the rate to ascertain the interest for one year, and the interest for one year used as a basis for ascertaining the interest for the number of months the principal has been used.

The following problems provide oral practice in ascertaining the interest when the principal, the rate, and the number of months are given.

1	2
<i>a.</i> \$300, 4 mos., 5%	<i>a.</i> \$2,000, 8 mos., 5%
<i>b.</i> 500, 6 mos., 4%	<i>b.</i> 3,000, 7 mos., 5%
<i>c.</i> 300, 2 mos., 7%	<i>c.</i> 5,000, 5 mos., 5%
<i>d.</i> 800, 3 mos., 8%	<i>d.</i> 1,000, 9 mos., 9%
<i>e.</i> 60, 6 mos., 7%	<i>e.</i> 2,000, 2 mos., 8%
<i>f.</i> 900, 1 mo., 8%	<i>f.</i> 6,000, 6 mos., $4\frac{1}{2}\%$
<i>g.</i> 100, 6 mos., 9%	<i>g.</i> 2,000, 2 mos., 8%
<i>h.</i> 200, 8 mos., 7%	<i>h.</i> 3,000, 5 mos., 7%
<i>i.</i> 300, 3 mos., 3%	<i>i.</i> 2,000, 8 mos., 7%
<i>j.</i> 800, 4 mos., $4\frac{1}{2}\%$	<i>j.</i> 9,000, 9 mos., 4%
<i>k.</i> 700, 3 mos., 7%	<i>k.</i> 2,500, 27 mos., 4%
<i>l.</i> 36, 6 mos., 5%	<i>l.</i> 2,000, 11 mos., 5%

EXERCISE No. 157, INTEREST FOR ONE OR MORE MONTHS

The following problems provide written practice in interest calculations when the principal, rate, and time in months are given; each year consisting of 12 months, and each month being $\frac{1}{12}$ of a year. The illustration below shows the method of calculating the interest on \$581.65 for 3 months at 7%.

$$\begin{array}{r} \$581.65 \\ .07 \\ \hline 4)40.7155 \\ \hline 10.1788 \end{array}$$

The principal is multiplied by .07 giving a product of 40.7155 which is the interest for one year; dividing this by 4 gives a quotient of 10.1788 or \$10.18, the interest for 3 months.

In all interest calculations carry the operations to four decimal places before discarding the remainders.

1

- a. \$565.15, 6 mos., 5%*
- b. 75.65, 3 mos., 7%
- c. 323.50, 7 mos., 8%
- d. 439.65, 1 mo., 9%
- e. 786.15, 6 mos., 7%
- f. 435.80, 1 mo., $4\frac{1}{2}\%$
- g. 219.55, 3 mos., 5%
- h. 398.95, 4 mos., 7%
- i. 456.65, 5 mos., 8%
- j. 275.50, 7 mos., 3%
- k. 316.25, 8 mos., 7%
- l. 599.50, 6 mos., 4%

2

- a. \$1,250.75, 7 mos., 5%
- b. 3,475.95, 2 mos., 9%
- c. 5,432.10, 3 mos., 7%
- d. 6,125.50, 7 mos., 4%
- e. 4,050.25, 6 mos., 8%
- f. 2,254.35, 1 mo., $4\frac{1}{2}\%$
- g. 6,250.85, 4 mos., 7%
- h. 4,125.65, 5 mos., 9%
- i. 4,450.55, 2 mos., 7%
- j. 3,130.30, 3 mos., 3%
- k. 5,300.75, 5 mos., 5%
- l. 4,198.45, 6 mos., 4%

*NOTE: If desired, the interest may be ascertained at 6% for the given number of months and this used as a basis for ascertaining the interest at a rate above or below 6%. If the rate is 8%, the interest would be $\frac{1}{3}$ more than it is at 6%; 9% would be $\frac{1}{2}$ more; $4\frac{1}{2}\%$ would be $\frac{1}{4}$ less, 5% would be $\frac{1}{8}$ less, and 7% would be $\frac{1}{8}$ more.

EXERCISE No. 158, INTEREST AT 6% FOR DAYS

When the time is less than a year and not an exact number of months, it is expressed in days. When the money is used for a designated number of days, it is necessary to use the number of days in a year in order to ascertain the fractional part of a year the money has been used. For convenience in interest calculations, it is customary in business to regard a year as 360 days composed of 12 months of 30 days each. This means that 20 days would be $\frac{20}{360}$ or $\frac{1}{18}$ of a year; 30 days, $\frac{30}{360}$ or $\frac{1}{12}$ of a year; 60 days, $\frac{60}{360}$ or $\frac{1}{6}$ of a year; 6 days, $\frac{6}{360}$ or $\frac{1}{60}$ of a year, etc.

Applying the above reasoning to find the interest on \$200 for 90 days at 6%, the process would be as follows: the interest on \$200 for one year at 6% is 200 times .06 or \$12; 90 days is $\frac{90}{360}$ or $\frac{1}{4}$ of a year; $\frac{1}{4}$ of \$12 is \$3, the interest on \$200 for 90 days at 6%.

If \$200 were used for 40 days the interest would be $\frac{1}{9}$ of \$12, or \$1.33, because 40 days is $\frac{1}{9}$ of 360 days. If the \$200 were used for 20 days the interest would be 67c, because 20 days is $\frac{1}{18}$ of 360 days.

The following problems provide oral practice in ascertaining the interest at 6%, when the time is expressed in days, and the days are regarded as a fractional part of 360.

1	2	3
a. \$300 for 60 days	a. \$1,200 for 12 days	a. \$22 for 60 days
b. 200 for 90 days	b. 2,250 for 20 days	b. 84 for 30 days
c. 450 for 30 days	c. 4,500 for 40 days	c. 36 for 20 days
d. 450 for 40 days	d. 7,500 for 80 days	d. 95 for 6 days
e. 200 for 30 days	e. 2,500 for 30 days	e. 49 for 12 days
f. 450 for 20 days	f. 3,900 for 40 days	f. 62 for 18 days
g. 800 for 40 days	g. 4,200 for 80 days	g. 65 for 90 days
h. 250 for 90 days	h. 6,000 for 60 days	h. 80 for 30 days
i. 375 for 20 days	i. 6,600 for 66 days	i. 75 for 20 days
j. 375 for 60 days	j. 7,200 for 72 days	j. 96 for 60 days
k. 300 for 40 days	k. 8,000 for 80 days	k. 72 for 12 days
l. 375 for 60 days	l. 3,600 for 36 days	l. 13 for 40 days

EXERCISE No. 159, INTEREST AT 6% FOR DAYS

The following problems provide written practice in ascertaining the interest when the time is expressed in days and the rate is 6%. The illustration at the left shows the method of ascertaining the interest on \$282.45 for 45 days at 6%. You will show the solution of each problem in the same form as in the illustration.

$$\begin{array}{r}
 \$282.45 \\
 .06 \\
 \hline
 16.9470 \\
 45 \text{ days} = \frac{45}{360} \text{ or } \frac{1}{8} \\
 \frac{1}{8} \text{ of } 16.9470 = 2.1183 \\
 \text{or } \$2.12
 \end{array}$$

The interest for one year would be .06 of \$282.45 or 16.9470 as shown in the solution at the left. 45 days is $\frac{45}{360}$ or $\frac{1}{8}$ of a year. If the interest on \$282.45 is 16.9470 for one year, for $\frac{1}{8}$ of a year, it would be $\frac{1}{8}$ of this amount or \$2.12.

1	2	3
a. \$299.75, 40 days	a. \$85.40, 54 days	a. \$2,311.55, 6 days
b. 377.16, 30 days	b. 99.18, 36 days	b. 3,265.70, 60 days
c. 704.45, 18 days	c. 52.40, 90 days	c. 2,291.90, 45 days
d. 199.25, 36 days	d. 78.65, 30 days	d. 4,402.11, 36 days
e. 462.75, 80 days	e. 15.50, 120 days	e. 9,841.40, 90 days
f. 281.90, 90 days	f. 77.75, 80 days	f. 7,208.16, 20 days
g. 616.65, 72 days	g. 36.85, 40 days	g. 3,303.48, 60 days
h. 109.14, 90 days	h. 82.65, 36 days	h. 2,908.65, 80 days
i. 200.20, 100 days	i. 62.95, 72 days	i. 8,414.15, 40 days
j. 411.15, 45 days	j. 44.88, 60 days	j. 7,209.19, 40 days
k. 214.55, 30 days	k. 27.25, 150 days	k. 4,000.05, 60 days
l. 175.75, 40 days	l. 69.65, 80 days	l. 2,218.19, 30 days

EXERCISE No. 160, SIXTY-DAY 6% METHOD

You will have observed from the calculations required in the interest problems given in the preceding exercises that the interest for 60 days at 6%, is $\frac{1}{100}$ of the principal, and for 6 days, $\frac{1}{1000}$ of the principal. The reason for this is because 60 days is $\frac{1}{6}$ of a year and multiplying the principal by $\frac{6}{100}$ and dividing by 6 is equivalent to dividing the principal by 100. 6 days is $\frac{1}{10}$ of 60 days, hence the interest for 6 days is $\frac{1}{10}$ of that for 60 days or $\frac{1}{1000}$ of the principal. Many interest calculations may involve a number of days which is not conveniently expressed as a fraction of a year, but which can be more conveniently expressed as a fraction of 60 days; and since the interest for 60 days is known without any calculations, it is advisable to use the 60 days as a basis for the fraction, instead of 360 days.

Applying the above reasoning to ascertain the interest on \$300 for 30 days at 6%, the process would be as follows: the interest on \$300 for 60 days is \$3; 30 days = $\frac{30}{60}$ or $\frac{1}{2}$ of 60 days; if the interest on \$300 for 60 days is \$3 and 30 days is $\frac{1}{2}$ of 60 days, the interest on \$300 for 30 days is \$1.50.

To find the interest on \$200 for 42 days the process would be as follows: the interest on \$200 for 60 days is \$2; 42 days = $\frac{42}{60}$ or $\frac{7}{10}$ of 60 days; $\frac{7}{10}$ of \$2 = \$1.40.

To find the interest on \$500 for 48 days the process would be as follows: the interest on \$500 for 60 days is \$5; 48 days = $\frac{48}{60}$ = $\frac{4}{5}$; $\frac{4}{5}$ of \$5 is \$4, the interest on \$500 for 48 days.

The following problems provide oral practice in finding the interest at 6%, using the sixty-day method explained in Exercise No. 160, or the six-day method explained in the preceding paragraph. In solving the problems, use the method which you consider the shorter.

1	2	3
a. \$200 for 24 days	a. \$ 85 for 24 days	a. \$2,000 for 27 days
b. 150 for 18 days	b. 550 for 48 days	b. 1,000 for 39 days
c. 300 for 36 days	c. 75 for 66 days	c. 3,000 for 86 days
d. 400 for 42 days	d. 300 for 90 days	d. 2,500 for 72 days
e. 50 for 45 days	e. 250 for 51 days	e. 1,500 for 66 days
f. 25 for 66 days	f. 99 for 48 days	f. 900 for 9 days
g. 375 for 24 days	g. 150 for 72 days	g. 9,000 for 72 days
h. 250 for 42 days	h. 500 for 78 days	h. 8,500 for 24 days
i. 500 for 48 days	i. 250 for 72 days	i. 5,500 for 33 days
j. 300 for 51 days	j. 95 for 36 days	j. 4,000 for 40 days
k. 450 for 57 days	k. 400 for 84 days	k. 3,000 for 44 days
l. 200 for 54 days	l. 350 for 60 days	l. 8,000 for 46 days

EXERCISE No. 163, SIX-DAY 6% METHOD

As explained in Exercise No. 162, the interest at 6% for 6 days is found by moving the decimal point three places to the left. The illustration below shows the method of ascertaining the interest on \$418.25 for 78 days at 6%, by the six-day method, and the proof by the sixty-day method.

$$\begin{array}{r}
 a. \\
 78 \div 6 = 13 \\
 .41825 = \text{interest for 6 days} \\
 \underline{13} \\
 \$5.43725 = \text{required interest}
 \end{array}$$

$$\begin{array}{r}
 b. \\
 78 \text{ days} = 60 + 12 + 6 \\
 \hline
 4.1825 = \text{interest for 60 days} \\
 .8365 = \text{interest for 12 days} \\
 .4182 = \text{interest for 6 days} \\
 \hline
 \$5.4372 = \text{interest for 78 days}
 \end{array}$$

Solution (a) shows the 6-day method, and solution (b) shows the 60-day method. According to the 6-day method, 78 is divided by 6, and the principal with the decimal point moved three places to the left is multiplied by the quotient. According to the 60-day method, the interest for 60 days is the principal with the decimal point moved two places to the left. The interest for 12 days is $\frac{1}{5}$ of the interest for 60 days, and the interest for 6 days is $\frac{1}{2}$ of the interest for 12 days. The sum of the interest for 60, 12, and 6 days is \$5.44, the interest for 78 days.

The following problems provide written practice in applying the six-day method at 6% as illustrated, and proof of the calculations by the sixty-day method. You are to show the calculations necessary to ascertain the desired interest and proof.

1	2	3
a. \$199.65, 51 days	a. \$55.40, 91 days	a. \$2,987.62, 59 days
b. 724.50, 69 days	b. 75.65, 38 days	b. 9,881.62, 49 days
c. 415.40, 33 days	c. 22.50, 102 days	c. 5,005.05, 92 days
d. 622.18, 81 days	d. 55.05, 78 days	d. 6,661.42, 17 days
e. 944.44, 21 days	e. 92.19, 57 days	e. 7,991.99, 74 days
f. 515.78, 27 days	f. 57.57, 45 days	f. 6,682.48, 65 days
g. 866.78, 99 days	g. 38.62, 93 days	g. 8,881.19, 11 days
h. 200.50, 45 days	h. 45.55, 87 days	h. 4,455.15, 95 days
i. 750.75, 84 days	i. 72.50, 78 days	i. 6,114.45, 46 days
j. 219.95, 48 days	j. 99.98, 96 days	j. 9,881.55, 39 days
k. 616.27, 18 days	k. 69.48, 27 days	k. 1,503.72, 71 days
l. 514.34, 21 days	l. 17.49, 49 days	l. 5,981.91, 95 days

EXERCISE No. 164, RATES OTHER THAN 6%

The interest on any principal is $\frac{1}{100}$ of the principal for the number of days obtained by dividing 360 by the rate. Thus the interest on any principal is $\frac{1}{100}$ of itself at:

6% for 60 days ($360 \div 6 = 60$); 8% for 45 days ($360 \div 8 = 45$);
 9% for 40 days ($360 \div 9 = 40$); 10% for 36 days ($360 \div 10 = 36$);
 15% for 24 days ($360 \div 15 = 24$); $4\frac{1}{2}\%$ for 80 days ($360 \div 4\frac{1}{2} = 80$);
 4% for 90 days ($360 \div 4 = 90$); 5% for 72 days ($360 \div 5 = 72$).

The number of days in which the principal will produce interest $\frac{1}{100}$ of itself at a given rate, can be used to advantage at rates other than 6%. Divide 360 by the rate, move the decimal point two places to the left in the principal, and you have the interest at the given rate, for the number of days obtained by dividing.

Applying the above facts, the interest on \$40 for 45 days at 8% is 40c; for 60 days it is $\frac{1}{2}$ more than 40 or 53c; and for 80 days it is two times 40c or 80c. The interest on \$50 at 9% for 40 days is 50c; for 80 days, is \$1; and for 20 days, is 25c. The interest on \$60 at $4\frac{1}{2}\%$ for 80 days is 60c; for 100 days it is one fourth more than for 80 days, or 75c; and for 40 days it is one half of 60c, or 30c.

The following problems provide oral practice in ascertaining the interest on a given sum at rates other than 6%. Divide 360 by the rate, to ascertain the number of days, and move the decimal point two places to the left to ascertain the interest for this number of days. This interest will be proportionately increased, if the number of days given is more, and decreased, if the number of days given is less.

1	2
a. \$30 for 40 days at 9%	a. \$ 90 for 60 days at 10%
b. 30 for 60 days at 9%	b. 90 for 20 days at 10%
c. 30 for 20 days at 9%	c. 90 for 30 days at 12%
d. 75 for 80 days at 8%	d. 48 for 72 days at 5%
e. 75 for 45 days at 8%	e. 48 for 36 days at 5%
f. 75 for 30 days at 8%	f. 48 for 18 days at 5%
g. 75 for 60 days at 8%	g. 48 for 90 days at 5%
h. 75 for 90 days at 8%	h. 48 for 144 days at 5%
i. 80 for 80 days at $4\frac{1}{2}\%$	i. 200 for 60 days at $4\frac{1}{2}\%$
j. 80 for 40 days at $4\frac{1}{2}\%$	j. 200 for 80 days at $4\frac{1}{2}\%$
k. 80 for 100 days at $4\frac{1}{2}\%$	k. 200 for 100 days at $4\frac{1}{2}\%$
l. 90 for 80 days at 12%	l. 200 for 60 days at $4\frac{1}{2}\%$

EXERCISE No. 165, RATES OTHER THAN 6%

The illustration below shows the method of ascertaining the interest on \$128.65 for 65 days at 8%. This is the same process as used in the oral solution of problems in the preceding exercise.

$$65 = 45 + 15 + 5$$

1.2865 = interest, 45 days

.4288 = interest, 15 days

.1429 = interest, 5 days

\$1.8582 = interest, 65 days

The 65 is divided into numbers, one of which is 45 and the others, factors of 45. The interest on the given principal for 45 days at 8% is one one-hundredth of the principal. The interest for 15 days is $\frac{1}{3}$ of that for 45 days, and the interest for 5 days is $\frac{1}{3}$ of that for 15 days. The sum of these three amounts

is \$1.86 which is the required interest at 8% for 65 days.

The problems at the top of page 221 provide written practice in ascertaining interest at rates other than 6% when the principal, rate and number of days are given. You are to show all the calculations necessary to find the required interest.

1

- a. \$429.85 for 100 days at 5%
- b. 98.42 for 99 days at 8%
- c. 964.86 for 128 days at $4\frac{1}{2}\%$
- d. 354.44 for 95 days at 8%
- e. 229.69 for 117 days at 10%
- f. 542.37 for 115 days at 8%
- g. 82.46 for 225 days at 8%
- h. 723.52 for 93 days at 10%
- i. 450.05 for 99 days at 5%
- j. 712.33 for 66 days at 8%
- k. 9.65 for 125 days at 9%
- l. 800.50 for 48 days at 10%

2

- a. \$2,546 for 60 days at 9%
- b. 1,758 for 150 days at 5%
- c. 9,500 for 225 days at 4%
- d. 7,768 for 220 days at $4\frac{1}{2}\%$
- e. 1,500 for 199 days at 5%
- f. 3,271 for 88 days at 8%
- g. 1,653 for 37 days at 10%
- h. 1,005 for 55 days at 12%
- i. 9,367 for 80 days at 8%
- j. 1,592 for 130 days at $4\frac{1}{2}\%$
- k. 1,632 for 128 days at 9%
- l. 2,500 for 125 days at 8%

EXERCISE No. 166, RATES OTHER THAN 6%

The interest at 6% may be used as a basis for ascertaining the interest at any rate other than 6%. When this plan is followed, the interest at 6% is divided by 6 to ascertain the rate at 1%, and this quotient multiplied by the given rate to ascertain the desired interest. Thus, if it is desired to ascertain the interest on \$600 at 8% for 60 days, the mental process would be as follows: interest on \$600 for 60 days at 6% is \$6; at 1% it is \$1; and at 8% it is \$8. If the method given in Exercise No. 164 is used, the mental process is as follows: the interest at 8% for 45 days is \$6; and the interest for 15 days more is $\frac{1}{3}$ more than \$6 or \$8.

The following problems provide oral practice in ascertaining the interest on a given principal, at a given rate, for a stated time. Use the mental process first suggested, but be prepared to use the second one.

1

- a. \$2,400 for 60 days at 8%
- b. 3,600 for 66 days at 8%
- c. 1,800 for 75 days at 5%
- d. 1,200 for 15 days at $4\frac{1}{2}\%$
- e. 3,000 for 45 days at 8%
- f. 4,200 for 12 days at 5%
- g. 2,400 for 72 days at 5%
- h. 3,600 for 24 days at 9%
- i. 3,000 for 30 days at 8%

2

- a. \$1,500 for 60 days at 4%
- b. 3,600 for 45 days at 5%
- c. 5,400 for 60 days at 9%
- d. 7,500 for 75 days at 4%
- e. 4,200 for 30 days at 8%
- f. 2,500 for 24 days at 9%
- g. 3,000 for 36 days at 5%
- h. 6,000 for 85 days at 4%
- i. 4,000 for 80 days at 8%

EXERCISE No. 167, RATES OTHER THAN 6%

The solutions below show the method of ascertaining the interest on \$421.60 for 69 days at 8% by one method, and the proof of the result by another method.

a.	b.
\$4.2160 = int. @ 6%, 60 days	\$4.2160 = int. @ 8%, 45 days
.4216 = int. @ 6%, 6 days	1.4053 = int. @ 8%, 15 days
.2108 = int. @ 6%, 3 days	.8432 = int. @ 8%, 9 days
4.8484 = int. @ 6%, 69 days	\$6.4645 = int. @ 8%, 69 days
.80806 = int. @ 1%, 69 days	
6.46448 = int. @ 8%, 69 days	

In the solution (a) the method explained in Exercise No. 161 is used. After obtaining the interest at 6% for 69 days, this is divided by 6 in order to ascertain the interest at 1%, and the quotient multiplied by 8 to ascertain the interest at 8%. The proof, solution (b), is based on the method explained in Exercise No. 164. You will observe in solution (a) that the fraction of a cent is not discarded until the final result is obtained.

The following problems provide written practice in ascertaining the interest at rates other than 6% using the 60-day method as a basis. You are to show on tablet paper the solution of each problem by one of the two methods illustrated and the proof of the solution by the other method.

1	2
a. \$721.65, 66 days, 7%	a. \$3,754.75, 94 days, 4%
b. 318.54, 49 days, 5%	b. 7,621.32, 72 days, 7%
c. 218.91, 66 days, 8%*	c. 3,381.85, 51 days, 5%
d. 714.45, 84 days, 9%	d. 2,216.48, 72 days, 7%
e. 216.72, 24 days, 10%	e. 9,861.75, 99 days, 3%
f. 327.92, 44 days, 7%	f. 2,221.25, 45 days, $4\frac{1}{2}\%$
g. 216.65, 55 days, 4%	g. 8,962.75, 44 days, $4\frac{1}{2}\%$
h. 986.55, 55 days, 5%	h. 6,241.45, 63 days, 5%
i. 221.25, 42 days, 9%	i. 7,219.95, 98 days, 9%
j. 316.36, 27 days, $7\frac{1}{2}\%$	j. 4,454.45, 78 days, 7%
k. 333.55, 48 days, 4%	k. 6,666.66, 79 days, 8%
l. 919.98, 84 days, 8%	l. 2,222.25, 18 days, 9%

*NOTE: When the rate is 8%, add $\frac{1}{2}$ to the interest at 6%; $7\frac{1}{2}\%$, add $\frac{1}{4}$; $\frac{3}{2}\%$, deduct $\frac{1}{4}$; 3%, deduct $\frac{1}{2}$, and 9%, add $\frac{1}{2}$.

EXERCISE No. 168, CANCELLATION METHOD

When the rate is other than 6%, the cancellation method can sometimes be used to advantage in ascertaining the amount of interest. The interest is the quotient resulting from the principal multiplied by the number of days, multiplied by the rate, divided by 360, the number of days in a year. The illustration below shows the application of the cancellation method in ascertaining the interest on \$829.48 for 90 days at 7%.

$$\begin{array}{r} 207.37 \\ \cancel{829.48} \times \cancel{90} \times .07 \\ \hline \quad \quad \quad \cancel{360} \\ \quad \quad \quad \cancel{4} \end{array} = 207.37 \times .07 = \$14.52$$

The principal, number of days, and rate are written above the line and the number of days in a year below the line. 90 is contained into 360, 4 times, hence the 90 is canceled and the 4 written

below the 360. 829.48 is divisible by 4, hence the 4 is canceled and 207.37, the result obtained by dividing by 4 is written above. The numbers below the line are all canceled, hence the product of $207.37 \times .07$ is the required interest.

The illustration at the right shows the process by the 60-day method. It is well to know both methods because sometimes the cancellation method is quicker for the solution or the proof. The result should always be proved by a different method.

$$\$829.48 = 8.2948 \text{ for 60 days}$$

$$90 \text{ days} = \frac{90}{60} \text{ or } 1\frac{1}{2}$$

$$\$8.2948 \times 1\frac{1}{2} = \$12.4422$$

$$\$12.4422 \div 6 = \$2.0737 \text{ interest at } 1\%$$

$$\$2.0737 \times 7 = \$14.52 \text{ interest at } 7\%$$

The following problems provide written practice in applying the cancellation method. You will show on tablet paper the solution by the cancellation method and the proof by either the 60-day method or the 6-day method.

1

- a. \$315.17, 55 days, 7%
- b. 221.75, 40 days, 8%
- c. 598.76, 54 days, 5%
- d. 665.45, 90 days, 3%
- e. 432.26, 75 days, 4%
- f. 850.50, 80 days, 9%
- g. 925.35, 30 days, 3%
- h. 475.65, 45 days, 7%
- i. 220.50, 70 days, 9%
- j. 766.66, 50 days, 5%

2

- a. \$2,356.75, 18 days, 10%
- b. 5,566.75, 12 days, 9%
- c. 3,789.21, 54 days, 7%
- d. 9,482.56, 90 days, 5%
- e. 8,521.10, 75 days, 4%
- f. 3,134.57, 80 days, $4\frac{1}{2}\%$
- g. 6,750.25, 24 days, 3%
- h. 4,156.30, 48 days, 9%
- i. 7,721.13, 50 days, 8%
- j. 1,090.80, 55 days, 7%

EXERCISE No. 169, TIME REQUIRED

In the solution of problems involving interest, it is necessary to ascertain the time the money has been in use in order to calculate the interest for its use. The time involves the number of years, months, or days between the date on which the interest begins, and the date on which it is desired to ascertain the amount of the interest. There are two methods of ascertaining the time between two given dates: (a) by subtracting the beginning date from the ending date, known as compound subtraction; and (b) by counting the actual number of days between the two dates.

The illustration below shows the method of ascertaining the time between March 16 and October 27, by compound subtraction; you will use this method only when instructed to do so.

Mo. Da.	October is the tenth month and March the
10 — 27	third, hence it is necessary to subtract three
3 — 16	months and sixteen days from ten months and
	twenty-seven days. The difference is seven
7 — 11	months and eleven days. It is customary to
$7 \times 30 = 210$ days	regard a year as twelve months of thirty days
$210 + 11 = 221$ days	each. In seven months there are 210 days; add-
	ing the eleven days makes a total of 221 days,
	the difference in time by compound subtraction.

The illustration below shows the method of ascertaining the time by counting the actual number of days between the two dates; you will use this method in solving problems, unless specifically instructed to ascertain the time by compound subtraction.

Mar. 31 days	March, the first month, has thirty-one days, but the interest does not
Mar. 16	begin until the sixteenth of March, hence subtracting sixteen from thirty-
	one leaves a balance of fifteen days. To this is added
15	the number of days in each of the succeeding months,
Apr. 30	and the twenty-seven days in October. The reason
May 31	the time by actual count is more than by the other
June 30	method is because some of the months have thirty-
July 31	one days. Because of the fact that all of the months
Aug. 31	do not have the same number of days, it is customary
Sept. 30	to count the number of days as illustrated, especially
Oct. 27	when the time involved is less than a year.
	Since the tendency in modern business is to ascer-
225	tain the time by counting the actual number of days,
	unless otherwise instructed, your interest calculations
	should be based on the actual number of days.

The following problems provide oral practice in ascertaining the time between two given dates. You are to state the mental process of ascertaining the time in each problem.

1

- a. May 10 to June 5
- b. November 6 to January 4
- c. October 7 to November 9
- d. July 1 to July 28
- e. July 16 to August 16
- f. August 16 to September 16
- g. September 16 to October 16
- h. March 1 to April 10
- i. January 5 to February 11
- j. November 9 to December 23
- k. May 18 to August 12
- l. May 12 to May 20

2

- a. June 15 to July 6
- b. August 7 to September 9
- c. May 4 to May 25
- d. February 9 to February 28
- e. March 16 to April 1
- f. July 1 to July 12
- g. May 16 to May 29
- h. April 1 to May 10
- i. May 11 to June 10
- j. July 6 to August 3
- k. November 9 to June 1
- l. January 11 to February 20

EXERCISE No. 170, TIME AND INTEREST REQUIRED

When the beginning and ending dates are given, it is customary to ascertain the time between these two dates by counting the actual number of days.

The illustration below shows one method of ascertaining the interest on \$425.50 from April 5 to June 13 at 6%, and the proof of this solution by another method.

Apr. 30	a. \$425.50 = .42550	b. 4.2550 = int. for 60 days
Apr. 5	69 ÷ 6 = 11½	.4255 = int. for 6 days
25	.42550 × 11½ =	.2127 = int. for 3 days
May 31	4.89325 or	
June 13	\$4.89	4.8932 = int. for 69 days
69		or \$4.89

In solution (a) the six-day 6% method is used. The interest on the principal for six days is ascertained by moving the decimal point three places to the left. The interest for the desired number of days is ascertained by multiplying the interest for six days by 11½.

The proof shown in illustration (b) is based on the sixty-day method. The number of days is divided into sixty, six, and three, and the interest ascertained as explained in previous exercises. You should always prove your solutions by applying two different methods.

The following problems provide written practice in: (a) ascertaining the time between two given dates; and (b) ascertaining the interest on a stated amount for this time. Use the 60-day method or the 6-day method for ascertaining the interest, and the cancellation method for proving your results.

1. \$ 275.65 from June 9 to November 12 at 6%
2. 741.50 from March 3 to June 16 at 6%
3. 5,000.00 from May 15 to October 21 at 5%
4. 118.62 from November 5 to February 28 at 6%
5. 10,000.00 from March 1 to October 1 at $4\frac{1}{2}\%$
6. 12.50 from June 16 to November 3 at 6%
7. 211.11 from April 9 to October 12 at 6%
8. 2,500.00 from July 1 to January 1 at 5%
9. 44.55 from May 9 to November 9 at 6%
10. 71.75 from May 17 to June 21 at 6%
11. 2,000.00 from July 1 to November 5 at 4%
12. 416.65 from November 9 to February 16 at 8%
13. 841.45 from June 21 to August 13 at 7%
14. 402.09 from November 1 to February 6 at 9%
15. 412.55 from May 11 to August 16 at 6%
16. 724.25 from March 1 to May 1 at 8%

CHAPTER XVIII

INTEREST (Concluded)

ACCURATE OR EXACT INTEREST

As previously explained the interest for one year is ascertained by multiplying the principal by the rate expressed as a decimal fraction; the product is the interest at the given rate for one year. It is customary in business to regard a year as 360 days, thus facilitating the calculations required to ascertain the interest when the time is expressed in days, but, if desired, the actual number of days in a year may be used as a basis. Simple interest on the principal, when reckoned on the basis of 365 days to a common year, and 366 days to a leap year, is referred to as *accurate interest*.

The illustrations below show the calculations necessary to secure the interest on \$721.68 for 48 days at 6% by simple interest (at the left) and by accurate interest (at the right).

\$721.68 .06 43.3008 48 3464064 1732032 2078.4384 5.7734 360)2078.4384 1800 2784 2520 2643 2520 1238 1080 1584 1440 The interest on the principal for one year is the principal multiplied by the rate expressed as a decimal fraction. The interest for one day is $\frac{1}{360}$ of a year by the simple interest method and $\frac{1}{365}$ of a year by the accurate interest method. The interest for 48 days would be 48 times the interest for one day. The simpler plan is to multiply the interest for a year by 48 and divide by 360 for simple interest, or by 365 for accurate interest as in the illustrations. The simple interest is \$5.77, and the accurate interest is \$5.69. The simple interest for any number of days less than a year, is $\frac{1}{72}$ greater than the accurate interest; and the accurate interest is $\frac{1}{72}$ less than the simple interest. \$721.68 .06 43.3008 48 3464064 1732032 2078.4384 5.6943 365)2078.4384 1825 2534 2190 3443 3285 1588 1460 1284 1095
--

When the principal has been used for a full year, the accurate interest and simple interest will be the same, but when the principal has been used for less than a year, the amount of the accurate interest will be $\frac{1}{73}$ less than the simple interest. The reason for this is because simple interest is reckoned on a basis of 360 days to a year instead of 365. This time is five days too short, and the interest is therefore $\frac{5}{365}$ or $\frac{1}{73}$ too much. When accurate interest is required, the better plan is to ascertain the simple interest and deduct $\frac{1}{73}$.

NOTE: Cancellation, as explained in Exercise No. 168, may be used when finding accurate interest by dividing by 365 for a common year, or by 366 for a leap year.

EXERCISE No. 171, ACCURATE OR EXACT INTEREST

The following problems provide written practice in ascertaining accurate interest. You will show on tablet paper: (a) the calculations necessary to find the time by counting the actual number of days; and (b) the accurate interest on the given principal and rate for this time.

1. Interest on \$1,560 at 4% from March 18 to December 9 of the same year.
2. Interest on \$960.65 at 6% from June 30 to September 20 of the same year.
3. Interest on \$5,000 at 8% from November 2 of one year to January 15 of the following year.
4. Interest on \$650 at 8% from October 14 to December 27 of the same year.
5. Interest on \$2,400.24 at $4\frac{1}{2}\%$ from August 23 in one year to March 20 of the following year (February, 28 days).
6. Interest on \$1,741.50 at 6% from July 29 to September 11 of the same year.
7. Interest on \$975.50 at 6% from January 3 to March 1 of the same year (February, 28 days).
8. Interest on \$800 at 5% from December 18 of one year to April 14 of the following year (February, 28 days).
9. Interest on \$22,500 from January 10 of one year up to and including July 1 of the following year at 6%.

Count the time from January 10 of one year, up to and including January 10 of the following year, as one full year; and the actual number of days of the remaining time (29 days in February) is the time for which accurate interest is computed.

EXERCISE No. 172, REVERSING THE OPERATIONS

Every mathematical operation has its reverse. Just as addition has its subtraction, multiplication its division, involution its evolution, etc., so interest has its reverse operations.

There are four elements involved in interest calculations: (a) the principal, (b) the rate, (c) the time, and (d) the interest. If any three of these elements are provided, the fourth may be obtained. The process may be explained as follows:

- Principal $\times$ (rate for the time) = interest
- Interest $\div$ (\$1 $\times$ rate for the time) = principal
- Interest $\div$ (principal $\times$ rate for 1 day) = time (in days)
- Interest $\div$ (principal $\times$ 1% for given time) = rate used

Substituting figures for the above you have:

- \$1,260 at 5% for 54 days = \$9.45
- $\$9.45 \div .007\frac{1}{2}$ (interest on \$1 for 54 days at 5%) = \$1,260
- $\$9.45 \div .17\frac{1}{2}$ (interest on \$1,260 for 1 day at 5%) = 54 days
- $\$9.45 \div \1.89 (interest on \$1,260 for 54 days 1%) = 5%

One day is used as the basis when it is desired to find the time, and \$1 is used as the basis when it is desired to find the principal.

The following problems provide practice in ascertaining one of the terms when the other three are given. You are to ascertain the information required in the columns in which the question marks appear.

PRINCIPAL		RATE	TIME	INTEREST		AMOUNT		PROCEEDS	
1.	\$800	7 %	4 mo., 3 days	?		?	..	...	.
2.	?	6 %	3 mo., 3 days	\$ 32	55	...	..	?	..
3.	325	50	11 mo., 9 days	24	52	...	..	...	..
4.	4,500		?	175	50	...	..	...	..
5.	1,846	24	1 mo., 5 days	?		...	..	?	
6.	?	5 %	33 days	3	08	...	..	...	..
7.	825	?	54 days	9	90	...	..	...	..
8.	960	5 %	?	18	80	...	..	...	..
9.	?	4 %	27 days	?		752	75	...	..
10.		..	2 mo., 6 days	?		...	..	354	72

The *amount* is the principal plus the interest, and the *proceeds* is the principal minus the interest.

EXERCISE No. 173, COMPOUND INTEREST

When a contract specifies that the accrued interest at the end of designated periods be added to the principal, and the interest for the next period calculated on the new principal, the interest is said to be compounded and the interest, *compound interest*. The word "compound" before interest does not change its meaning but simply refers to a method of ascertaining the amount of interest. The contract will determine the length of each interest period. Thus the period may be monthly, quarterly, semi-annually, annually, etc., as the contracting parties may agree upon. The calculations below show how to ascertain the amount of interest on a \$5,000 note, dated January 1, due in one year, with interest at 6% to be compounded quarterly.

Principal, the first quarter, \$5,000
 Interest on \$5,000 for $\frac{1}{4}$ year = \$75
 Principal, second quarter, \$5,075
 Interest on \$5,075 for $\frac{1}{4}$ of a year = \$76.13
 Principal, third quarter, \$5,151.13
 Interest on \$5,151.13 for $\frac{1}{4}$ of a year = \$77.27
 Principal, fourth quarter, \$5,228.40
 Interest on \$5,228.40 for $\frac{1}{4}$ of a year = \$78.43
 Amount due January 1, \$5,306.83

The principal for the first quarter is \$5,000, but beginning with April 1, the second quarter, it is increased by the interest on the principal for a quarter; hence during the second quarter the principal is \$5,075. Beginning with July 1, the principal is increased by the interest on the principal for the second quarter, or one fourth of a year. The same plan is followed for the third and fourth quarters. You will observe that the difference between the compound and simple interest is \$6.83.

The following problems provide written practice in ascertaining the amount of interest when the contract is to compound the interest at stated periods. When the period for compounding is less than a year, you may regard it as a fractional part of a year. Thus the interest for 1 month would be $\frac{1}{12}$ of the interest for a year; quarterly, $\frac{1}{4}$ of the interest for a year; semi-annually, $\frac{1}{2}$ of the interest for a year, etc.

	<i>Principal</i>	<i>Rate</i>	<i>Time</i>	<i>Compounded</i>
1.	\$ 313.20	6%	1 year	Quarterly
2.	200.50	4%	2 years	Semi-annually
3.	1,050.00	3%	3 years	Annually

	<i>Principal</i>	<i>Rate</i>	<i>Time</i>	<i>Compounded</i>
4.	\$ 725.80	6%	2 years	Semi-annually
5.	5,000.00	5%	1 year	Quarterly
6.	460.20	8%	1 year	Monthly
7.	10,000.00	4½%	3 years	Annually
8.	375.80	7%	1 year	Monthly
9.	1,500.00	5%	5 years	Annually
10.	10,000.00	4%	4 years	Semi-annually
11.	2,000.00	6%	2 years	Quarterly
12.	3,500.00	4½%	4 years	Annually
13.	2,000.00	6%	1 year	Quarterly
14.	186.75	6%	1 year	Quarterly
15.	20,000.00	3%	10 years	Annually

EXERCISE No. 174, COMPOUND INTEREST TABLES

The table on page 232 shows the amount of \$1.00 compounded annually at 3%, 4%, 5%, 6%, 7%, 8%, and 10% for 40 years. The use of this table is illustrated below. The calculations refer to the interest on \$5,000 compounded annually for a period of 5 years at 5%.

Amount of \$1.00 for 5 years (see table) = 1.2762816

\$5,000 × 1.2762816 = \$6,381.41, the amount of \$5,000 for 5 years

\$6,381.41 - \$5,000 = \$1,381.41, the compound interest

The following problems provide practice in using the information given in the Compound Interest Table on page 232. You will arrange your calculations on tablet paper in the same order as in the illustration.

1	2
a. \$ 2,500, 5 years, 6%	a. \$25,000, 15 years, 4%
b. 1,800, 9 years, 3%	b. 400, 9 years, 8%
c. 100, 20 years, 8%	c. 3,750, 12 years, 5%
d. 500, 6 years, 7%	d. 450, 4 years, 10%
e. 30,000, 40 years, 8%	e. 7,000, 5 years, 5%
f. 9,500, 8 years, 4%	f. 2,250, 12 years, 6%
g. 2,100, 7 years, 5%	g. 4,275, 9 years, 8%
h. 50, 31 years, 6%	h. 2,175, 21 years, 5%
i. 2,000, 10 years, 8%	i. 10,000, 10 years, 4%
j. 3,500, 7 years, 7%	j. 2,500, 35 years, 7%

COMPOUND INTEREST TABLE

Showing what \$1.00 will amount to at compound interest, at various rates per cent in any number of years, from 1 to 40 inclusive.

Yrs.	3 Per cent	4 Per cent	5 Per cent	6 Per cent	7 Per cent	8 Per cent	10 Per cent	Yrs.
1	1.0300 0000	1.0400 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.1000 000	1
2	1.0609 0000	1.0816 0000	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.2100 000	2
3	1.0927 2700	1.1248 6400	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.3310 000	3
4	1.1255 0881	1.1698 5856	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4641 000	4
5	1.1592 7407	1.2166 5290	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.6105 100	5
6	1.1940 5230	1.2653 1902	1.3400 956	1.4185 191	1.5007 304	1.5668 743	1.7715 610	6
7	1.2298 7387	1.3159 3178	1.4071 004	1.5036 303	1.6057 815	1.7138.243	1.9487 171	7
8	1.2667 7008	1.3685 6905	1.4774 554	1.5938 481	1.7181 862	1.8509 302	2.1435 888	8
9	1.3047 7318	1.4233 1181	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.3579 477	9
10	1.3439 1638	1.4802 4428	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.5937 425	10
11	1.3842 3387	1.5394 5406	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.8531 167	11
12	1.4257 6089	1.6010 3222	1.7958 563	2.0121 965	2.2521 916	2.5181 701	3.1384 284	12
13	1.4685 3371	1.6650 7351	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.4522 712	13
14	1.5125 8972	1.7316 7645	1.9799 316	2.2609 040	2.6785 342	2.9371 936	3.7974 983	14
15	1.5579 6742	1.8009 4351	2.0789 282	2.3965 582	2.7590 315	3.1721 691	4.1772 482	15
16	1.6047 0644	1.8729 8125	2.1828 746	2.5403 517	2.9521 638	3.4259 426	4.5949 730	16
17	1.6528 4763	1.9479 0050	2.2920 183	2.6927 728	3.1588 152	3.7000 181	5.0544 703	17
18	1.7024 3306	2.0258 1652	2.4066 192	2.8543 392	3.3799 323	3.9960 195	5.5599 173	18
19	1.7535 0605	2.1068 4918	2.5269 502	3.0255 995	3.6165 275	4.3157 011	6.1159 390	19
20	1.8061 1123	2.1911 2314	2.6532 977	3.2071 355	3.8696 845	4.6609 571	6.7275 000	20
21	1.8602 9457	2.2787 0807	2.7859 626	3.3995 636	4.1405 624	5.0338 337	7.4002 499	21
22	1.9161 0341	2.3699 1879	2.9252 607	3.6035 374	4.4304 017	5.4365 404	8.1402 749	22
23	1.9735 8651	2.4647 1555	3.0715 238	3.8197 497	4.7405 299	5.8714 637	8.9453 024	23
24	2.0327 9111	2.5633 0417	3.2250 999	4.0489 346	5.0723 670	6.3411 807	9.8497 327	24
25	2.0937 7793	2.6658 3633	3.3863 549	4.2918 707	5.4274 326	6.8484 752	10.8347 059	25
26	2.1565 9127	2.7724 6979	3.5556 727	4.5493 830	5.8073 529	7.3963 532	11.9181 765	26
27	2.2212 8901	2.8833 6858	3.7334 563	4.8223 459	6.2138 676	7.9880 615	13.1099 942	27
28	2.2879 2768	2.9987 0332	3.9201 291	5.1116 867	6.6488 384	8.6271 064	14.4209 936	28
29	2.3565 6551	3.1186 5145	4.1161 356	5.4183 879	7.1142 571	9.3172 749	15.8630 930	29
30	2.4272 6247	3.2433 9751	4.3219 424	5.7434 912	7.6122 550	10.0626 569	17.4494 023	30
31	2.5000 8035	3.3731 3341	4.5380 395	6.0881 006	8.1451 129	10.8676 694	19.1943 425	31
32	2.5750 8276	3.5080 5875	4.7649 415	6.4533 867	8.7152 708	11.7370 830	21.1137 768	32
33	2.6523 3524	3.6483 8110	5.0031 885	6.8405 899	9.3253 398	12.6760 496	23.2251 544	33
34	2.7319 0530	3.7943 1634	5.2533 480	7.2510 253	9.9781 135	13.6901 336	25.5476 699	34
35	2.8138 6245	3.9460 8899	5.5160 154	7.6860 868	10.6765 815	14.7853 443	28.1024 369	35
36	2.8982 7833	4.1039 3255	5.7918 161	8.1472 520	11.4239 422	15.9681 718	30.9126 805	36
37	2.9852 2668	4.2680 8986	6.0814 069	8.6360 871	12.2236 181	17.2456 256	34.0039 486	37
38	3.0747 8348	4.4388 1345	6.3854 773	9.1542 524	13.0792 714	18.6252 756	37.4043 434	38
39	3.1670 2698	4.6163 6599	6.7047 512	9.7035 075	13.9948 204	20.1152 977	41.1447 778	39
40	3.2620 3779	4.8010 2063	7.0399 887	10.2857 179	14.9744 578	21.7245 215	45.2592 556	40

EXERCISE No. 175, INSTALLMENT PAYMENTS

When merchandise is sold on the installment plan, the purchaser pays a part of the purchase price when the sale is made and the remainder in equal installments, usually payable monthly. It is customary to collect interest on the unpaid balance at the time each monthly payment is collected. This has the same effect as compound interest and permits the merchant to secure compensation for the use of his money monthly, instead of waiting until the contract ends.

July 1, J. C. Shook purchased a piano from the Star Piano Company for \$900. He paid \$300 cash and is to pay the balance in monthly installments of \$100 with interest at 6%. This means that Mr. Shook will pay 6% interest on the amount of money due the Star Piano Company for the time he is using it, paying the interest on the first of each month.

The payments for this piano when the account is closed on January 1 of the following year will appear as follows:

Due, July 1, \$600

Interest for July on \$600 = \$3

August 1, Due, \$603

August 1, received Cash, \$103

Interest for August on \$500 = \$2.50

September 1, Due, \$502.50

September 1, received Cash, \$102.50

Interest for September on \$400 = \$2

October 1, Due, \$402

October 1, received Cash, \$102

Interest for October on \$300 = \$1.50

November 1, Due, \$301.50

November 1, received Cash, \$101.50

Interest for November on \$200 = \$1

December 1, Due, \$201

December 1, received Cash, \$101

Interest for December on \$100 = \$0.50

January 1, Due, \$100.50

January 1, received Cash, \$100.50

NOTE: Each month is treated as 30 days and the interest is calculated by the 60-day method.

The following problems provide written practice in calculating interest on deferred or installment payments.

1. March 1, The Music Shop sold Robert Downing a phonograph for \$375. Mr. Downing paid \$75 cash, and is to pay the balance in monthly installments of \$25 with 6% interest on the balance payable on the first of the month when each monthly installment is payable. You are to show a record of the transactions with Mr. Downing as they relate to the sale of this phonograph, showing the amount of each payment.

2. May 1, J. L. Jones purchased from the Citizens Motor Car Company a used automobile for \$1,000. He paid \$400 cash and the balance to be paid in monthly installments of \$100 with interest at 6% on the balance payable on the first of the month at the time each installment is paid. Show a record of the transactions and the amount of each payment.

3. May 12, C. C. Maynard purchased from the Doyle Realty Company a house and lot for \$10,000. He paid \$2,000 cash and executed sixteen notes for \$500 each due November 12 and May 12 respectively in each year, and sixteen interest notes. The first interest note was due November 12 and was for interest on \$8,000 at 6% for six months; the second interest note was due May 12 of the following year and was for interest on \$7,500 at 6% for six months; the third interest note was due on the following November 12, and was for interest at 6% on \$7,000 for six months. The other interest notes were for the interest due at the time of each semi-annual payment. You are to show on tablet paper all the calculations necessary to find the face of each interest note.

4. March 16, E. H. Love purchased a piano from the Baldwin Piano Company for \$2,100. He paid \$300 cash, and was allowed a credit of \$600 on his used piano accepted in exchange. The balance was to be paid in monthly installments of \$100 per month. At the time each monthly payment is paid on the piano, Mr. Love is to pay in addition to the \$100, interest on the balance at 6% for one month, a month to be regarded as $\frac{1}{12}$ of a year. The first installment is to be paid on June 1, and no interest is to be charged on the balance until May 1, interest beginning on that date. Find the amount of each monthly installment.

5. June 17, O. A. Muir purchased a piano from the Wurlitzer Piano Company for \$1,500. He paid \$300 cash and agreed to pay the balance of \$1,200 in monthly installments of \$50 each. At the time he pays each monthly installment, he is also to pay interest on the balance at 6% for $\frac{1}{12}$ of a year. The first monthly installment is due August 1, at which time he pays interest for one month on \$1,200. He pays the required monthly installment and interest August 1, September 1, and October 1. November 1, he pays \$200 and the required interest; December 1, \$150 and required interest; and January 1, the balance due on the piano

and the interest. You are to show all the calculations required to find the balance due, January 1.

6. March 1, Joseph A. Smith purchased a house and lot from the Brown Real Estate Company for \$12,500. He paid \$2,500 cash and executed four notes of \$2,500 each due in three, six, nine, and twelve months respectively; and four notes for the interest due at 6% at the time each of the four notes is due. The contract provides that Mr. Smith can pay as much as he desires in excess of \$2,500 at the maturity of any of the notes, and that proper credit is to be made on the interest notes affected. June 1, he paid the first \$2,500 note, the first interest note, and \$500 on the note maturing September 1. September 1, he paid the interest note due with the proper credit, the balance of the second note, and \$2,000 on the third note. December 1, he paid the balance on the third note, the balance of the interest due on this note, and \$2,000 on the fourth note. You are to show: (a) the amount of each of the four interest notes; (b) the amount of the credit due on the second interest note because of the excess payment on June 1; (c) the amount of the credit on the third interest note due to the excess payment, September 1; and (d) the amount of the credit on the interest note due to the excess payment, December 1; and (e) the amount paid, March 1.

NOTE: If you will prepare the eight notes, and make the credits as the payments are made, it will help you in solving this problem and others like it which occur quite frequently in business due to the deferred payment plan of selling real estate.

EXERCISE No. 176, ACCOUNTS RECEIVABLE

The merchant sells merchandise for cash and on account. A sale *on account* means that the customer to whom the merchandise is sold will pay for it either at a stated time or on the first of the following month. The *due date* of the purchase is indicated on the invoice sent to the customer at the time the merchandise is shipped or delivered to him. "Net 30 days" written after the terms indicates that the due date is 30 days from the date of the invoice; "net 60 days", that the invoice is due 60 days from the date of the invoice. If no specific time is stated in the terms, it is understood that the customer is to pay for the merchandise on the first of the following month.

A record of the sales on account to a customer and the cash received from the customer is made in account form. This account form is so arranged that sales and payments are known by their position. The illustration below shows a record of the transactions in account form with C. H. Middleton as it appears in the ledger of the Davis Manufacturing Company. The figures on the left of the double line in the center refer to sales and those on the right of the line to cash received from Mr. Middleton.

C. H. MIDDLETON

Mar.	1	30 days	\$287	60	April	1	Cash	\$200
Mar.	10	60 days	194	85	June	1	Cash	100
April	7	60 days	513	38	Aug.	1	Cash	300
May	2	90 days	407	20				

If Mr. Middleton wishes to pay this account in full September 1, and it is agreed that he should pay interest on past due bills, the custom is to add to the amounts on the left side of the account interest from the due date of each bill up to and including the date of settlement; and to add to the amounts on the right side interest from the date of each payment up to and including the date of settlement. The balance of the account is the difference between the total sales plus interest, and the total payments received from the customer plus interest.

Assuming that Mr. Middleton does pay this account on September 1, and that he agrees to pay interest on the past due bills at 6%, the method of finding the amount he owes is illustrated below.

Interest on \$287.60 from March 31 to September 1, 154 days, is \$7.38.

Interest on \$194.85 from May 9 to September 1, 115 days, is \$3.73.

Interest on \$513.38 from June 6 to September 1, 87 days, is \$7.44.

Interest on \$407.20 from July 31 to September 1, 32 days, is \$2.17.

Interest on \$200 from April 1 to September 1, 153 days, is \$5.10.

Interest on \$100 from June 1 to September 1, 92 days, is \$1.53.

Interest on \$300 from August 1 to September 1, 31 days, is \$1.55.

Total of the left side, \$1,403.03, + the interest, \$20.72, = \$1,423.75.

Total of the right side, \$600, + the interest, \$8.18, = \$608.18.

Balance due, September 1 = \$815.57.

You will observe that the due date of a bill is ascertained by adding to the date of the invoice the number of days in which the customer is to pay the bill. In other words, the bill is past due the day after the due date.

The following problems in account form provide written practice in applying the use of interest in settling accounts. When a bill is paid within its terms it is customary not to include interest on either the bill or the payment, but when payments are made

only on account or if a bill is paid in full a week or more after it is due, the interest is considered and calculated as in the illustration.

It is not always the best policy to insist on a customer paying interest on past due bills. When the agreement is to pay interest, the plan of settlement suggested is the most satisfactory one.

1.

C. J. LAWSON

Aug.	7	60 days	\$185	15	Oct.	1	Cash	\$200	15
Aug.	31	30 days	207	55	Nov.	1	Cash	150	75
Sept.	12	60 days	81	25	Dec.	1	Cash	300	25
Sept.	25	30 days	322	50					
Oct.	19	60 days	406	21					

Ascertain the amount due January 1, interest, 6%.

2.

W. B. WIBERLING

Jan.	2	60 days	\$321	20	Mar.	1	Cash	\$321	20
Feb.	12	60 days	255	40	April	1	Cash	150	40
April	19	60 days	422	75	June	1	Cash	100	60
May	31	60 days	718	91	June	18	Cash	422	75
July	25	60 days	275	55	Sept.	1	Cash	350	25
Sept.	10	60 days	192	90	Oct.	1	Cash	100	60
Oct.	1	60 days	255	07	Dec.	1	Cash	100	80

Ascertain the balance due January 1, interest at 6%.

3.

A. L. FIELD

July	2		\$ 39	87	Aug.	1	Cash	\$250	30
July	12		142	50	Sept.	1	Cash	250	50
July	20		78	91	Oct.	1	Cash	100	25
July	31		107	36	Nov.	1	Cash	100	75
Aug.	9		151	75					
Aug.	25		172	36					
Sept.	20		202	32					

It is desired to know the balance due on December 1, interest at $5\frac{1}{2}\%$, assuming that each invoice was due on the first of the following month after the sale, and interest is to be collected accordingly.

4.

J. C. BARCLAY

Mar.	7	30 days	\$250	15	April	6	Cash	\$250	15
April	10	30 days	327	50	May	1	Cash	150	90
May	25	30 days	500	35	June	1	Cash	200	85
June	17	30 days	181	25	July	1	Cash	100	60

It is desired to know the balance due September 1, interest at 8%.

5.

D. H. SCHURMAN

May	4	60 days	\$ 71	56	July	1	Cash	\$ 50	75
June	25	60 days	151	40	Sept.	1	Cash	100	15
July	31	60 days	192	27	Oct.	1	Cash	100	35
Sept.	9	60 days	201	21	Nov.	1	Cash	150	50
Oct.	7	60 days	89	56					

It is desired to ascertain the balance due January 1, interest at 6%.

CHAPTER XIX

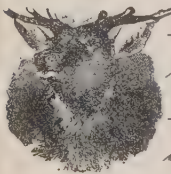
PRACTICAL PROBLEMS

The following problems provide written practice in applying the principles discussed in the preceding chapters. They include the application of the four fundamental operations in arithmetic to integers, common fractions, decimal fractions, percentage, and interest. You are to show on tablet paper all the calculations necessary to ascertain the desired results.

1. Smith Bros. owe the Regal Shoe Company \$2,762.85 but cannot pay the amount now. The Regal Shoe Company agree to allow 60 days provided Smith Bros. will give them a note for an amount which when discounted at their bank at 6% will produce the balance of the account. Smith Bros. accept the proposition and execute the note on May 16. Find the face of this note.

NOTE: When no reference is made to interest in connection with a note, it is to be interpreted as a non-interest bearing note.

2. The note shown below was discounted at the First National Bank, April 27, at 8%, and the proceeds placed to the credit of Mr. Ingleke. What amount was credited to his account?

	\$ 361.27	Denver, Colo.,	April 4, 19
	Three months after date I promise to pay to		
	the order of J. C. Ingleke		
	Three hundred sixty-one and - - - - - 27/100		
	Dollars		
at First National Bank			
Value received with interest at 6%			
No 123 Due		Laura Guildford	

It is the usual custom with banks to count the actual number of days between the date and maturity of a note to ascertain the maturity value, and the actual number of days between the date the note was discounted and maturity, in ascertaining the discount. This note matures on a holiday, hence cannot be collected until the following day.

3. In a certain school 4% of the total number of students were absent on January 26. There were 576 students present on that day. (a) What was the total enrollment, and (b) how many students were absent?

4. Distribute \$2,346.25 expenses over the following departments using the yearly sales expressed in hundred dollar units as a basis. Department A, \$67,896.23; B, \$55,234.89; C, \$45,123.45; D, \$19,456.34. The sales of the four departments are the total sales for the year.

In using \$100 units as a basis, treat Department A as \$67,900, Department B as \$55,200, Department C as \$45,100, and Department D as \$19,500.

5. January 16, O. C. Barber of Huntsville, Ala., borrowed \$500 from the First National Bank of Huntsville and gave in evidence of the loan an interest-bearing note due in 90 days. The note was not paid when due and legal proceedings were necessary to collect it. A judgment was awarded the First National Bank for the note and accrued interest on October 16 of the same year. What was the amount of this judgment if it was rendered for the present value of the note on that date, with legal interest from the date of the note up to and including the day the judgment was rendered, if the court costs were \$27.50?

6. G. B. Dougherty paid the R. & V. Motor Car Company \$2,532 for an automobile. This selling price included: (a) freight and tax, \$287; and (b) the dealer's commission of 20% on the selling price of the car at the factory. (a) How much did the R. & V. Motor Car Company pay the factory for this automobile? (b) How much commission did the dealer receive? (c) What per cent of the factory price was the freight and tax?

7. An automobile tire cost \$36.74 and was run 14,600 miles before it was discarded. What was the cost per mile in cents and fractions of a cent expressed in ten-thousandths?

8. November 2, C. E. Wright bought two \$1,000 Inland Steel Company $5\frac{1}{2}\%$ bonds at $95\frac{1}{2}$. He sold them November 2 of the following year for $98\frac{7}{8}$. (a) How much was paid for the bonds? (b) How much was received for them when sold? (c) What was the per cent of profit on the investment?

9. A boy had \$500 deposited to his credit in the Citizens Mutual Savings Bank on his tenth birthday. The bank pays $5\frac{1}{2}\%$, compounded semi-annually. What will be the amount to the young man's credit on his fifteenth birthday, the principal and interest having remained undisturbed in the bank?

10. The following commissions were paid during the business year: Brown, \$189.55 first quarter, \$202.10 second quarter, \$197.75 third quarter, and \$266.86 fourth quarter; Clark, \$245.50 first quarter, \$284.20, second quarter, \$288.45 third quarter, \$297.75 fourth quarter; Doyle, \$132.20 first quarter, \$128.70 second quarter, \$156.35 third quarter, and \$144.65 fourth quarter; Smith, \$200.55 first quarter, \$214 second quarter, \$224.25 third quarter, \$249.80 fourth quarter; Rey, \$185.60 first quarter, \$236.75 second quarter, \$202.45, third quarter, and \$110.80 fourth quarter; Jones, \$257.65 first quarter, \$240.25 second quarter, \$308.95 third quarter, \$311.30 fourth quarter. Tabulate this information so as to show: (a) each salesman's commission in each quarter; (b) total commission paid each salesman; (c) total commissions paid for each quarter; and (d) total commissions paid to all salesmen.

11. You are to ascertain the amount of the gas bill for the gas consumed; use the rates given in the schedule.

MARCH 2, 1926		
No Discount Allowed on this Bill after MARCH 8		1
To Union Gas & Electric Co., Dr. FOURTH AND PLUM STREETS		The J. F. Symmes Co. 109 West Vine St.
For Gas Service:		
Feb. 24	296000	
Jan. 25	261000	
Cubic Ft. Consumed 35000		
RATE SCHEDULE—EFFECTIVE JUNE 1, 1925 First 5000 Cu. Ft. @ 80c per M Cu. Ft. Next 5000 " " @ 75c " " " " " 5000 " " @ 70c " " " " " 5000 " " @ 65c " " " " " 5000 " " @ 60c " " " " All over 25000 " " @ 55c " " " " Subject to a discount of 5c per M cu ft., if paid during discount period. Minimum Charge 80c Gross, 75c Net, per Month.		
PLEASE BRING THIS BILL TO BE RECEIPTED. OFFICE HOURS: 8 A. M. to 5 P. M.; SATURDAY to 12 M. Closed on Legal Holidays.		

12. J. H. Johnson sold a house and lot to G. E. Smith for \$5,500. He received cash, \$3,800, the balance to be paid in

monthly installments of \$100 each on the principal, plus the accrued interest at 6% on the unpaid balance. Mr. Smith was given permission to pay the full amount due on the first of any month and receive a deed for the property. He paid the balance in full when making the sixth payment. What was the amount of that payment?

You may assume that each month is $\frac{1}{12}$ of a year and the first monthly payment includes the interest on \$1,700 for $\frac{1}{12}$ of a year, the second monthly payment includes the interest on the new principal for $\frac{1}{12}$ of a year, etc.

13. What will be the cost of covering a roof each side of which is 42 feet long, and 18 feet from the eaves to the ridge, at \$3.75 per square of 100 square feet for the material, and \$1.25 per square for labor?

14. A teacher offered a prize during the month of April, 1927, to three classes, to be awarded to the one which had the best percentage of attendance. The enrollment is as follows: Class A, 35; Class B, 31; Class C, 28. During the month of the contest Class A had 8 absentees; class B had 7 absentees; class C had 6 absentees. (a) Which class won the prize; and (b) what was the per cent of attendance in each class?

15. June 1, James Dooley purchased a home from Mayo Realty Co. for \$7,000. He paid \$2,500 cash and agreed to pay the balance in installments of \$75 per month. The monthly payments were to include 6% interest on the balance of the principal; by agreement the interest was to be paid quarterly on July 1, October 1, January 1, and April 1. Mr Dooley was to have the privilege of paying any amount he desired in excess of \$75 on July 1 and January 1. The following is a list of the payments made by Mr. Dooley: July, August, September, October, November, and December, \$75 each; January, \$500; February, March, April, May, and June, \$75 each; July, \$300; August, September, October, November, and December, \$75 each; January, in full. What amounts were paid on July 1 and January 1 of each year?

16. March 6, Robert Downing purchased from Moore Bros. on 30 days time merchandise to the value of \$412.85. This bill was not paid at maturity and it was necessary to bring suit to make collection. The court rendered judgment on October 12 of the same year for the amount of the claim and interest at 6% from maturity. What was the amount of the judgment, if it included court costs of \$21.80?

17. How long will it take (a) \$3,627.92 to produce \$55.63 interest at 6%, (b) at 8%, and (c) at 4%?

18. The annual dues in a certain association are \$15, and life membership dues, \$150. How long must a life member live to secure his annual membership at the same rate as those who pay by the year provided money is worth 6%?

19. A. C. Clark purchased a new automobile from the Model Motor Car Company for \$2,400. He paid \$1,000 in cash and contracted to pay the balance in monthly installments of \$200, payable on the first of each month. At the time he paid each monthly installment, he was to pay in addition to this the interest at 6% on the balance. (a) What was the amount of each monthly payment, and (b) how many months did it take him to pay the balance due?

20. A merchant lost 12% of his capital through the failure of one debtor, $7\frac{1}{2}\%$ of the remainder through the failure of another debtor, and had \$60,500.55 left. What was his original capital?

21. The following account appears in the ledger of Roe Bros.

A. L. ROBINSON

Mar.	1	30 days	\$137	85	Mar.	31	Cash	\$137	85
Mar.	20	60 days	208	12	June	1	Cash	100	
April	10	30 days	185	16	July	1	Cash	200	
May	12	60 days	321	90	Aug.	1	Cash	100	

October 1, Mr. Robinson gave a 90-day non-interest-bearing note for an amount which when discounted at 8% would pay the balance of this account with interest at 6%. What was the face of this note?

22. J. T. Burnet owned four notes of \$500 each, received by him in payment of a house and lot sold to E. H. Fagley. Each note was dated January 1, and they were due in six, twelve, eighteen, and twenty-four months respectively with interest at 6%. Mr. Fagley had the privilege of paying more than \$500 at the maturity of any one of the notes. July 1, when the first note was due he paid it in full. January 1, when the second note was due, he paid it in full and \$100 on the third note. July 1, when the third note was due, he paid the balance on it, and \$200 on the fourth note. January 1, when the fourth note was due, he bor-

rowed sufficient money from his bank to pay the balance due Mr. Burnet. You are to show on tablet paper: (a) the amount paid at the time the first note was due; (b) the amount paid at the time the second note was due; (c) the amount paid at the time the third note was due; (d) the amount of the ninety-day note which Mr. Fagley gave to his bank to secure funds to pay the balance on the fourth note, the face of this note being for an amount which, when discounted at 8% for ninety days would produce the amount due Mr. Burnet.

23. During a business year, the facts regarding purchases and sales as recorded at the Fair Company were as follows: Beginning inventory, \$8,627.92; Purchases during the year, \$12,444.81; Freight cost, \$867.42; Sales during the year, \$21,648.55; Merchandise returned by customers, \$375.80; Inventory of merchandise on hand at the end of the year, \$8,221.65. (a) What was the profit on sales, and (b) the per cent of profit based on net sales?

Net sales is the gross sales less returns by customers.

24. T. A. Bauer and W. J. Smith bought an apartment house containing 21 apartments, for \$175,000. They paid \$75,000 cash and executed a mortgage for \$100,000 payable in ten years with interest at 6%, payable July 1 and January 1 of each year. Nine of the apartments rent for \$85 each per month, six, for \$100 each, and six, for \$125 each per month. The cost for janitor service, heating, repairs, taxes, insurance, etc., amounts to \$10,225 a year. What is the net rate of income on the purchase price?

25. The Hillside Creamery, buying milk during the month of April received the following supply: R. A. Smith, 546 lbs.; H. L. Brooks, 1,595 lbs.; R. W. Downing, 2,380 lbs.; C. C. Davis, 4,540 lbs.; J. M. Jones, 3,150 lbs. The creamery paid for the milk at the rate of $32\frac{1}{2}$ c per pound of butter fat. In milk tests the milk was found to grade as follows: R. A. Smith, 3.96% butter fat; H. L. Brooks, 3.87%; R. W. Downing, 3.65%; C. C. Davis, 3.88%; J. M. Jones, 3.72%. (a) How much did each farmer receive, and (b) how much did the creamery pay for all the milk?

26. At the present time I am earning \$2,500 a year and am living in a 3-family house paying \$50 per month rent. I have been considering for some time the advisability of buying a home but have only \$3,000 in available funds. I have been looking

at a house which I can buy for \$6,000. The Co-operative Bank agrees to lend me the additional \$3,000 at $6\frac{1}{2}\%$. The assessed valuation on the house and lot is \$4,500; tax rate, \$24 per thousand; water rate, \$15 per year; and insurance on the house, \$25 per year. How much will it cost me per month to own this house, allowing me 6% interest for my \$3,000?

27. C. C. Carpenter presented his daughter with ten \$1,000, 10-year, $5\frac{1}{2}\%$ bonds on her tenth birthday, July 1. How much will she have at maturity of these bonds which will be her twentieth birthday, if the semi-annual interest payable July 1 and January 1 is deposited in a bank where interest at 5% compounded annually is allowed, the principal and interest remaining in the bank undisturbed until the end of the 10 years?

28. What amount will Mr. Phillip receive for note shown below if it is discounted at the First National Bank July 1 at 8% ?

\$ 427.80	EASTON, PA.	May 16, 192
	Ninety days	after date I promise to pay to
	the order of	L. A. Phillip
	Four hundred twenty-seven and	80/100 Dollars
	at First National Bank	
	Value received	with interest at 6%
No 146	Due	G. C. Atwater

29. August 6, the Miller Bros. Co. sold A. L. Jenkins merchandise for \$781.65. August 10, Mr. Jenkins accepted a 90-day trade acceptance for an amount which when discounted at 6% would pay the amount of this bill less 2% . What was the face of the trade acceptance?

30. The Jones Brick Company of Oakland, California, drew a 60-day sight draft, dated August 20, on A. C. Brown of Cleveland for \$375.50, balance of Brown's account. Brown accepted the draft, August 25. September 1, the Jones Brick Company discounted this draft at their bank at 8% . What were the proceeds?

31. The Daisy Manufacturing Company holds a note of C. W. Worth & Company for \$2,000 due in one year from September

5, interest at 7% from date. January 1, of the next year, The Daisy Manufacturing Company sells this note to a note broker at a discount of 8%. How much was received from the broker?

32. The Lyons Company paid \$3,000 for a delivery truck January 22. At the end of the first year it was depreciated 25%, the second year, 20%, and the third year, 15%. At the beginning of the fourth year it was exchanged for a new truck with an exchange allowance of \$1,000. (a) What was the amount of the depreciation; (b) the difference between the exchange value and the present value; and (c) what per cent of the cost is the difference between the present value and the exchange value?

33. J. H. Snyder purchased a radio for \$350 including the cost of installation. He paid \$50 cash and the balance, \$25 per month on the first of each month; interest at 6% on the principal was paid in addition to the monthly payments. You are to ascertain: (a) the amount paid each month; and (b) the total cost of the radio outfit including the purchase price and interest.

34. May 12, James Slocum paid \$5,400 for an automobile. He sold it on April 12 of the fourth year for \$1,800. (a) What was the average monthly cost because of the depreciation? (b) What was the average monthly interest cost, money being worth 6%?

35. A bank discounted the following non-interest-bearing notes on May 2, rate of discount 6%:

Date	Time	Face
1. April 1	60 days	\$5,000
2. May 2	60 days	2,000
3. April 16	90 days	1,200
4. April 2	60 days	2,500
5. April 28	30 days	4,000
6. April 26	90 days	3,000
7. April 17	30 days	5,000

You are to ascertain: (a) the discount on each note; (b) the proceeds of each note; (c) the total discount; (d) the total proceeds; and (e) the total amount to be collected at maturity.

36. If the rate is $9\frac{1}{2}\%$ and the percentage is \$126.55, what is the base?

37. If the base is \$2,521.25 and the rate is $7\frac{1}{2}\%$, what is the percentage?

38. If the base is \$1,645 and the percentage is \$451.42, what is the rate?

39. The monthly rent of the McAlpin Store is \$5,525. It is desired to distribute this over five departments on a basis of square feet as follows: Department A, 3,500 square feet; Department B, 2,500 square feet; Department C, 3,750 square feet; Department D, 1,250 square feet; Department E, 2,000 square feet. You are to ascertain the amount of rent chargeable to each of the five departments.

40. The First National Bank offered to discount this note on August 1 at 8% . The discount was to be calculated on the maturity value of the note counting the actual number of days between the date and maturity. The Citizens Bank offers to discount the note at the same rate and consider the maturity value as the face plus interest for 4 months, or $\frac{1}{3}$ of a year. You are to ascertain the proceeds of the note if discounted: (a) at the First National Bank; (b) at the Citizens Bank; and (c) the difference between the two offers.

	\$ 2000.00	MINNEAPOLIS, MINN.,	July 1, 19
	Four months	after date I promise to pay to	
	the order of	C. C. Duggan	
	Two thousand	Dollars	
	at	First National Bank	
	Value received with interest at 6%		
No. 221	Due	M. D. Person	

41. December 9, the A. W. Shaw Co. sold Donaldson Bros. merchandise for \$287.38. December 12, Donaldson Bros. accepted a 60-day trade acceptance for an amount which when discounted at 8% , would pay the amount of this purchase. You are to ascertain the face of the trade acceptance.

42. The population of a certain city was 42,500 at one census period and 45,000 at the next period. (a) What was the increase of population; and (b) the per cent of increase?

43. The freight rate between two points when shipped by rail is \$1.275 per cwt. and when shipped by water, \$1.1275 per cwt. (a) What is the saving in the water rate on a shipment of 5,280 pounds; and (b) what is the per cent of saving?

44. A. L. Hodge, a traveling salesman, received \$9,876.55 as compensation for his service in a certain year. What were his total sales if he receives $7\frac{1}{2}\%$ as his commission?

45. How much would A. L. Peterson, a salesman, receive as his compensation, if he was allowed $9\frac{1}{2}\%$ on his sales and the total volume of sales during a given period was \$13,781.65?

46. James Brown borrowed from the Central Trust Company \$25,000 giving as security a mortgage on a flat building which he owned. He is to pay 5% interest semi-annually and to execute a note for the principal due in 5 years, and 10 notes each for the semi-annual interest on the entire loan. You are to ascertain: (a) the face of each interest note; and (b) the total amount of interest paid for this loan.

47. R. A. Albers purchased the property at 916 Mason Street for \$12,500. He paid \$4,500 cash and assumed a mortgage of \$8,000, with interest at 7%. The property is on the tax list at \$9,000 and the tax rate is \$27.50 per thousand. The insurance on the house is \$45 per year. If he rents the house for \$100 per month, what per cent of income will he receive on the purchase price?

48. J. J. Alderson inherits the property at 729 May Street. This property contains six apartments which rent as follows: Apartments A, B, and C, \$85 each per month; Apartments D and E, \$75 each; and Apartment F, \$65. The average expenses of maintaining the property during the past 5 years has been \$52.50 per month. What should be the market value of this property if Mr. Alderson wishes his money to earn 8%?

49. L. D. Mason paid \$10,000 cash for a farm. How much rent per year should he receive for the farm if the only expenses are the taxes of \$175.60 and he desires that his money earn $7\frac{1}{2}\%$?

50. The market quotation of corn on January 9, was $67\frac{3}{8}$ c per bushel, and on February 9, $66\frac{7}{8}$ c per bushel. If A. J. Jardine, a farmer, had 1,287 bushels sacked ready to sell on January 9, but held it for a higher price and finally sold it February 9 at the market quotation on that date, how much did he (a) receive for the corn; (b) lose by holding it; and (c) what was the per cent of his loss?

CHAPTER XX

BANKING AND INTEREST

As explained in Chapter II, the principal income of a bank is the interest received for the use of the money which belongs to the bank, or to its depositors who have left it with the bank for safe-keeping. The interest earned by a bank does not differ from the interest earned by an individual; it is income earned through the use of money loaned.

The method of ascertaining the time for which the money has been used is usually the same when the money is loaned by a bank, as when it is loaned by an individual. This time is the number of days the money has been used, omitting the day on which the money is borrowed and counting the day on which it is paid or the loan renewed. Loans are made by banks for only a short time, usually 30, 60, or 90 days. Some banks collect the interest in advance, while others collect it at maturity, but this does not change the amount of the interest.

The bank will require a written evidence of a loan. This is usually in the form of a promissory note or accepted draft. A promissory note is illustrated on page 250; an accepted draft will be illustrated later.

The *face* of a note is the amount stated in it. The *maturity value* is the amount that can be collected at maturity. A note containing the words "with interest" is interest-bearing; if "with interest" is not contained in the note, it will not bear interest until after maturity. The rate of interest is determined by contract, or by the law of the state in which the note is issued; unless the rate is specified in the note, the legal rate fixed by the state applies.

The maturity value of a \$500 non-interest-bearing note, dated May 23 and due in 90 days is \$500 on August 21. The face value of the note up to and including August 21 is \$500. If the legal rate of interest in the state in which it is executed is 6%, the face value of this note, 30 days after maturity, will be \$502.50; 60 days after maturity, \$505.

\$500.00	Kansas City, Mo.	Mar 25 192
	Ninety days after date I promise to pay to	
	the order of Merchants National Bank	
	Five Hundred Dollars	
at	Value received with interest at 6%	
No. 5	Due June 23	W. C. Williamson

This note may be interpreted as follows: W. C. Williamson, known as the *maker*, borrowed \$500 from the Merchants National Bank, the *payee*, March 25, and agreed to pay the \$500 with 6% interest, 90 days from the date he borrowed it, or on June 23. This means that Mr. Williamson will pay the bank \$507.50 on June 23, the \$500 he borrowed, and \$7.50 for its use.

The maturity value of an interest-bearing note is the face, plus the interest at maturity. An interest-bearing note bears interest to maturity at the contract rate specified in the note, and thereafter at the legal rate in the state in which it is issued.

The face of a \$500, 6% interest-bearing note, dated June 10, and due in 90 days, is \$500. The maturity value is \$507.50, the face of the note, plus \$7.50 interest.

One who wishes to borrow money should obtain it from a bank, because it is the bank's business to lend money. It is just as inconsistent to ask a friend or relative who is not in the drug business, for a loan of drugs, as it is to ask a friend or relative who is not in the banking business, for a loan of money. No higher tribute can be paid to any young man or young woman than to have an established credit at a bank, a credit which will secure a loan when needed.

Business concerns desiring a loan are given a credit rating, based on the net value of the property owned by the business, and the financial standing of the business. Member banks of the Federal Reserve System are required to have on file a financial statement from each business concern to whom they lend money similar to that in the illustration on page 251. The information in this form is secured from the accounting records of the business concern and must be correct. Should a business concern establish a credit rating with a bank on a statement which did not contain

FINANCIAL STATEMENT To THE FIFTH-THIRD NATIONAL BANK

(Corporation)

CORPORATE NAME The Keystone Manufacturing CompanyBUSINESS Manufacturers of automobilesMAIN OFFICE Detroit, MichiganLOCATION OF PLANT Detroit, Mich.BRANCHES Cincinnati, Ohio St. Louis, Mo.

FOR THE PURPOSE OF PROCURING CREDIT FROM TIME TO TIME WITH YOU, FOR OUR NEGOTIABLE PAPER OR OTHERWISE, WE FURNISH THE FOLLOWING AS A TRUE AND CORRECT STATEMENT OF OUR FINANCIAL CONDITION ON December 31 19.....

AND HEREBY AGREE TO NOTIFY YOU IMMEDIATELY IN WRITING, OF ANY MATERIALLY UNFAVORABLE CHANGE IN OUR FINANCIAL CONDITION; IN THE ABSENCE OF SUCH NOTICE OR A NEW AND FULL WRITTEN STATEMENT, THIS IS TO BE CONSIDERED AS A CONTINUING STATEMENT, AND THAT OUR PECUNIARY RESPONSIBILITY HAS NOT FALLEN BELOW THE CONDITION HEREIN SET FORTH AND IT IS HEREBY EXPRESSLY AGREED THAT UPON APPLICATION FOR ADDITIONAL CREDIT OR RENEWAL THIS STATEMENT AND INFORMATION SHALL HAVE THE SAME FORCE AND EFFECT AS IF DELIVERED AS AN ORIGINAL REPORT OF OUR FINANCIAL CONDITION AT THE TIME SUCH ADDITIONAL CREDIT OR RENEWAL IS REQUESTED.

ASSETS				LIABILITIES			
CASH ON HAND		105	00	BILLS PAYABLE (NOTES FOR MERCHANDISE)		5	000 00
CASH IN BANK		7	845 00	BILLS PAYABLE FOR BORROWED MONEY TO OWN BANKS			
BILLS RECEIVABLE OF CUSTOMERS, DUE WITHIN 90 DAYS		6	805 18	BILLS PAYABLE FOR BORROWED MONEY FOR PAPER SOLD			
BILLS RECEIVABLE OF CUSTOMERS, DUE BEYOND 90 DAYS		1	444 82	BILLS PAYABLE TO DIRECTORS OR STOCKHOLDERS			
ACCOUNTS RECEIVABLE AT A GOOD, OWING FROM CUSTOMERS		25	000 00	OPEN ACCOUNTS NOT DUE		15	000 00
MERCHANDISE, FINISHED, AT ACTUAL PRESENT CASH VALUE		20	000 00	OPEN ACCOUNTS PAST DUE			
MERCHANDISE, UNFINISHED, AT ACTUAL PRESENT CASH VALUE		20	000 00	ACCOUNTS OWING DIRECTORS OR STOCKHOLDERS			
RAW MATERIAL		18	000 00	DEPOSITS AND OTHER TRUST FUNDS			
TRADE ACCEPTANCES OWNED		2	290 00	TRADE ACCEPTANCES PAYABLE		1	107 45
				Accrued Payroll		1	592 55
				Accrued Interest		8	000 00
TOTAL QUICK ASSETS		99	490 00	TOTAL CURRENT LIABILITIES		30	700 00
STOCKS, BONDS AND INVESTMENTS				MORTGAGES AND LIENS ON REAL ESTATE DUE			
MACHINERY AND FIXTURES		98	500 00	BONDS DUE		116	000 00
REAL ESTATE		122	500 00	CHattel MORTGAGES			
ACCOUNTS AND BILLS RECEIVABLE OWING FROM OFFICERS AND EMPLOYEES				DUE TO CONTROLLED OR ALLIED CONCERNS, For Merchandise			
BILLS RECEIVABLE OWING FROM DIRECTORS OR STOCKHOLDERS				DUE TO CONTROLLED OR ALLIED CONCERNS, For Advances			
ACCOUNTS RECEIVABLE OWING FROM DIRECTORS OR STOCKHOLDERS		1	000 00				
DUE FROM CONTROLLED OR ALLIED CONCERNS, FOR MERCHANDISE							
DUE FROM CONTROLLED OR ALLIED CONCERNS, FOR ADVANCES				CAPITAL STOCK OUTSTANDING PREFERRED			
NOTES AND ACCOUNTS RECEIVABLE, 6 MONTHS OR MORE PAST DUE		1	235 65	CAPITAL STOCK OUTSTANDING COMMON		150	000 00
Deferred Charges			764 35	SURPLUS		12	790 00
Goodwill		50	000 00	RESERVES { UNDIVIDED PROFITS		40	000 00
				DEPRECIATION		24	000 00
TOTAL		373	490 00	TOTAL		373	490 00

I, HEREBY SOLEMNLY DECLARE AND CERTIFY THAT THE ABOVE IS A TRUE STATEMENT OF OUR BUSINESS.

CORPORATE NAME Keystone Manufacturing Co.DATE SIGNED January 25

19.....

BY J. W. Baker

(Officers Title Must Be Given)

President

NOTE--IF YOU HAVE ANY DIFFICULTY IN FILLING IN THE FOREGOING STATEMENTS AND QUESTIONARY, THE CREDIT DEPARTMENT OR ANY OTHER OFFICER OF THIS BANK IS PREPARED TO EXTEND YOU COURTEOUS AND INTELLIGENT SERVICE, IN FACT, WE SOLICIT YOUR CO-OPERATION AND ACQUAINTANCE WITH OUR CREDIT METHODS.

Financial statement submitted to a bank for financial rating.

facts, and fail to pay its obligations, the individual responsible for this incorrect statement would be subject to prosecution.

When the financial rating of a business or an individual cannot be established, or when either has borrowed in excess of the credit rating, the bank may make a loan on proper security. This security may be personal endorsement, or collateral. Personal indorsement is effected by the borrower having someone to sign with him on the face of the note or by signing his name on the back of the note. Collateral security is effected by the borrower depositing with the bank, notes, shares of stock, bonds, or other intangible property as security for the loan.

EXERCISE No. 177, MATURITY AND PRESENT VALUE

The only profit accruing to a bank through a loan of money is the interest collected at the time the loan is made or at maturity. The bank is interested in knowing (a) the maturity of the loan, and (b) the amount that will be collected at maturity, which includes the principal, and the interest if this were not included at the time the loan was made.

The maturity of a note is ascertained by adding to the date of the note, the number of days or months stated in it.

The following problems provide oral practice in determining the maturity, present value, and maturity value of a note; *present value* is the amount the borrower receives. In each of these problems it is assumed that the note is received by the bank as evidence of a loan made on the date of the note, and that when the note is interest-bearing, the borrower received the face of the note; if it is non-interest-bearing, the borrower received the face of the note less the interest at the agreed rate for the number of days stated in the note.

	FACE	DATE	TIME	MA- TURITY	INT. RATE	PRESENT VALUE	MATURITY VALUE
1.	\$ 500.00	May 11	60 days				
2.	300.00	Mar. 6	90 days		6%		
3.	1,000.00	April 1	60 days				
4.	600.00	July 2	90 days		6%		
5.	400.00	July 19	60 days				
6.	250.00	Nov. 16	90 days				
7.	100.00	Oct. 3	90 days				
8.	9,000.00	Sept. 1	90 days				
9.	700.00	June 2	60 days		8%		
10.	500.00	May 7	90 days		6%		
11.	800.00	Oct. 3	90 days				

EXERCISE No. 178, INTEREST ON BANK LOANS

The following problems provide additional oral practice in ascertaining the amount of money a borrower will receive on the paper given to a bank as evidence of a loan. As stated in the preceding exercise, some banks collect the interest in advance, hence accept non-interest-bearing notes, while other banks collect the interest at maturity, hence accept interest-bearing notes.

1. James Brown applied to the First National Bank for a loan of \$400. He was told to present his 60-day note, with interest at 6% from date, for this amount. He presented the note, May 19 and received \$400 cash. When was the note due, and how much was he required to pay on the due date?

2. Robert Downing presented to the City National Bank his note for \$800, dated June 17, due in 90 days, and received cash from the bank, less the interest at 8% for 90 days. (a) How much cash did he receive from the bank, (b) what was the maturity date of the note, and (c) what was the maturity value of the note?

3. W. L. Douglas applied to the Citizens National Bank for a loan of \$1,000 and was told to present his note, with interest at 6%, payable in 90 days. He presented the note on November 4 and received \$1,000 cash. (a) When did this note mature, and (b) what amount did he owe the bank on that date?

4. March 5, C. L. Lloyd borrowed from the Merchants National Bank \$2,500 on his 90-day note. The bank gave him cash for the face of the note, less interest at 6% for 90 days. (a) How much cash did he receive, (b) when did the note mature, and (c) how much cash did he pay the bank at maturity?

5. May 27, James King borrowed \$250 from the Union Trust Company on his 60-day interest-bearing note. He received \$250 cash from the bank. When did this loan mature and what amount of cash did he pay the bank at maturity provided the rate of interest in the note was 9%?

6. May 17, the Union Manufacturing Company borrowed \$10,000 from the Second National Bank at 5% for 90 days. They issued a note in favor of the bank for \$10,000 and received cash for the difference between the face of the note and the interest agreed upon. (a) How much cash did they receive from the bank? (b) When did the loan mature?

7. D. H. Mosier borrowed \$100 from the Citizens Bank and Trust Company. He gave his 30-day note payable to the bank for this amount, and received cash, less the interest for 30 days, at 8%. How much did he receive?

8. Albert Donaldson arranged with the Third National Bank for a loan of \$5,000, on his 90-day note, with interest at 6%. He presented his note to the bank on May 16, and received \$5,000 cash. At maturity, he paid half of the note and interest, and renewed the other half on a non-interest-bearing note due in 90 days and paid the interest in advance. (a) When was the first note due; (b) how much interest did Mr. Donaldson pay the bank at the time the second loan was made; (c) what was the amount of the new note given to renew a part of the first note; (d) how much interest did Mr. Donaldson pay on this new note; (e) how much cash did he pay the bank at the maturity of the first loan?

EXERCISE No. 179, INTEREST ON BANK LOANS

When a note is given as evidence of a loan, the borrower or maker of the note receives cash or credit for the face value, provided it bears interest. If it does not bear interest he receives cash or credit for the face of the note less the agreed interest for the time it has to run. This condition applies only when the note is given in evidence of a loan and not where a note of another is discounted at the bank.

March 13, James Brown borrowed \$375.50 from the City National Bank and gave in evidence of this loan a 90-day note with interest at 8%. The calculations below show one method of ascertaining: (a) the due date, and (b) the amount Mr. Brown will pay the bank at the maturity of the note.

31 - 13 = 18	remaining days in Mar.	Face of note, \$375.50	
April	30 days	3.7550	Int. 8% for 45 days
May	31 days	3.7550	Int. 8% for 45 days
June	11 days	7.5100	Int. 8% for 90 days
	90	375.50	Cash rec'd by borrower
		383.01	Maturity value

The borrower receives the face of the note, because it was interest-bearing. At maturity he pays the bank the face of the note and interest at the agreed rate. Had the borrower given a non-interest-bearing note as evidence of the indebtedness, he would have received \$367.99 and paid \$375.50 at maturity.

The following problems provide written practice in discounting notes. The note in each problem was given as evidence of a loan made on the date mentioned. The borrower received the face of the note, if it was interest-bearing; if non-interest-bearing, the face of the note less interest at 8% for the time.

1. Face of note, \$273.50, dated March 9 for 90 days, rate of interest, 8%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

2. Face of note, \$542.50, dated May 1 for 60 days, rate of interest, 7%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

3. Face of note, \$85.50, dated July 1 for 120 days, rate of interest, 9%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

4. Face of note, \$2,500, dated August 1 for 75 days, rate of interest, 5%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

5. Face of non-interest-bearing note, \$752.65, dated March 1 for 90 days. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

6. Face of note, \$96.95, dated July 12 for 66 days, rate of interest, 8%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

7. Face of non-interest-bearing note, \$726.25, dated March 5 for 60 days. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

8. Face of note, \$244.55, dated January 1 for 30 days, rate of interest, 8%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

9. Face of non-interest-bearing note, \$1,500, dated March 15 for 120 days. Desired to know: (a) the due date; (b) amount

of interest; (c) cash received by the borrower; and (d) amount due at maturity.

10. Face of note, \$400, dated May 7 for 120 days, rate of interest, 9%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

11. Face of note, \$204.50, dated June 14 for 60 days, rate of interest, 7%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

12. Face of note, \$1,200, dated May 1 for 120 days, rate of interest, 7%. Desired to know: (a) the due date; (b) amount of interest; (c) cash received by the borrower; and (d) amount due at maturity.

EXERCISE No. 180, INTEREST ON BANK LOANS

The following problems provide additional written practice in ascertaining the information needed by a bank, (a) at the time a loan is made, and (b) at maturity. Arrange your work on tablet paper in seven columns showing all the facts given below and in addition the desired facts in the columns that are blank. Use 7% in finding the present value of the non-interest-bearing notes.

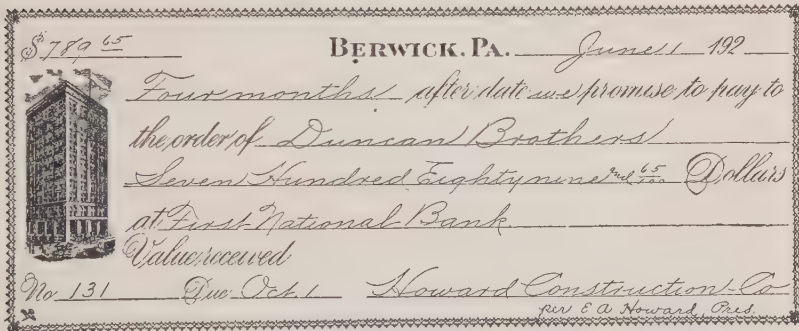
	FACE	DATE	TIME	MA- TURITY	INT. RATE	PRES- ENT VALUE	MA- TURITY VALUE
1.	\$1,245.55	Sept. 21	30 days		6%		
2.	225.45	Nov. 15	90 days				
3.	2,110.50	May 1	60 days		5%		
4.	3,500.15	Jan. 12	90 days		8%		
5.	550.65	Mar. 15	30 days				
6.	1,035.16	June 5	60 days		6%		
7.	672.35	Aug. 16	90 days		6%		
8.	2,156.45	Sept. 12	30 days				
9.	1,123.65	Feb. 12	90 days		7%		
10.	335.15	Dec. 13	60 days				
11.	125.16	April 8	30 days		6%		
12.	3,412.76	May 11	90 days				
13.	1,444.25	Mar. 3	60 days		6%		
14.	655.75	Dec. 15	30 days				
15.	225.50	June 10	30 days				
16.	1,006.75	July 6	60 days		6%		

EXERCISE No. 181, DISCOUNTING NON-INTEREST-BEARING NOTES

It is sometimes necessary and often advisable for a merchant or manufacturer to accept notes from customers in payment of merchandise sold, or in settlement of an account, also to discount such notes at the bank. Discounting the note of another at the bank is the same process as borrowing on one's own note, except it is necessary to determine the present value of the note at a time other than the date of maturity.

The bank receives interest for the use of its money; this interest is calculated on the amount due at maturity, and for the number of days the bank must wait to collect the loan.

Duncan Bros., wholesale hardware merchants, received from the Howard Construction Company the note illustrated below. It is desired to borrow from the City National Bank the present value of the note on July 1, less 6%. The calculations necessary to ascertain the amount of cash Duncan Bros. will receive from the bank as a loan on this note are given below the note.



Amt. due at maturity, Oct. 1,	\$789.65
Time from July 1 to Oct. 1, 92 da.	
6% Int. on \$789.65 for 92 days,	12.11
Present value of note July 1,	\$777.54

The note does not bear interest, hence only the face value can be collected at maturity. The number of days between July 1 and October 1 is ascertained by counting the actual number

of days as in previous exercises. The amount due at maturity, less 6% interest from the date of discount to the date of maturity is the present value of the note, which is the amount Duncan Bros. will receive in cash or have credited to their account.

The notes illustrated on this, and the following pages provide problems for written practice in ascertaining the present value of a non-interest-bearing note which is discounted at a date subsequent to the date of the note. The present value of the note in each case will be the face, less the interest for the number of days between the discount date and the maturity date. You are to show on tablet paper the calculations necessary to ascertain the desired information in each note, arranging your work in the same order as illustrated on page 257.

\$361 ⁰⁰	DOVER, OHIO,	May 11 192
	Ninety days after date I promise to pay to	
	the order of Adler Bros.	
	Three Hundred Sixty-one ⁰⁰ / ₁₀₀ Dollars	
	at Fifth Third National Bank	
Value received		
No 12	Due	James A. Bushbach

1. You are to ascertain the amount of credit received by Adler Bros. if this note was discounted on (a) June 16, less 8% discount, (b) July 1, less 7% discount, or (c) July 15, less 6% discount. The maturity date of this note is 90 days after May 11, counting the actual days.

\$250 ⁰⁰	EVANSVILLE, IND.,	Oct 4 192
	Four months after date I promise to pay to	
	the order of E B Thomas	
	Two Hundred Fifty ⁰⁰ / ₁₀₀ Dollars	
	at Citizens Bank & Trust Co	
Value received		
No 111	Due	Paul A. Kiltman

2. You are to ascertain the amount of cash received by Mr. Thomas if this note was discounted at the bank on (a) Decem-

ber 1, less 6% discount; (b) December 24, less 7% discount, or (c) January 6, less 8% discount.

A note due 4 months after date is due on the same day of the fourth month after the one in which it is dated.

\$122.50 Cincinnati, Ohio, Mar. 1, 1912

Sixty days after date I promise to pay to the order of G. L. Flanigan One Hundred Twenty-two and 5/100 Dollars at Fourth National Bank Value received

No. 12 Due L. M. Guilfoile



3. You are to ascertain the amount of cash received by Mr. Flanigan if this note was discounted at the bank on (a) March 5, at 9%; (b) April 1, at 7%; or (c) April 5, at 8%.

\$72.15 LYNCHBURG, VA., Mar. 16, 1912

Ninety days after date I promise to pay to the order of E. R. Duncan Seventy-two and 15/100 Dollars at Fifth Third National Bank Value received

No. 21 Due Harrison A. Moore



4. You are to ascertain the amount of cash received by Mr. Duncan if this note was discounted at the bank on (a) April 1, at 6%; (b) May 1, at 8%; or (c) May 31, at 9%.

\$101⁰⁰ YORK, PA. June 5 1912
 Three months after date we promise to pay to
 the order of Johnson Bros.
 One Hundred One and ⁰⁰/₁₀₀ Dollars
 at their office
 Value received The Moores Coney Co.
 No. 101 Due per E. A. Bock

5. You are to ascertain the amount of cash Johnson Bros. received if this note was discounted at the bank on (a) July 3, less 7% discount; (b) July 25, less 8% discount; or (c) July 30, less 9% discount.

\$75⁰⁰ HUNTINGTON, W. VA. Mar 1 1919
 Six months after date I promise to pay to
 the order of A. F. Harman
 Seventy-five Dollars
 at Provident Savings Bank
 Value received.
 No. 23 Due Donald L. Baughle

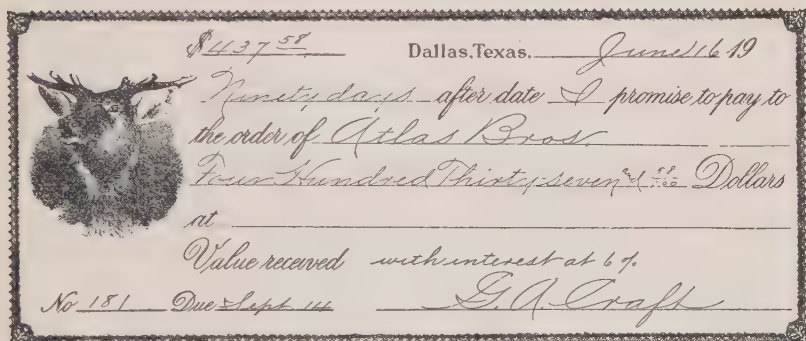
6. You are to ascertain the amount of cash that would be received from the bank by Mr. Harman if this note was discounted on (a) March 10, at 6%; (b) April 1, at 8%; or (c) May 1, at 7%.

EXERCISE No. 182, DISCOUNTING INTEREST-BEARING NOTES

When an interest-bearing note is discounted, it is necessary to ascertain the maturity value by adding to the face of the note,

the interest for the number of days between the date and maturity of the note at the designated rate. It is necessary to know the maturity value, because the present value is always calculated on the maturity value.

Atlas Bros. received from G. A. Craft the note illustrated below and discounted it at 8% at the Merchants National Bank on July 2. The calculations necessary to ascertain (a) the maturity value, and (b) the present value are shown below.



Face of the note, \$437.58
 Interest at 6% for 90 days =
 \$6.56

Maturity value = \$437.58 +
 \$6.56 = \$444.14

Date of note, June 16

Due date (14 + 31 + 31 + 14)
 September 14

July 2 to September 14 = (29 +
 31 + 14) 74 days

Interest on \$444.14 for 74
 days at 6% ($.44414 \times 12\frac{1}{3}$) =
 5.47772

8% = $1\frac{1}{3}$ times 6%
 $5.47772 \times 1\frac{1}{3}$ = \$7.30 the dis-
 count

\$444.14 - \$7.30 = \$436.84 =
 cash received by Atlas Bros.

The first information necessary is the maturity value of the note, which is the face of the note plus 6% interest for 90 days. The number of days the bank will have to wait before maturity is the difference in time between July 2 and September 14, or 90 days minus the time between the date of the note and the date discounted. The present value of the note is the difference between the maturity value of the note and the interest on the maturity value at 8% for the number of days Atlas Bros. will use the money before the bank makes collection.

\$112²¹ ASHLAND, PA. Oct 1 19



Ninety days after date I promise to pay to
the order of R. R. Hardin
One Hundred Twelve^{and 21/100} Dollars
at Merchants National Bank
Value received with interest at 6%
No. 57 Due E. W. Church

1. You are to ascertain the amount of cash Mr. Hardin would receive from the bank if this note was discounted (a) October 15, at 8%; (b) November 1, at 7%; or (c) December 11, at 9%.

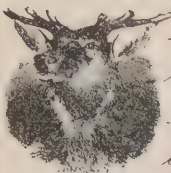
\$300⁰⁰ ATCHISON, KANS. Jan 30 19



Four months after date I promise to pay to
the order of Black Bros.
Three Hundred Dollars
at their office
Value received with interest at 6%
No. 30 Due J. H. Reid

2. You are to ascertain the amount of cash received from the bank by Black Bros. if this note was discounted at the bank on (a) February 5, at 7%; (b) March 1, at 8%; or (c) April 1, at 6%.

The due date is 4 months from January 30 or May 30, but this being a legal holiday it cannot be collected until May 31. The number of days between January 30 and May 31 counting 28 days in February is 121 days. The maturity value of this note will be the face plus interest at 6% for 121 days. Some banks would add to the face of this note the interest for four months or 120 days disregarding the actual number of days. However, the tendency in modern banking is to use the actual number of days in interest calculations and unless otherwise instructed you will follow this plan when the time of an interest-bearing-note is a designated number of months.


 \$27⁵⁰ Syracuse, N.Y. Apr 1 19
 Thirty days after date I promise to pay to
 the order of *The Fair Store*
Thirty seven and ⁵⁰/₁₀₀ *Dollars*
 at *Union Trust Co*
 Value received Int. 6%
 No. 11 Due *P. H. Slack*

3. You are to ascertain the amount of cash The Fair Store would receive from the bank if this note was discounted (a) April 20, at 8%; (b) May 1, at 7%; or (c) May 15, at 6%.


 \$126⁵⁰ KNOXVILLE, TENN. Sept 25 19
 Three months after date I promise to pay to
 the order of *Lesser Bros.*
One Hundred Twenty six and ⁵⁰/₁₀₀ *Dollars*
 at *Brighton Bank & Trust Co*
 Value received Int. 6%
 No. 21 Due *U. R. Moore*

4. You are to ascertain the amount of cash received by Lesser Bros. if this note was discounted (a) October 1, at 7%; (b) November 1, at 8%; or (c) November 15, at 8%.

CHAPTER XXI

TRADE AND CASH DISCOUNTS

Trade discount is a deduction from the catalog or list price, usually stated in the form of one or more discounts. Its purpose is to provide for the fluctuation in the cost of raw material, labor, etc., in order that standard list prices may be provided.

The Standard Manufacturing Company, manufacturers of plumbers' supplies, issued a catalog in which were listed the various articles offered for sale to plumbers. The list price of a certain type of bath tub as shown in this catalog was \$160. Before a customer has a tub installed, he usually wishes to make a selection, and the plumber permits him to do this from the catalog. The price given in the catalog, \$160, is the price at which the plumber is supposed to sell it. However, the price to the plumber is given in a special confidential price list which shows the catalog price and the trade discount or discounts allowed.

Assuming that the discounts allowed by the Standard Manufacturing Company are 20% and 10%, the net cost of the bath tub to the plumber would be \$160 less 20%, or \$128 less 10%, or \$115.20. Each discount is deducted separately. The results are the same in this problem, whether the 10% is deducted first and then the 20%, or vice versa.

EXERCISE No. 183, TRADE DISCOUNT

The following problems provide oral practice in deducting trade discount.

1

- a. \$100 less 10% and 10%
- b. 60 less 25% and 10%
- c. 35 less 40% and 10%
- d. 100 less 20% and 10%
- e. 100 less 20% and 10%
- f. 200 less 25% and 10%
- g. 400 less 40% and 20%
- h. 75 less 20% and 20%
- i. 80 less 10% and 10%
- j. 90 less 20% and 20%
- k. 75 less $33\frac{1}{3}\%$ and 10%

2

- a. \$100 less 20%, 10% and 5%
- b. 80 less $12\frac{1}{2}\%$, 10% and 10%
- c. 150 less 20%, 20% and 20%
- d. 200 less 20%, 10% and 10%
- e. 100 less 15% and 5%
- f. 125 less 20%, 20% and 10%
- g. 150 less $33\frac{1}{3}\%$, 20% and 10%
- h. 200 less 50%, 40% and 10%
- i. 60 less 50%, 40% and 50%
- j. 30 less 10%, 10% and 10%
- k. 40 less 50%, 20% and 10%

EXERCISE No. 184, TRADE DISCOUNT

When a written solution is necessary to ascertain the net price when trade discounts are involved, one of the four methods illustrated in this and the succeeding exercises may be used. It is customary in business, to consider five mills or more in the discount as a cent; if less than five mills, to discard it. You will also observe that the final results are the same regardless of the order in which the discounts are deducted.

The illustration below shows the method of ascertaining the net price when each discount is deducted separately. The solution shows the method of ascertaining the net price of six machines listed at \$140 each, with trade discounts of 25% and 15%.

\$840	\$630
.25	.15
4200	3150
1680	630
\$210.00	\$94.50

\$840	\$630.00
\$210	94.50
\$630	\$535.50

The first discount is obtained by multiplying the total list price by the discount expressed as a decimal fraction. The net price after deducting the first discount is \$630. The second discount is obtained by multiplying the second list price by the second discount. The net price after deducting the second discount is \$535.50, the net price of all the machines. When more than one item is involved, calculate the discounts on the total list price of all the items.

The following problems provide written practice in finding the net price when the list price and the trade discounts are given. Show on tablet paper all of your calculations necessary to ascertain the net price in each problem, arranging your work in the same order as in the illustration. Prove the correctness of your solution by deducting the discounts in the reverse order.

1. List price, \$140, less 25% and 15%. What is the net price?

2. What is the net cost of 25 stoves listed at \$22.40 each, with discounts of 18% and 10%?

3. List price, \$44.50, discounts, 15% and $12\frac{1}{2}\%$. What is the net price?

4. What is the net cost of a radio which is listed in a catalog at \$92.50 less discounts of 25%, 5% and 5%?

5. List price, \$196, discounts, 20%, $12\frac{1}{2}\%$ and 5%. What is the net price?

6. What is the net price of 8 wash stands listed at \$32.50 each, with discounts of 25%, 20%, 10%, $7\frac{1}{2}\%$ and 5%?

EXERCISE No. 185, TRADE DISCOUNT

When the rate of discount is an aliquot part of a dollar the discount may be ascertained by dividing the list price by the denominator of the fraction representing the aliquot part. Deducting the discount obtained by this operation from the list price will give the net price.

The illustration below shows the method of applying aliquot parts to ascertain the net price of a radio outfit listed at \$290, with discounts of 25%, $16\frac{2}{3}\%$, and $12\frac{1}{2}\%$.

4) <u>\$290.00</u>	\$217.50
72.50	36.25
	8) <u>\$181.25</u>
\$290.00	22.66
72.50	\$181.25
6) <u>\$217.50</u>	22.66
36.25	\$158.59

The first net price is ascertained by dividing the list price by 4 (25% is $\frac{1}{4}$ of 100), and subtracting the quotient from the list price. The second net price is ascertained by dividing the first net price by 6 ($16\frac{2}{3}\%$ is $\frac{1}{6}$ of 100), and subtracting the quotient from the first net price. The third net price is ascertained by dividing the second net price by 8 ($12\frac{1}{2}\%$ is $\frac{1}{8}$ of 100), and subtracting the quotient from the second net price. This gives the desired net price, because there are only three discounts allowed.

The following problems provide written practice in ascertaining the net price when the list price is given, and the trade discounts are an aliquot part of a dollar. A combination of this method with the one given in the preceding exercise will give the same results. Thus, if only one of the discounts in the series is an aliquot part of a dollar, division may be used instead of multiplication. Prove your calculations by the method illustrated in Exercise No. 197.

1. The list price of a stove was \$162.50, with trade discounts of $16\frac{2}{3}\%$ and 5%. What was the net price?
2. What was the net cost of three radio outfits listed at \$350 each, with trade discounts of $12\frac{1}{2}\%$, 5% and 2%?
3. The list price of a bookcase was \$185, less 20%, $12\frac{1}{2}\%$ and 5%. What was the net price?
4. What was the net cost of five suites of furniture listed at \$476 each, with trade discounts of 20%, $16\frac{2}{3}\%$, 5% and 2%?
5. What would be the net cost of two pianos listed at \$1,680 each, with trade discounts of 25%, 20%, $12\frac{1}{2}\%$ and 5%?
6. The list price of an automobile was \$1,540, with trade discounts of 20%, $16\frac{2}{3}\%$, $12\frac{1}{2}\%$ and 5%. What was the net price?

EXERCISE No. 186, TRADE DISCOUNT

If desired, the list price may be multiplied by the difference between 100 and the first rate; this product multiplied by the difference between 100 and the second rate, etc., until all the rates have been used; the final product will be the desired net price. The final results are the same with any of the four methods if the fractions of a cent are retained through all of the operations. When solving problems by this and the following method the results obtained in some problems may vary a few cents from the two preceding methods because of discarding and changing the fractions of a cent in the discounts.

The illustration below shows the process explained above applied to finding the net price of a refrigerator listed at \$240 with trade discounts at 35% and 15%.

\$240	\$156
.65	.85
1200	780
1440	1248
\$156.00	\$132.60

The first net price is ascertained by multiplying the list price by .65, which is 100% minus 35%. The second net price is the product of \$156 multiplied by .85, which is the difference between 100% and 15%. This product is the desired net price, because only two discounts are allowed.

The following problems provide written practice in finding the net price when the list price and the trade discounts are given. Show on tablet paper all of your calculations necessary to ascertain the net price in each problem, arranging your work in the same order as in the illustration. Prove your solution by using the method illustrated in Exercise No. 197.

1. List price, \$4.50, discounts, 15%, 10%, 10%, 5%, 5% and 5%. What is the net price?

2. What is the net price of 32 dozen hatchets listed at \$9.75 per dozen, with discounts of 20%, 12½%, 10%, 5% and 5%?

3. List price, \$182.50, discounts, 25%, 15%, 12½%, 10%, 5% and 5%. What is the net price?

4. What is the net cost of 9 refrigerators listed at \$65.50 each, with discounts of 20%, 15%, 12½% and 10%?

5. List price, \$48.50, discounts, 25%, 15% and 12½%. What is the net price?

6. What is the net cost of 6 automobile tires listed at \$47.50 each, with discounts of 40%, 33⅓%, 5% and 5%?

EXERCISE No. 187, TRADE DISCOUNT

The per cent value of the list price after deducting a series of discounts may be ascertained and the list price multiplied by this per cent value to obtain the net price.

The illustration below shows the method of finding the net price of a desk listed at \$165 with discounts of 15% and 15%.

1.00	\$165
<u>.15</u>	<u>.7225</u>
.85	825
<u>.85</u>	<u>330</u>
425	330
<u>680</u>	<u>1155</u>
.7225	\$119.2125

Each rate is deducted separately from 100%. The differences are multiplied together. The product will give the equivalent per cent value of the list price. Multiplying the list price by this result gives the desired net price. Subtracting the per cent value obtained from 100% will give the net value of the series of discounts. $100\% - 72.25\% = 27.75\%$; this is the one discount equivalent to the discount series.

The following problems provide written practice in applying the above method. Show on tablet paper all of the calculations necessary to ascertain the result of each problem. Arrange your work in the same order as in the illustration. Show proof of your results by using one of the methods given in the preceding exercises.

1. What would be the cost of six kitchen cabinets listed at \$78.50 each, with discounts of 15% and 5%?
2. What single discount is equivalent to 20%, 15% and $12\frac{1}{2}\%$?
3. What is the net price of eight overstuffed chairs listed at \$125 each, with discounts of $16\frac{2}{3}\%$, 10% and 5%?
4. A dealer paid \$861.84 for an automobile on which he was allowed discounts of 20%, 10% and 5%. What was the list price?
5. What would be the cost of three vacuum cleaners listed at \$57.80 each, with discounts of 15% and 2%?
6. The list price of a rug was \$360, with discounts of $16\frac{2}{3}\%$, $12\frac{1}{2}\%$ and 5%. What was the net price?
7. What one discount will express the value of: (a) 65%, 35%, and 10%; (b) 90%, 10% and 5%; (c) 75%, 75% and 10%; (d) 25%, 25%, 10% and 5%; (e) 45%, 40%, 30% and 25%; (f) 50%, $33\frac{1}{3}\%$, 25%, 20%, $16\frac{2}{3}\%$, $12\frac{1}{2}\%$ and 10%?
8. What would be the cost of a typewriter listed at \$120 with discounts of 20% and 5%?

(a)

EXERCISE No. 188, CASH DISCOUNT

Cash discount refers to a discount allowed for prompt payment. It is assumed that a customer will make an effort to pay his bill within a specified time, if by so doing he can secure a deduction from the amount through a cash discount allowed. Cash discount has no connection with trade discount. If there are trade discounts applicable to the list price of an article, the net price which is the list price less the discounts, is used in the extension of the invoice.

The cash discount is usually indicated on the invoice as a part of the terms of purchase. Thus, "2/10, n/30" indicates that the purchaser may deduct 2% from the amount of the invoice, if he pays within ten days from the date of the invoice; but if he does not pay within that time, the full amount will be due 30 days from the date of the invoice.

The following problems provide oral practice in ascertaining the amount of the cash discount. This is ascertained by multiplying the amount of the invoice by the rate expressed as a decimal fraction. The amount of the invoice is given in the first column and the rate per cent in the second.

1	2	3
a. \$180.00, 2%	a. \$212.50, 2%	a. \$ 97.50, 1%
b. 250.00, 1%	b. 300.00, 4%	b. 98.15, 1%
c. 35.00, 2%	c. 125.00, 5%	c. 27.65, 2%
d. 120.00, 3%	d. 160.00, $2\frac{1}{2}\%$	d. 36.00, $2\frac{1}{2}\%$
e. 225.00, 1%	e. 180.00, $1\frac{1}{2}\%$	e. 88.50, 1%
f. 38.50, 1%	f. 22.50, 2%	f. 19.29, 1%
g. 209.65, 1%	g. 35.65, 1%	g. 212.50, 2%
h. 9.85, 1%	h. 204.00, 2%	h. 99.00, 2%
i. 35.25, 2%	i. 308.00, $1\frac{1}{2}\%$	i. 75.50, 1%
j. 150.00, 3%	j. 115.00, 3%	j. 99.80, 1%

EXERCISE No. 189, CASH DISCOUNT

When an invoice subject to discount is paid in full, the amount required to pay it is ascertained by multiplying the amount of the invoice by the rate per cent stated in the terms, and deducting the discount from the amount of the invoice. It is quite evident that the purchaser expects the amount he pays to cancel the full

amount of the invoice, hence his account would be credited with the cash and the cash discount. When only a part of an invoice, subject to discount, is paid within the discount period, the purchaser is entitled to a discount on the amount paid. If the terms are, 2%, 10 days, and he pays part of his account within that time, each 98c he pays entitles him to a credit of \$1.00. This means that the amount of the payment is divided by .98, to ascertain the amount with which his account will be credited. The difference between the amount he pays and the amount with which he is credited is the cash discount.

The following problems provide written practice in the application of cash discount. The first column shows the date; the second, the amount of the invoice; the third, the terms; the fourth, the date of payment; and the fifth, the amount paid. You are required to show the amount of the cash discount.

	Date	Amount	Terms	Payment	Amount	
					Paid	Discount
1.	Jan. 2	\$216.40	2/10, n/60	Jan. 9	In full	
2.	May 4	909.16	1/30, n/60	June 3	In full	
3.	Feb. 9	475.85	3/10, n/30	Feb. 19	\$300	
4.	Oct. 6	98.60	2/30, n/60	Nov. 5	In full	
5.	Nov. 2	752.60	3/10, n/30	Nov. 10	\$500	
6.	Jan. 11	55.65	2/10, n/30	Jan. 20	In full	
7.	Apr. 4	317.65	1½/10, n/30	Apr. 13	\$200	
8.	May 9	455.08	1/30, n/90	June 7	In full	
9.	June 4	396.55	2/30, n/60	July 2	\$300	
10.	Dec. 1	508.16	1/10, n/30	Dec. 10	In full	
11.	Feb. 11	87.55	3/30, n/60	Mar. 8	In full	
12.	Sept. 6	115.55	2/30, n/60	Oct. 1	In full	

EXERCISE No. 190, TRADE AND CASH DISCOUNTS

The following invoices provide further practice in (a) auditing extensions, (b) auditing trade discounts, and (c) ascertaining the amounts necessary to pay the invoices in accordance with the terms. Show on tablet paper all the calculations necessary to verify the extensions and trade discount, and ascertain the net amount necessary to pay each invoice, less the cash discount.

(a)

1. **Cutlery**
 Revolvers
 Fire Arms
 Ammunition
 Bicycle Sundries
 Fishing Tackle
 Hardware Specialties
 Paints-Varnishes
 House Furnishings
 Sargent's Builders' Hardware

TELEPHONES | MAIN 2805
 | MAIN 2806

The Evans Bros. Hardware Co.

FINE TOOLS
 :: FOR ALL TRADES ::

Fifth and Elm Streets, Cincinnati,
 Nov. 20, 1926

Sold to G. W. Harvey
 Okeana, Wis.

TERMS 2/10, n/30

17½	Doz. Miners Hoes	\$6.40 per doz.	112		
	Less 20% and 5%		26	98	85 12
500	Coal Picks, special	7.80 " "	325		
	Less 5%, 2% and 1%		25	46	299 54
1420	Just Right Shovels	125.40 per C	1750	68	
	Less 33 1/3% and 2½%		622	24	1157 44
	Freight Prepaid				1542 10
					140 27
					1682 37

2. 

E. C. SIMMONS
HARDWARE CO.

CUTLERY
 REVOLVERS
 FIRE ARMS
 AMMUNITION
 BICYCLES
 SUNDRIES

FINE TOOLS
 FOR ALL TRADES

Oct. 11, 1926 ST. LOUIS

Sold to C. R. Bader,
 Cincinnati, Ohio.

Shipped via Frt. Prepaid
 Terms 2/10, n/30

24½	Doz. X. L. Garden Hoes	\$5.50	134	75	
120	16" Steel Rakes each	.86 2/3	104		
1 5/6	Doz. Lawn Mowers	22.75	500	50	739 25
	Less 10, 5 and 2%				119 84
	Freight Prepaid and charged				619 41
					15 20
					634 61

3. **A. M. BYERS COMPANY**
 ESTABLISHED 1864
 MANUFACTURERS OF
 GENUINE WROUGHT IRON PIPE AND OIL COUNTRY TUBULAR PRODUCTS
 PITTSBURGH, PA.

CONSIGNEE
 ADDRESS
 VIA

INVOICE NO.

DATE August 27, 1926.

SOLD TO: E. H. Mason & Son,
 Sharonville, Ohio.

TERMS Draft 30 days.

CAR NO. 88300 WEIGHT P. O. B. Pgh.

CUSTOMER'S ORDER NO.	QDLS.	PIECES	SIZE	DESCRIPTION	FEET	IN.	PRICE	AMOUNT	TOTAL AMOUNT
		4	2	Ex. Hvy Gal Ran Pipe	80		.50½	40.40	
				7.5 & 5%				3.03	36.37
		2	1½	DO Pe Sw	114		.30	34.20	
				8.5 & 5%				4.49	29.73
		8	1	DO Pe Sw	810		.22	178.20	
				8.5 & 5%				23.20	155.80
		3	3/4	DO Pe Sw	416		.15	62.40	
				8.5 & 6%				8.15	54.25
									275.15

J. A. CHURCH, President

A. T. HEDDITT, Vice President

E. W. CHURCH, Secy.

L. L. EVANS, Treas.

4.

The Cincinnati Cordage & Paper Co.



Paper and Twine

CINCINNATI Nov. 12, 1926

 NO 128 EAST SIXTH ST
CINCINNATI
NO 535 EAST THIRD ST
DAYTON

 TERMS: 30 days SOLD TO: Standard Publishing Co.,
Ninth & Plum Sts.,
Chicago, Ill.

Accounts to be settled Monthly

SHIPPED VIA:

F.O.B. Cin.

 No Claims allowed on this Bill unless made within Five Days after receipt of Goods.
Pay nothing to Agents unless by our written Authority.

25	M American Linen Billheads	2.35 per M	58	75	
	Less 5%		2	94	55.81
25	M 4 ply #117 Corner Cards	2.10 " "	52	50	
	Less 5%		2	73	49.77
50	M 5 ply #88 Round Corner Cards	1.05 " "	52	50	
	Less 10 & 5%		7	62	43.88
12	M #10 White Wave Envelopes 28 lb.	2.28 " "	27	36	
	Less 5%		12	63	14.68
					164.14

5.

ESTABLISHED 1864

A. M. BYERS COMPANY

MANUFACTURERS OF

 GENUINE WROUGHT IRON PIPE AND OIL COUNTRY TUBULAR PRODUCTS
PITTSBURGH, PA.

 CONSIGNEE
ADDRESS
VIA

INVOICE NO

 SOLD TO: Wroth & Co.,
Pueblo, Colo.

DATE July 20, 1926.

TERMS Draft 30 days

CAR NO. CNJ 88299

WEIGHT

F.O.B. Pgh.

CUSTOMERS ORDER NO	BOLS	PIECES	SIZE	DESCRIPTION	FEET	IN	PRICE	AMOUNT	TOTAL AMOUNT
		30	3	Blk Ran Pipe T&C	636		.76½	486	67
				30½%				148	45
		20	10"	-35# DO	408		3.50	1420	
				27½%				291	70
		20	12"	-45# DO	397		4.50	1786	20
				25%				446	63
									1339.87
									2754.41

EXERCISE No. 191, CASH DISCOUNT

The following problems provide written practice in applying the principles of cash discount, and the application of other principles discussed in previous chapters.

1. May 16, The Holmes Furniture Company purchased from the Grand Rapids Furniture Company a bill of furniture amounting to \$1,847.65. According to the terms of this purchase the Grand Rapids Furniture Company was to prepay the freight and charge the Holmes Furniture Company with only one half of it. The invoice mentioned includes the value of the merchandise and one half of the freight, \$188.56. The terms of the bill were 3/30, n/60. June 14, the cashier of The Holmes Furniture Company issued a check in payment of this bill in accordance with the terms. You are required to show on tablet paper all of the calculations necessary to determine the amount of this check. You may assume that The Holmes Furniture Company understands it would not be fair to deduct discount on prepaid freight.

2. August 19, The Central Hardware Company purchased hardware invoiced at \$1,627.56, from the Simmons Hardware Company, terms, 2/30, n/60. When the merchandise was delivered, the management of The Central Hardware Company discovered that an error had been made in ordering additional stock of one item when this was not needed, and asked the Simmons Hardware Company for the privilege of returning merchandise, the value of which was \$248.50. September 10, a credit bill was issued for the value of this returned merchandise. September 16, the cashier of The Central Hardware Company issued a check in payment of the amount due the Simmons Hardware Company. You are to show on tablet paper all the calculations necessary to determine the amount of this check.

3. June 12, The Globe Clothing Company purchased clothing amounting to \$3,856.90, from The Crown Clothing Company. The terms of the invoice rendered by The Crown Clothing Company were October 1, 3/10, 2/30, n/60; these terms indicate that 3% may be deducted if payment is made on or before October 11, 2% on or before October 31, and no discount thereafter. The Globe Clothing Company could not pay the bill in full, October 9, but sent a check for \$2,000 to apply as part payment. October 30, they sent another check for \$1,000, and December 1, a check for

the balance. You are to show on tablet paper all the calculations necessary to ascertain the amount of the last check.

4. July 9, The Watson Supply Company purchased 6 desks from the Dietz Desk Company at \$148.50 each, with trade discounts of 15%, 10%, and 5%. The desks were billed at the net price, subject to 3% cash discount, if paid within 10 days. July 18, a check was issued in payment of these 6 desks. August 1, the receiving clerk of The Watson Supply Company reported to the manager that one of the desks was in bad condition. The manager wrote to the Dietz Desk Company and asked for an adjustment. The Dietz Desk Company suggested that the desk be returned and they would send a check for the amount The Watson Supply Company had paid for it. This suggestion was made by the Dietz Desk Company because they did not have in stock another desk to replace the one damaged and it seemed only fair that The Watson Supply Company should have a check to close the account. You are to show (a) all the calculations necessary to ascertain the amount of the check issued by The Watson Supply Company, July 18, and (b) the amount of the check received from the Dietz Desk Company in payment of the desk which was returned per agreement.

5. April 6, The Central Publishing Company purchased paper billed at \$2,122.65, from The Woods Paper Company. The terms were July 1, 3/10, 2/30, n/60, paper delivered at the freight station of The Central Publishing Company. April 9, the paper was delivered with a freight bill of \$209.65. The notation on the invoice of The Woods Paper Company, "receipted freight bill will be accepted as part payment of this invoice," authorized the Central Publishing Company to pay the freight and charge it to the Woods Paper Company. July 9, The Central Publishing Company paid \$1,200 on this purchase. July 25, the Woods Paper Company asked for the privilege of drawing a 60-day draft for an amount, which when discounted at 7% will produce the balance due. July 27, permission was granted, and the draft was drawn payable 60 days after date. The Central Publishing Company accepted this draft on July 29. You are to show on tablet paper all the calculations necessary to determine the amount of this draft.

CHAPTER XXII

COMMISSION AND BROKERAGE

A commission merchant is one who sells merchandise for others on consignment. He accepts the merchandise, sells it, and remits to the owner in accordance with the agreement. The owner of consigned merchandise is known as the *consignor*, and the commission merchant to whom it is consigned, as the *consignee*. The agreed rate per cent the consignee is to receive for distributing the merchandise is known as the *commission*. The report of the consignee to the consignor setting forth the disposition of the goods consigned to him is known as an *account sales*.

The tendency in modern business is for the distributor to buy the merchandise which he distributes. However there are certain classes of merchandise which may best be handled on consignment. Cotton, grain, vegetables, and poultry are illustrations of the class of merchandise sometimes handled on consignment. There are firms in nearly all of the large cities who handle merchandise on consignment; they receive the goods, sell them, and remit to the consignee the net proceeds, which is the amount of the sales, less their commission and expenses.

As a student of arithmetic, you are interested only in the mathematical calculations necessary to arrive at the facts in which the consignor and consignee are both interested. You will learn more about consignments in the study of bookkeeping and commercial law.

EXERCISE No. 192, COMMISSION ON CONSIGNMENTS

A *consignment* is a quantity of merchandise owned by one person or firm and sold by another on a commission basis. The title to the consignment remains in the consignor until the consignee has sold it. When a sale has been made, the consignee is then responsible to the consignor for the sale price of the merchandise, less the commission due the consignee and any expenses which the consignee may have incurred in making the sale.

An *account sales* is the business form used by the consignee for reporting to the consignor the sales and the expenses incurred in handling the consignment. The account sales illustrated below shows a record of the various charges and sales made by the Ferguson Commission Company.

July 2, the Ferguson Commission Company paid \$48.62, freight on 200 crates of berries, owned by Robert C. Harmon of Bowling Green, to be sold on commission. July 2, they sold 100 crates of the berries at \$3.25, and on July 3, they sold 100 crates at \$3.20. July 5, they sent Mr. Harmon a check for the net proceeds of the sale of the berries less the freight, drayage at 5c per crate, storage at 5c per crate, and a commission of 10% of the net sales.

Account Sales							
				Cincinnati, Ohio July 5, 19__			
M. Robert C. Harmon				Address Bowling Green, Ky.			
Below please find account sales of 200 crates Berries							
Sold by Ferguson Commission Co. Commission Merchant							
Received July 2 191__ and sold for account of yourselves							
DATE	CHARGES	Folio	AMOUNT	DATE	SALES	Folio	AMOUNT
July 2	Freight	C3	48 62	July 2	50 crates at 3.25	S1	162 50
5	Drayage	J1	10	2	50 " " 3.25	S1	162 50
	Returns			3	100 " " 3.20	C2	320
	"						
	Advances						
	"						
5	Storage	J1	10				
	Insurance						
5	Commission	J1	64 50				
5	Net Proceeds	C2	511 88				
			645				645

The following problems provide written practice in ascertaining the net amount due the consignor for merchandise sold. Show on tablet paper (a) the total expenses, (b) the total sales, (c) the commission, and (d) the amount of the check required to pay the net proceeds. Arrange your work as in the illustration.

1. June 5, R. F. Sadler and Company paid \$170.54, freight on 300 crates of berries, 200 baskets of green peas, 100 crates of cantaloupes, and 100 crates of tomatoes received from F. A. Allen of Lexington, Kentucky. July 7, they sold 300 crates of

(a)

berries at \$2.80, 200 baskets of green peas at \$1.35, and 100 crates of tomatoes at \$1.16. July 8, they sold 100 crates of cantaloupes at \$2.30. July 10, they sent Mr. Allen a check for the sale of the produce less the freight, drayage at 5c per crate and basket, storage, 4c and a commission of 10% of the net sales.

2. July 15, the C. C. Seal Company received from Embry Green & Company of Atlanta, Georgia, 200 crates of peaches and 100 baskets of green peas; they gave the L. & N. R. R. a check for \$52.64 in payment of the freight on the shipment. July 16, they sold 150 crates of peaches at \$2.50, and 72 baskets of green peas at \$1.75. July 17, they sold 50 crates of peaches at \$2.50, and 28 baskets of green peas at \$1.75. What was the amount of the check sent to Embry Green & Company, after deducting 2c for each crate and basket for drayage, 2c storage, and 10% of the net sales as commission.

3. May 8, the Producers Co-Operative Company received from C. A. Carter of Augusta, Georgia, 100 crates of tomatoes, and paid the Southern R. R. \$30.85 for freight. May 9, they sold 50 crates at \$2.25. May 10, they sold 25 crates at \$2.15. May 11, they sold 25 crates at \$2.00. May 15, they sent Mr. Carter a check for the sale of the produce less the freight, drayage at 4c per crate, 5c per crate for storage, and 10% of the net sales as commission.

4. July 5, J. N. Richey & Company received from Hubbard & Moore of Denver, 25 crates of peaches and 200 sacks of corn. They gave the C. & C. R. R. \$36.80 to pay for the freight charges. July 6, J. N. Richey & Company sold 25 crates of peaches at \$2.50. July 7, they sold 200 sacks of corn at \$1.75 per sack. July 15, they sent Hubbard & Moore a check for the sale of the produce less the freight, 2c for each sack and crate for drayage, $\frac{3}{4}$ c each for storage, and 10% of the net sales as commission.

5. July 12, T. B. Jones paid \$162.04, freight on a consignment of 500 crates of tomatoes and 200 crates of cantaloupes from the Consolidated Fruit Company of Savannah. July 12, he sold 400 crates of tomatoes at \$2.25, and 100 crates of cantaloupes at \$2.00. July 13, he sold 100 crates of tomatoes at \$2.15, and 100 crates of cantaloupes at \$1.80. July 15, a check was sent to the Consolidated Fruit Company for the sale of the consignment

less the freight, 4c per crate for drayage, 5c per crate for storage and a commission of 10% of the net sales.

6. June 6, the Cordes Commission Company paid \$86.06, freight on 200 crates of tomatoes and 300 crates of berries received from H. C. Shafer of Charleston. June 6, they sold 50 crates of tomatoes at \$2.00, 125 crates of berries at \$2.75. June 7, they sold 150 crates of tomatoes at \$1.90, and 175 crates of berries at \$2.75. June 10, a check was sent to the Cordes Commission Company for the sale of the tomatoes and berries less the freight, drayage of 5c per crate, storage of 4c per crate, and 10% commission on the net sales.

7. July 29, Long, West & Company received from T. Knoll, Augusta, 200 baskets of peas and 100 baskets of cucumbers. They gave the C. R. S. & P. R. R. a check for \$40.52 in payment of the freight. July 29, they sold 100 baskets of peas at \$1.65 and 50 baskets of cucumbers at \$1.90. July 30, they sold 100 baskets of peas at \$1.65 and 50 baskets of cucumbers at \$2.00. August 1, a check was mailed to T. Knoll for \$434.23 the net proceeds of the consignment after deducting the freight, drayage at 2c per basket, storage at $\frac{3}{4}$ c per basket, and the commission on the net sales. What was the rate of commission?

EXERCISE No. 193, BROKERAGE ON SALES

A *merchandise broker* is one who sells merchandise for a manufacturer or jobber. The merchandise broker sells on a commission basis, but does not carry the merchandise in stock. He takes orders and these are filled from a central warehouse or by direct shipment from the factory. The collection is usually made by the factory, but sometimes it is made by the broker.

The compensation of the merchandise broker is an agreed rate per cent of the sales. This is paid to him by the owner of the goods after the sale has been effected. Applying percentage, the sales is the *base*, the per cent of commission, the *rate*, and the broker's income, the *percentage*.

The following problems provide written practice in solving the problems of a merchandise broker. You are to show on tablet paper all the calculations necessary to ascertain the desired facts.

1. During the month of March, C. A. Bates, a merchandise broker, sold the Hannaford Manufacturing Company merchandise

(a)

to the value of \$11,865.86. He is to have a commission of $2\frac{1}{2}\%$ on his sales. What was the amount of his compensation?

2. July 1, L. C. Riley, a merchandise broker, received a check from the Daffley Manufacturing Company for \$187.64, the commission due him for sales of merchandise during June, on a basis of $3\frac{3}{4}\%$ of the net sales. What was the amount of the gross sales if the value of the returned sales was \$382.65?

3. W. H. Carter, a merchandise broker, sold merchandise to the amount of \$3,781.65 for the Downing Manufacturing Company during the month of May. If the check received for his services was \$132.36, what was the rate of his commission?

4. During a certain year D. D. Fields, a merchandise broker, sold merchandise manufactured by the Nassau Manufacturing Company amounting to \$12,461.96. During the year the returned sales amounted to \$475.81. If Mr. Field's commission was $4\frac{1}{4}\%$ on the net sales, what was his compensation?

5. J. C. Strong, a merchandise broker, received 200 cases of condensed milk from the Peerless Manufacturing Company to be placed in the Central Warehouse. These were sold to various customers at \$4.55 per case. Mr. Strong was to receive a commission of $5\frac{1}{2}\%$ on sales, and to be reimbursed for any expenses in handling the milk. His report to the Peerless Manufacturing Company showed the following expenses: delivery to the warehouse, 10c per case; delivery to customers, $12\frac{1}{2}$ c per case; storage charges by the warehouse, 9c per case; commission on the sales, $5\frac{1}{2}\%$. What was the amount of the check received by Mr. Strong from the Peerless Manufacturing Company in payment of his commission and expenses?

6. What per cent of profit was made by the Peerless Manufacturing Company on the sale of the condensed milk in problem 5, if the cost of production was \$2.87 per case?

7. J. L. Darby, a merchandise broker, sold to the Birch Manufacturing Company 500 cases of canned corn at \$2.65 per case. (a) What was his commission on a basis of $5\frac{1}{2}\%$ of net sales, and (b) what would have been his commission if 5 cases were returned for credit?

EXERCISE No. 194, STOCK BROKERS

A stock broker is one who buys or sells corporation stocks through a stock exchange. The commission charged by the broker, usually referred to as *brokerage* is an agreed amount for each share of stock purchased or sold.

Brokerage is sometimes stated as a rate per cent; that is, $\frac{1}{8}\%$ would be $12\frac{1}{2}c$, $\frac{1}{4}\%$ would be $25c$, $\frac{1}{2}\%$ would be $50c$, etc.

Walter Cole instructed Robert Adair, a stock broker, to purchase for him ten shares of American Locomotive at the current market price. The market price of this stock when purchased was $113\frac{1}{2}$; he received a bill for ten shares at \$113.50 per share, or \$1,135 plus \$2.50, this being the brokerage fee at 25c per share.

The following problems provide written practice in ascertaining the sale or purchase value of stock at a stated market price, and the broker's commission or brokerage for making the sale or purchase. You are to show on tablet paper the calculations necessary to arrive at: (a) the value of the stock, (b) the brokerage; and (c) the amount the purchaser will pay, or the seller will receive from the broker as a result of the transaction.

1. May 3, A. C. Johnson gave his broker an order to purchase twenty-one shares of American Linseed. The broker purchased the stock at $46\frac{7}{8}$ per share and charged 15c per share brokerage. What was the amount of the check Mr. Johnson gave the broker to pay for this purchase?

2. C. J. Arnold gave his broker twenty-five shares of Baldwin Locomotive with instructions to sell. The broker sold this at $126\frac{7}{8}$ and gave him a check for the proceeds less 25c brokerage. What was the amount of the check?

3. J. L. Dodd purchased through his brokers twenty-nine shares of Chandler Motors at $51\frac{5}{8}$; brokerage, $17\frac{1}{2}c$. What was the amount of the check to pay for this stock?

4. T. H. Barton, a broker, sold sixty shares of Crucible Steel at $77\frac{1}{2}$. What was the amount of the check received by his customer, after the brokerage charge of 20c was deducted?

5. A. J. Frank, a broker, purchased the following stock for a customer: 15 shares of General Electric at $315\frac{3}{4}$; brokerage, 40c; 10 shares of General Motors at $129\frac{3}{4}$, brokerage, 25c; 10 shares of General Asphalt at $68\frac{1}{4}c$, brokerage, $17\frac{1}{2}c$; 25 shares of Illinois Central Railroad at $120\frac{1}{8}$, brokerage, 25c; 15 shares of Mont-

(a)

gomery Ward at $76\frac{7}{8}$, brokerage, 20c; 10 shares of New York Central Railroad at $129\frac{1}{4}$, brokerage, 25c; 20 shares of American Can at $54\frac{7}{8}$, brokerage, $17\frac{1}{2}$ c. What was the amount of the check received from his customer?

6. A. L. Webb instructed his broker to sell 50 shares of Standard Milling, and received a check from the broker for \$4,327.50, the proceeds of the sale less 20c brokerage. (a) What was the amount of the brokerage charges, and (b) what was the selling price of the stock per share?

7. R. L. Haskins, a broker, purchased for J. H. Simms, 25 shares of United States Rubber, common at $85\frac{3}{4}$, brokerage, 20c; and 25 shares of United States Rubber, preferred at $108\frac{1}{2}$, brokerage, 25c. What was the amount of the check received from Mr. Simms in payment of this stock?

8. O. D. Carter instructed his broker to sell 125 shares of Union Pacific, and received a check from the broker for \$18,659.37 in payment of the stock, less the brokerage of $22\frac{1}{2}$ c. At what price was the stock sold per share?

9. A. L. Pease sold through his broker 125 shares of Underwood Typewriter at $61\frac{1}{2}$. How much did he receive from the sale of this stock, the brokerage being $17\frac{1}{2}$ c a share?

10. L. W. Singer instructed his broker to sell 75 shares of Chicago and Alton, preferred at $17\frac{3}{4}$, brokerage $12\frac{1}{2}$ c. What amount of cash did he receive from his broker for this stock?

11. T. C. Johnson purchased 125 shares of United Fruit at 293 per share, brokerage 35c. How much did he receive from his customer to pay for this stock?

12. During a certain day a brokerage firm bought and sold the stocks listed below; what was the commission on the business transacted during the day?

1,416 shares.....	brokerage $7\frac{1}{2}$ c per share
781 shares.....	brokerage $12\frac{1}{2}$ c per share
1,645 shares.....	brokerage $17\frac{1}{2}$ c per share
942 shares.....	brokerage 20c per share
540 shares.....	brokerage 25c per share
275 shares.....	brokerage 35c per share

EXERCISE No. 195, COMMISSION ON BONDS

Bonds are sold by banks, bond houses, and brokerage firms. The commission is based on the par value of the bonds. Usually the same rate is charged for buying and selling.

James Watson purchased from the First National Bank ten \$1,000 Chesapeake and Ohio $4\frac{1}{2}$ s at $94\frac{3}{8}$; maturing in ten years; brokerage, \$2 per bond. This means that he will pay the bank \$9,437.50, plus \$20 brokerage, or a total of \$9,457.50.

1. May 1, the City National Bank sold the bonds listed below. You are to ascertain (a) the amount of cash received by the bond department for each sale, and (b) the total amount of the bonds sold, and (c) the total brokerage collected by the bond department for the sales made on this day.

- a. To J. H. Brown, twenty-five \$500 B. & O. R. R. 6s at $106\frac{3}{4}$, brokerage \$1 per bond.
- b. To Robert Young, ten \$1,000 Bethlehem Steel 5s at $95\frac{1}{2}$, brokerage \$2 per bond.
- c. To Mays Brothers, fifteen \$500 Terminal Building 5s at $98\frac{1}{8}$, brokerage \$1.60 per bond.
- d. To J. H. Sanders, twelve \$1,000 Bell Telephone of Pa. 5s at 102, brokerage \$2 per bond.
- e. To Mrs. Johnson twenty-five \$500 C. & N. W. R. R. 5s at 101, brokerage \$1.60 per bond.
- f. To C. H. Stephens, seven \$1,000 C. C. C. & St. L. 6s at $103\frac{3}{8}$, brokerage \$2 per bond.
- g. To J. H. Long, fifty \$1,000 Erie R. R. 4s at $83\frac{7}{8}$, brokerage \$1.50 per bond.
- h. To W. O. Crosswhite, five \$1,000 fourth $4\frac{1}{4}$ Liberty bonds *102-17, brokerage 50c per bond.
- i. To E. H. Crabbe, five \$1,000 Anaconda Copper 6s at $103\frac{1}{4}$, brokerage \$2 per bond.
- j. To W. H. Roberts, fifteen \$1,000 New England Telephone 5s at $102\frac{7}{8}$, brokerage \$2 per bond.
- k. To C. C. Duggan, twenty-five \$500 North American Cement $6\frac{1}{2}$ s at $98\frac{3}{8}$, brokerage \$1.60 per bond.

*Quotations on Liberty bonds are in thirty-seconds. 102-17 means $102\frac{17}{32}$ for each \$100 of the face of the bond.

- l. To Mrs. C. H. Crowley, five \$1,000 Bethlehem Steel Corporation 6s at $98\frac{1}{8}$, brokerage \$2 per bond.
- m. To O. A. Crandall, ten \$1,000 U. S. Treasury $4\frac{1}{4}$ s at 107-26, brokerage \$1.25 per bond.
- n. To A. H. Craft, thirty-five \$1,000 D. R. & G. 5s at $70\frac{1}{8}$, brokerage \$2 per bond.
- o. To O. H. Brown, fifteen \$1,000 Anaconda Copper 6s at $105\frac{1}{4}$, brokerage \$2 per bond.
- p. To P. E. Gartin twenty-five \$1,000 Bell Telephone of Pa. 5s at 102, brokerage \$2 per bond.
- q. To P. J. Carroll, twenty-five \$1,000 Dodge Bros. 6s at $96\frac{3}{4}$, brokerage \$2 per bond.
- r. To C. B. Glover, two \$1,000 Southern Bell Telephone Company 5s at $102\frac{3}{8}$, brokerage \$1.50 per bond.
- s. To J. L. Dunn, eight \$1,000 New Orleans Public Service Ref. 5s A, at $93\frac{1}{4}$, brokerage \$2 per bond.
- t. To E. K. Field, fifty-five \$1,000 Utah L. & P. at $85\frac{1}{4}$, brokerage \$2 per bond.

EXERCISE No. 196, BUYING AND SELLING ON COMMISSION

The following problems provide written practice in ascertaining the compensation when a purchase or sale of property is made on a commission basis. The amount of the sale or purchase is the base, and the compensation is the result obtained by multiplying the base by the agreed rate per cent. You are to show on tablet paper the desired information in each problem.

1. An auctioneer is to receive $4\frac{1}{2}\%$ of the sales made at an auction advertised for May 16. The total sales made at the auction were \$2,146.85. What was the amount of his commission?

2. The buyer for a cotton mill purchased 50 bales of cotton, total weight, 25,962 pounds, at $21\frac{7}{8}^c$ per pound on a commission basis of $1\frac{3}{4}\%$. What was his commission?

3. A real estate agent works on a commission basis of $2\frac{1}{2}\%$ of his sales. He sold a house and lot for \$9,650. What was the amount of his commission?

4. An attorney collected a note for \$2,500 and paid the owner of the note \$2,437.50. What was the rate of his commission?

5. O. Thornton, a salesman, receives \$5 a day and commission as follows: 1% of his daily sales between \$100 and \$250, and 2% of all daily sales over \$250. His daily sales record for the week is: \$125.50, \$240.85, \$75.10, \$464.90, \$252.10, and \$720.50. What is Mr. Thornton's salary for the week?

6. R. J. Malcolm sent C. H. Wasson a New York draft for \$10,000 with instructions to buy wheat, but to deduct his commission for buying from the remittance. How many bushels and pounds of wheat would he buy at \$1.03 $\frac{7}{8}$ per bushel if his commission for buying was 1 $\frac{3}{4}$ %?

7. Russell Mann sent William Dale, a broker, a \$4,000 New York draft with instructions to purchase lumber. His commission was 2 $\frac{1}{2}$ %. How many feet of lumber will he purchase if the price is \$62.40 per M?

8. A farmer received \$597.75 from his agent as the net proceeds on a shipment of eggs. The agent charged 3% commission for selling, and paid \$8.50 expenses. How many dozen eggs were there, if they were sold for 25c per dozen?

9. A commission agent received \$828 with which to buy berries, after deducting his commission of 2 $\frac{1}{2}$ %. If he paid 7 $\frac{1}{2}$ c a box for the berries, how many boxes did he buy?

10. S. R. Stickley placed a claim of \$3,500 in the hands of his attorney for collection. Before collection could be made, the customer went into bankruptcy. The final adjustment of this case in the court resulted in a cash fund of \$23,400 to be distributed to the creditors in payment for claims amounting to \$39,672.40. How much of his claim did Mr. Stickley receive from his attorney, if the attorney's commission was 5% of the amount collected?

11. Charles Daniels received a check for \$19,500 in payment of a farm. What was the selling price of the farm, if the real estate agent's commission for selling was 2%?

12. Thomas Clark contracted to sell lots Nos. 283, 284, 285, 286, 287, and 288 in a certain subdivision on a commission basis of 3 $\frac{3}{4}$ %. He sold the lots for the following prices: No. 283 for \$1,250; No. 284 for \$1,300; No. 285 for \$2,130; No. 286 for \$2,275; No. 287 for \$1,150; and No. 288 for \$1,625. What was the amount due Mr. Clark on the day of settlement, if he had previously received \$450 to apply on his commission?

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